

# **A Fuzzy AHP & Extent Analysis Based Approach for Commercial Software Evaluation**

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## **Abstract**

The commercial availability of numerous software tools has increased its accessibility to many companies. However, at times, companies do not have the requisite resources to perform in-depth analysis for identifying the best software according to their requirements. Acquiring a software tool in such a situation might lead to a wrong or inferior decision. In this paper, we strive to overcome this difficulty by identifying the decision attributes for computational fluid dynamics simulation software. Moreover, the inter-attributes preferences are defined in qualitative manner, and fuzzy analytic hierarchy process is employed to prioritize these decision attributes. Whereas it has been proved that analytical hierarchy process improves the decision making process, its application to prioritizing decision attributes for software selection minimizes the errors that may occur if the prioritization is merely based on intuition.

## **Keywords**

Analytical hierarchy process; Computational fluid dynamics software; Decision attributes; Extent analysis; Fuzzy numbers.

## **Introduction**

Computational fluid dynamics (CFD) is used as a design tool to predict the fluid flow, heat transfer, mass transfer and chemical reactions within a process. It solves the governing mathematical equations using numerical algorithms and computer software. Engineering applications of CFD ranges from the flow around automobiles and airplanes to fluid-structure interaction and manufacturing processes. It is a powerful tool for performing analysis and simulation prior to a physical prototype being constructed. For the last few years, there has been continuous progress in the development of CFD [1]. Because of these developments and the growth in computational power accompanied by the reduced cost of computer hardware, today CFD is being widely used by many industries.

CFD software can help cope with complex problems in many fields. Numerous commercial CFD software tools have been developed, which increased the accessibility of many companies to CFD. A difficulty encountered by various companies is how to select an appropriate CFD software tool for use. The choice of the best suited CFD software enables a company user to reduce its design cycles and costs.

However, companies sometimes do not have the time or the requisite capabilities to perform in-depth analysis for identifying the best software tool according to their requirements. The problem is made more complicated with the rapid evolution of new software tools with new features. It is in this context that the present study contributes to identify and prioritize the relevant decision attributes for evaluating commercially available off-the-shelf (COTS) CFD software tools. A particular aspect of the study is the identification of the functional needs of an **External Aerodynamics** design group towards CFD software. This study will help the group to efficiently select the CFD software that best suits their needs, while also benefiting from its utilities and features.

The rest of the paper is organized as follows. In Section 2, a literature review about software attribute evaluation and decision making is briefly given. Section 3 presents the concept of fuzzy sets and the analytic hierarchy process (AHP) with fuzzy comparison extent analysis. Section 4 presents attributes and their hierarchy. Section 5 shows the empirical study of prioritizing CFD software attributes. Section 6 concludes this study.

## **Literature Survey**

According to [2], multiple attributes need to be carefully considered while selecting an appropriate commercial off-the-shelf (COTS) product. Lin, Ping, and Sheen [3] have observed that in addition to a large number of software

products in the market and the ongoing improvements in information technology, the presence of multiple attributes have made the software selection decision more complex. Software evaluation has been the focus of many researchers, e.g. Morisio and Tsoukias [4] have proposed a framework based on Multi Criteria Decision Analysis (MCDA) for the evaluation and selection of software products. A formal way is presented for an evaluation model with judgment and measurement. Vlahavas, Stamelos, Refanidisa, and Tsoukias [5] have proposed a prototype expert System (ESSE) for software evaluation that embodies the MCDA methodology. ESSE helps the evaluator to select the appropriate criteria and construct the evaluation model. The evaluator can re-use the past evaluation models from the knowledge base for new problems with flexibility of adding, removing and modifying attributes. Stamelos and Tsoukias [6] have emphasized on the lack of any formal aid in software selection and have introduced components that may be established to identify the problem situation.

From the review of contemporary literature it is evident that the software evaluation and selection decision, like any other complex decision making situation, is characterized by the presence of multiple and often conflicting attributes. It is also observed that considerable research efforts are dedicated towards software attributes evaluation and decision making. However, the present study is based upon the assertion of Carney and Wallnau [7] that it is unlikely that any single evaluation tool will cover a sufficient range of attributes to cope with the associated diversity in software evaluation. The fact is that there exist certain user and software specific attributes which needs to be identified before a selection decision is made. Prioritizing those attributes will help to arrive at a rational decision, thus resulting in best possible use of commercially available CFD software tools.

Operationally, the CFD software inter-attribute preferences can be defined in qualitative terms only. Also the subjective qualitative judgment is often vague and imprecise in nature. Therefore the prioritization methodology to be used must be able to deal with qualitative terms and to resolve the vagueness, ambiguity and subjectivity of human judgment. In view of these foreseen difficulties, the Analytical Hierarchy Process (AHP) method proposed by Saaty [8] in combination with Fuzzy sets theory is used in present work. While, AHP solicit the preferences through pair wise comparison in a hierarchical way the fuzzy methods are purposely designed for complex and ill defined problems.

## Methodology

### Fuzzy sets and numbers

Many real world assessment and evaluation problems are characterized by the unavailability of precise quantitative data. The decision makers in such situations are unable to assign exact numerical values to the comparison judgment and hence prefer to use natural language expression [9]. Fuzzy set theory, introduced by Zadeh [10], can effectively deal with the imprecise human judgment where the linguistic variables can be shown by fuzzy number.

Some basic definitions and notations of fuzzy sets are given briefly [11].

Triangular fuzzy numbers (TFN)

Let  $M \in F(R)$  be called a fuzzy number if:

(1) Exists  $x_0 \in R$  such that  $\mu_M(x_0) = 1$ .

(2) For any  $\alpha \in [0,1]$ ,

$A\alpha = [x, \mu_{A_\alpha} \geq a]$  is a closed interval. Here  $F(R)$  represents all fuzzy sets and  $R$  is the set of real numbers. A fuzzy number  $M$  on  $R$  is to be a triangular fuzzy number if its membership function  $\mu_M(x) : R \rightarrow [0,1]$  is equal to

$$\mu_M(x) = \left\{ \begin{array}{ll} \frac{x}{m-l} - \frac{l}{m-l}, & x \in [l, m], \\ \frac{x}{m-u} - \frac{u}{m-u}, & x \in [m, u], \\ 0, & \text{otherwise} \end{array} \right\} \quad (1)$$

Where  $l \leq m \leq u$ ,  $l$  and  $u$  stand for the lower and upper value of the support of  $M$  respectively, and  $m$  for the middle value. The triangular fuzzy number can be denoted by  $(l, m, u)$ . The support of  $M$  is the set of elements  $\{x \in R / l < x < u\}$  When  $l = m = u$ , it is a non-fuzzy number by convention.

Consider two TFN  $M_1$  and  $M_2$  such that  $M_1 = (l_1, m_1, u_1)$  and  $M_2 = (l_2, m_2, u_2)$ , their operations laws are as follows:

$$(l_1, m_1, u_1) \oplus (l_2, m_2, u_2) = (l_1 + l_2, m_1 + m_2, u_1 + u_2) \quad (2)$$

$$(l_1, m_1, u_1) \otimes (l_2, m_2, u_2) = (l_1 \times l_2, m_1 \times m_2, u_1 \times u_2) \quad (3)$$

$$(l_1, m_1, u_1)^{-1} = (1/u_1, 1/m_1, 1/l_1) \quad (4)$$

Moreover, according to Yong [12], a TFN  $M_1 = (l_1, m_1, u_1)$  can be transformed into a crisp number as

$$P(M_1) = \frac{l_1 + 4 \times m_1 + u_1}{6} \quad (5)$$

### Fuzzy AHP and extent analysis

The analytic hierarchy process (AHP), developed by Saaty [8], is a decision aiding tool to resolve unstructured evaluation and complicated decision problems. In the past several decades, AHP has been successfully applied to a diverse array of problems by many researchers e.g. [13, 14, 15]. In the traditional formulation of AHP, human judgment is represented as exact numbers. However, in many practical situations, the human preference model is uncertain and a decision maker finds it extremely difficult to express the strength of his/her preferences and to provide exact pair-wise comparison judgments [16]. Hence, to cater for this shortfall in traditional AHP, many scholars have combined the Fuzzy theory with AHP called fuzzy AHP. In fuzzy AHP, fuzzy numbers are substituted into pair-wise comparison matrix to deal with attribute evaluation. In application Fuzzy AHP has been adapted to many problems by a number of researchers, such as in [17, 18, 19].

The fuzzy AHP approach is based on pair wise comparisons and allows the utilization of linguistic variables. Although the pair wise comparison approach is the most demanding in terms of solicited input from the experts, it offers maximum insight, particularly in terms of assessing consistency of the experts' judgment [19]. In this context, this technique is ideal for prioritizing CFD software attributes. In present study, the linguistic pair wise comparisons are expressed by fuzzy numbers with the help of the fuzzy scale as shown in Table 1.

Let  $A = (a_{ij})_{n \times m}$  be a fuzzy pair wise comparison judgments matrix and  $M = (l_{ij}, m_{ij}, u_{ij})$  be a TFN. The procedure of fuzzy AHP is as follows [11, 20].

Step 1: A pair wise comparison of attributes is made by using the fuzzy numbers in the same level of hierarchy structure.

Step 2: The value of fuzzy synthetic extent with respect to the  $i$ th object is defined as:

$$S_i = \sum_{j=1}^m M_{ij} \otimes \left[ \sum_{j=1}^n \sum_{j=1}^m M_{ij} \right]^{-1} \quad (6)$$

In our case,  $n=m$  since a comparison matrix for criteria always has to be a square matrix.

$$\text{Where } \sum_{j=1}^m M_{ij} = \left( \sum_{j=1}^m l_{ij}, \sum_{j=1}^m m_{ij}, \sum_{j=1}^m u_{ij} \right), i = 1, 2, 3, \dots, n \quad (7)$$

$$\sum_{j=1}^n \sum_{j=1}^m M_{ij} = \left( \sum_{j=1}^n \sum_{j=1}^m l_{ij}, \sum_{j=1}^n \sum_{j=1}^m m_{ij}, \sum_{j=1}^n \sum_{j=1}^m u_{ij} \right) \quad (8)$$

$$\left[ \sum_{j=1}^n \sum_{j=1}^m M_{ij} \right]^{-1} = \left( \frac{1}{\sum_{j=1}^n \sum_{j=1}^m u_{ij}}, \frac{1}{\sum_{j=1}^n \sum_{j=1}^m m_{ij}}, \frac{1}{\sum_{j=1}^n \sum_{j=1}^m l_{ij}} \right) \quad (9)$$

Table 1: Fuzzy Scale

Preference of pair wise comparison	Triangular fuzzy Scale	Triangular fuzzy reciprocal scale
Equally Important	(1,1,2)	(1/2,1,1)
Moderately more important	(1,3/2,2)	(1/2,2/3,1)
Strongly more important	(1,2,3)	(1/3,1/2,1)
Very strongly more important	(3/2,5/2,7/2)	(2/7,2/5,2/3)

The TFN value of  $S_i = (l_i, m_i, u_i)$  are calculated by formula (6)-(9)

Step 3: The respective  $S_i$  values are compared and the degree of possibility of  $S_j = (l_j, m_j, u_j) \geq S_i = (l_i, m_i, u_i)$  is calculated as

$$V(S_j \geq S_i) = \text{height}(S_i \cap S_j) = u_{S_j}(d) = \begin{cases} 1, & \text{if } m_j \geq m_i \\ 0, & \text{if } l_i \geq u_j \\ \frac{l_i - u_j}{(m_j - u_j) - (m_i - l_j)} & \text{otherwise} \end{cases} \quad (10)$$

Step 4: The minimum degree possibility  $d(i)$  of  $V(S_j \geq S_i)$  for  $i, j = 1, 2, \dots, k$  is calculated as

$$\begin{aligned} & V(S_j \geq S_1, S_2, S_3, \dots, S_k) \text{ for } i, j = 1, 2, \dots, k \\ & = \sqrt[\left[ \begin{matrix} (S \geq S_1) \text{ and } (S \geq S_2) \\ \text{and } \dots (S \geq S_k) \end{matrix} \right]} = \min V(S \geq S_2) \\ & = d'(A_i) \end{aligned} \quad (11)$$

The weight vector is then calculated as

$$W' = (d'(A_1), d'(A_2), \dots, d'(A_n))^T \quad (12)$$

Where  $A_i (i = 1, 2, \dots, n)$  is the n element. Step 5: The weight vectors are normalized as

$$W = (d(A_1), d(A_2), \dots, d(A_n))^T \quad (13)$$

Thus resulting in non-fuzzy attribute weight.

Above steps are performed for pair-wise comparison matrix of top-level as well as those of all sub-attributes. The relative weight  $W_{ij}$  of a sub-attribute  $A_{ij}$  is computed as  $W_{ij} = W_i * w_{ij}$  where  $W_i$  is the relative weight of top-level attribute  $A_i$  relevant to sub-attribute  $A_{ij}$  and  $w_{ij}$  is the weight of the sub-attribute obtained from (13).

In order to check the consistency of evaluation, the values of  $\lambda_{\max}$  and  $CI$  for the pair-wise comparison matrix are calculated as follows.

$$\lambda_{\max} \text{ is determined by solving the equation } Aw = \lambda_{\max} w \quad (14)$$

where, A is the crisp version of the pair wise matrix and w is a column matrix of relative importance of attributes. Similarly, CI and CR are determined using the following formulas

$$CI = \frac{\lambda_{\max} - n}{CR} \quad (15)$$

Where RI is a known random consistency index depending upon the order of matrix.

Table 2: Value of the random consistency index

Size	RI
1	0.0
2	0.0
3	0.52
4	.89
5	1.11
6	1.25
7	1.35
8	1.40
9	1.45
10	1.49

Table 2 shows the value of the random consistency index (RI) for matrices of order 1 to 10 obtained by approximating random indices using a sample size of 500 [8].

$$CR = \frac{CI}{RI} \quad (16)$$

## CFD SOFTWARE ATTRIBUTES

To prioritize the attribute weights of commercially available off-the-shelf (COTS) CFD software tools, 6 top level attributes is composed including price, user support, pre-processing capabilities, physical modeling capabilities, the solver and the post-processing capabilities. The top level attributes are further decomposed into sub-attributes and the hierarchy is shown in fig.1.

A brief explanation of the attributes is presented below.

The price attribute indicate the total incurring price for acquiring the CFD software. It includes the licensing, training, documentation and any other related expenses to be paid to the supplier.

By user support is meant the arrangement of training sessions, answering the user queries and providing software related support when needed.

The pre-processing attribute includes a number of sub-attributes as can be seen in fig.1. The body fitted coordinate system sought in CFD software enables the user to distort the grid to fit the geometry. This capability is particularly useful while dealing with complex geometries. The mesh size limit is dictated by the maximum mesh size that CFD software or associated hardware is capable of handling. CFD software capable of incorporating the user defined routines enable the user to define own boundary conditions and incorporate other inputs.

In physical modeling unsteady flow modeling capability is desirable in order to be able to simulate time varying flow problems. Most turbulent flow problems involves transition from laminar to turbulent flow, hence modeling laminar-turbulent transition increases the accuracy of the results. Various turbulence models as illustrated in literature e.g. [21, 22] are proposed in the context of Reynolds averaged Navier-Stokes (RANS) equation. Successful modeling of turbulence greatly increases the quality of numerical simulations so CFD software must be equipped with variety of turbulence models. Multi-phase flow refers to the flow involving mixtures of different fluids having different phases, such as air and water and flow of combustion gasses where there might be a significant fraction of solids. The boundary layer treatment is also a desirable feature in CFD software to resolve the flow in the thin boundary layer region which increases the accuracy of the flow simulation.

Discretisation is the technique of reducing the governing CFD equations to a numerical form. Two main Discretisation methods namely finite element method and finite volume method exists. Finite volume method is discussed in detail in [23] and is the most popular method which divides the flow domain in control volumes (defined through mesh cells) and the governing conservation equations are integrated over each cell.

In the post-processing attributes, access to output data in ASCII format is necessary to compare results of CFD simulations with the experimental data or other CFD. Similarly CFD software with the possibility of interfacing with other post processing and visualization packages is preferred.

## Empirical Study

The linguistic expressions for the CFD software attributes are transformed into triangular fuzzy numbers (TFN) with the help of fuzzy scale shown in Table 1. The fuzzy evaluation matrix for pair wise comparison of the top-level attributes is shown in Table 3.

The consistency ratio (CR) for the crisp version of above matrix is  $0.07 < 0.10$ , hence the pair wise comparison is consistent.

The corresponding  $S_i$  values calculated through formula (6)-(9) are compared and the degree of possibility of  $S_j = (l_j, m_j, u_j) \geq S_i = (l_i, m_i, u_i)$  is calculated by formula (10). Table 4 shows the values of the degree of possibility of  $S_j \geq S_i$  i.e.  $V(S_j \geq S_i)$  for the top-level attributes.

The pair-wise comparison is done for all the sub-attributes in a similar way and the corresponding degree of possibilities of  $S_j = (l_j, m_j, u_j) \geq S_i = (l_i, m_i, u_i)$  is calculated. The final results of the overall prioritization weight for each attribute of CFD software are presented in Table 5.

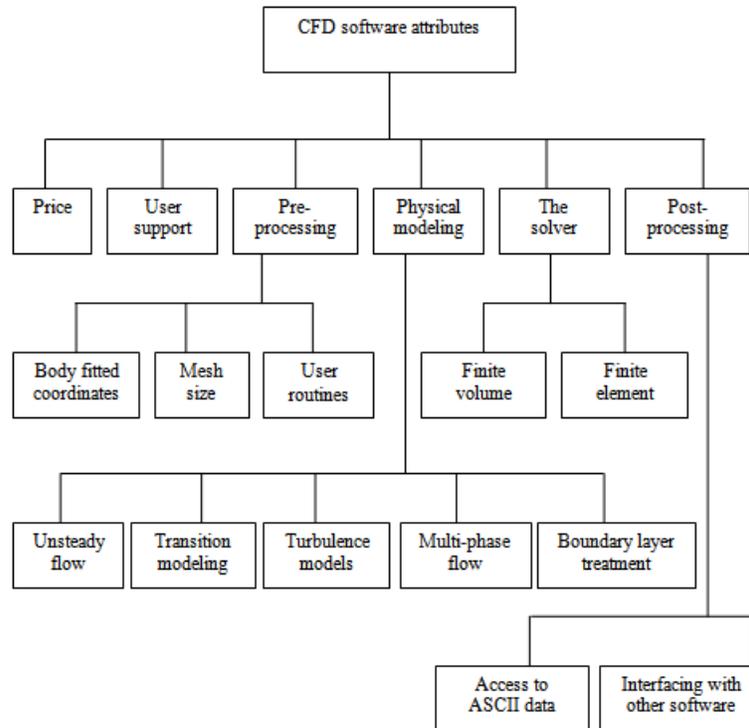


Figure 1: Hierarchy of attributes

Table 3: Pair-Wise Comparison of Top-Level Attributes

	Price	User support	Pre-processing	Physical modeling	The solver	Post-processing
Price	(1,1,1)	(1,1,2)	(1,1,2)	(1/3,1/2,1)	(1,1,2)	(1,3/2,2)
User support	(1/2,1,1)	(1,1,1)	(1,1,2)	(1/2,2/3,1)	(1,3/2,2)	(1,3/2,2)
Pre-processing	(1/2,1,1)	(1/2,2/3,1)	(1,1,1)	(1/2,2/3,1)	(3/2,5/2,7/2)	(1,2,3)
Physical modeling	(1,2,3)	(1,3/2,2)	(1,3/2,2)	(1,1,1)	(3/2,5/2,7/2)	(3/2,5/2,7/2)
The solver	(1/2,1,1)	(1/2,3/2,1)	(2/7,2/5,2/3)	(2/7,2/5,2/3)	(1,1,1)	(3/2,5/2,7/2)
Post-processing	(1/2,2/3,1)	(1/2,3/2,1)	(1/3,1/2,1)	(2/7,2/5,2/3)	(2/7,2/5,2/3)	(1,1,1)

Table 4: The Degree of Possibility of  $S_j > S_i$

V(S1>S2)	0.94	V(S2>S1)	1.00	V(S3>S1)	1.00	V(S4>S1)	1.00	V(S5>S1)	1.00	V(S6>S1)	0.61
V(S1>S3)	0.83	V(S2>S3)	0.86	V(S3>S2)	1.00	V(S4>S2)	1.00	V(S5>S2)	0.91	V(S6>S2)	0.57
V(S1>S4)	0.65	V(S2>S4)	0.64	V(S3>S4)	0.78	V(S4>S3)	1.00	V(S5>S3)	0.77	V(S6>S3)	0.47
V(S1>S5)	1.00	V(S2>S5)	1.00	V(S3>S5)	1.00	V(S4>S5)	1.00	V(S5>S4)	0.55	V(S6>S4)	0.25
V(S1>S6)	1.00	V(S2>S6)	1.00	V(S3>S6)	1.00	V(S4>S6)	1.00	V(S5>S6)	1.00	V(S6>S5)	0.66

Table 5: Prioritization Weights for CFD Software Selection Attributes

Top level	Relative	Sub	Relative
Price	0.167		
User support	0.166		
Pre-processing capabilities	0.201	A31	0.086
		A32	0.074
		A33	0.042
Physical modeling	0.259	A41	0.067
		A42	0.023
		A43	0.066
		A44	0.034
		A45	0.069
The solver	0.141	A51	0.098
		A52	0.043
Post-processing capabilities	0.066	A61	0.046
		A62	0.020

## Conclusions

In this paper, fuzzy AHP methodology is used to prioritize decision attributes for the selection of the commercial CFD software tools. The AHP approach is found to be useful because it accomplishes pair wise comparisons and provides information about the consistency of the judgment. Moreover, the fuzzy approach allows the utilization of linguistic variables in calculations.

The present study concludes that the attribute ‘physical modeling’ with relative importance of 26%, should be given the highest priority while selecting commercial off-the-shelf (COTS) CFD software. Among the 6 top-level attributes identified and considered in present study, the second most important CFD software attribute is the ‘pre-processing capabilities’ having relative importance of 20% followed by the ‘price’ and ‘user support’ attributes with relative importance of 16.5% each. The ‘solver’ attribute is second last with the relative importance of 14%. It is also observed that the attribute ‘post-processing capabilities’ has got the least relative importance of 6.5%. Further, it is observed that the hierarchical structure of AHP provides a systematic way to deal with decision attributes at different levels of hierarchy.

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