

Bayesian Approach for Early Stage Reliability Prediction of Evolutionary Products

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Abstract

New product reliability prediction has been center of research studies for many years. Reliability prediction focuses on developing a proper reliability model based on available data. Early life reliability prediction is challenging due to the lack of available data thus it is important to use a proper method for reducing this uncertainty. Bayesian method has been used widely in different area of studies to model uncertainty exist in a probabilistic way. This paper proposes a novel model to facilitate reliability prediction of non-repairable evolutionary product early after production using Bayesian method. The proposed Bayesian model joins prior information on past product failure and sparse few field data on new product's performance. This approach helps us to overcome one of the important obstacle in the new product early reliability prediction which is lack of data. This study shows that Bayesian model outcomes are more accurate and logical compared to classical methods and indications are favorable regarding the model's practicality in industry applications.

Keywords

Reliability prediction, Bayesian theory, Weibull distribution, prior information, sparse field data

1. Introduction

Today's fast development and improvement in industries is vital. Increasing competition among industries and growing customers demand in terms of functionality and reliability of products makes it important for companies to shorten the development phase of their production (Molenaar et al. 2002). Manufactures also have a greater responsibility for their product's behavior over a longer period, as warranty legislation for the customer. Hence having a quick and reliable prediction method for estimating length of warranty period and cost of that is very important from manager prospective.

There are different methods for reliability predictions based on product's life cycle (Coit 2000). In the design phase even the best computer systems can't estimate the reliability of the product but some systematic methods have been

implemented to predict reliability (Fajdiga and Jurejevcic 1996). To have an accurate and credible reliability prediction, it is important to have data available early in the product life (Usher et al. 1990). Prediction can be made only based on field data that available early after production but due to the information absence at this stage, it is more likely for statistical uncertainty to happen. In other words when sufficient data is not available, using common statistical methods for predicting reliability, may provide inaccurate results (Siu and Kelly 1998). In that case Bayesian estimation method is more appropriate to obtain acceptable results (Ion et al. 2006).

The main objective of this paper is to construct a methodology to estimate reliability of evolutionary product early after introduction to market. For this reason a novel model is introduced to merge limited failure information available for new product with historical data of previous generation of that product to have more realistic prediction of failure. Bayesian statistic is used to join prior information with sparse few field data on current product's performance. Different from other studies, our model provides more accurate estimation that cover existing uncertainty.

The outline of the present paper is the following. In Section 2, a background for different available method of reliability prediction is given. Proposed Bayesian model with different prior distribution is introduced in section 3. Section 4 demonstrates the applicability of a proposed model through synthetic and case study example. Finally the contribution will be finished in section 5 by conclusion and suggestion for future studies.

2. Background

Reliability prediction has become an important area of research for many decades. Many studies have been done to develop quantitative approaches for reliability estimation (Blanks 1998; Johannesson et al. 2013). When a new product has a long life time or it quickly moves from design phase to production line, accelerated life test is used to predict reliability (Nelson 1980). Another way for reliability prediction is physics of failure models such as block diagram or fault tree analysis (Denson 1998). With today's computers' development using computational methods in the field of reliability prediction increase. Monte Carlo simulation (Ormon et al. 2002), Neural Networks (Lolas and Olatunbosun 2008; Tong and Liang 2005), Support vector machine (SVM) (Lins et al. 2012) and hierarchical methods like Genetic Algorithms (Liang 2008) develop for this purpose. None of these methods consider uncertainty exist in the development phase of product's life cycle. Also information exist from previous product generation does not take in to account.

Bayesian models have been used widely in different area of research studies (Singh et al. 2001). The motivation for almost any statistical analyses is that some target population is not well understood or some aspects of it are unknown or unsure. So the idea of Bayesian methods is to say that any uncertainty can be modeled in a probabilistic way. Yadav et al. (2003) suggest a general framework for reliability prediction in the development phase of product based on Bayesian methods. Pan (2009) presents a new model for reliability prediction, combined accelerated life test information and new product field data using the Bayesian approach. Houben et al. (2009) model reliability prediction of product change through Bayesian inference. Wang et al. (2013) develop a Bayesian model for condition monitoring of units with heterogeneous degradation rates and imperfect inspections. Ye et al. (2013) propose a Bayesian evaluation method to integrate the Accelerated degradation testing (ADT) data from laboratory with the field failure data. The Recent study by Alzbutas and Iešmantas (2014) present an application of Bayesian methods for age-dependent reliability analysis. Although these approaches try to solve uncertainty problem exist in early life reliability prediction by using Bayesian methods but none of them consider past generation reliability information for model building. Stochastic modelling for degradation of highly reliable products has been proposed (Ye and Xie 2015).

Bayesian method has been developed for product reliability prediction using Weibull distribution (Canavos et al. 1973). Ion et al. (2006) suggests a Bayesian model for product that time to failure has Weibull distribution while scale and shape parameters has normal distribution. Even though this method overcome some of the problems exist in previous model but it is not ideal. The reason is in Weibull distribution scale and shape parameters both are greater than zero while normal distribution covers all real numbers both negative and positive. In this paper we develop new Bayesian model to predict reliability of evolutionary product when time to failure has Weibull distribution and scale parameter of it follow different prior distribution. To best of our knowledge this study is first attempt to build a Bayesian model for reliability prediction in early life of evolutionary product using both historical information and available failure data of new product.

3. Methodology

Reliability is generally defined as the probability that a system performs its intended function under operating conditions for a specified period of time (Dupow and Blount 1997). If $F(t)$ define as cumulative distribution and $f(t)$ define as density distribution of failure rate of one product below equation can be driven (Zacks 1992)

$$R(t) = P(T > t) = 1 - F(t) \quad (1)$$

In the field of reliability, prediction plays an important role. Reliability prediction means to use statistical models and empirical data to estimate product reliability before real data of new product is available (Denson 1998). In this paper it is assumed that product has Weibull distribution time to failure which is one of the most flexible distributions for any kind of data. The following cumulative distribution function shows as;

$$F(t | \alpha, \beta) = 1 - e^{-\frac{t^\beta}{\alpha}} \quad (2)$$

α is the scale parameter and β is the shape parameter of Weibull distribution. The density function and mean time to failure (MTTF) of this distribution are

$$f(t | \alpha, \beta) = \frac{\beta}{\alpha} t^{\beta-1} e^{-\frac{t^\beta}{\alpha}} \quad (3)$$

In the Weibull distribution β represents the failure rate. If $\beta < 1$, the product has a decreasing failure rate. If $\beta > 1$, the product has an increasing failure rate or it can be said that the product is depreciated and generally this type of failure happen for mechanical products. Finally, if $\beta = 1$, the distribution is the exponential and the product failure rate is constant and does not depend on time.

Bayesian method uses the prior information along with the field failure data to predict reliability of the products. The key parts of a Bayesian model are the likelihood function, which reflects information about the parameters contained in the data, and the prior distribution, which quantifies what known about the parameters before observing data. The prior distribution and likelihood can be easily combined to form the posterior distribution, which represents total knowledge about the parameters after data have been observed.

3.1 Proposed Bayesian model

In this study, it is assumed that shape parameter of Weibull distribution is known. Prior distribution for scale parameter will be estimated through the maximum likelihood method (Ion and Sander 2005). Four different prior distributions for scale parameter such as Gamma, Exponential, Inverted Gamma and truncated Normal are studied. The results are summarized in Table 1. As an example for Gamma prior distribution with the probability density function given as

$$g(\alpha) = \frac{1}{\gamma^\kappa \Gamma(\kappa)} \alpha^{\kappa-1} e^{-\frac{\alpha}{\gamma}} \quad (4)$$

It is easy to verify that for n as number of failure data, with assumption of having Weibull distribution, likelihood function can be expressed as

$$l(\underline{x} | \alpha, \beta) = \left(\frac{\beta}{\alpha}\right)^n \prod_{i=1}^n x_i^{\beta-1} \exp\left[-\frac{1}{\alpha} \sum_{i=1}^n x_i^\beta\right] \quad (5)$$

Posterior distribution based on the Bayesian approach can be obtain as

$$h(\alpha | \underline{x}) = \frac{g(\alpha) \times l(\underline{x} | \alpha, \beta)}{\int_0^\infty g(\alpha) \times l(\underline{x} | \alpha, \beta) d\alpha} \quad (6)$$

Assuming that β is known, posterior distribution for prior gamma distribution can be derived as below

$$h(\alpha | \underline{x}) = \frac{\frac{1}{\alpha^{n-\kappa+1}} e^{-\left(\frac{1}{\alpha} \sum_{i=1}^n x_i^\beta + \frac{\alpha}{\gamma}\right)}}{\int_0^\infty \frac{1}{\alpha^{n-\kappa+1}} e^{-\left(\frac{1}{\alpha} \sum_{i=1}^n x_i^\beta + \frac{\alpha}{\gamma}\right)} d\alpha} \quad (7)$$

Which n represents total number of new products' failure data, and x_i shows time to failure for i 's product. With respect to a quadratic loss function, the Bayesian estimator of α is determined by the following posterior expectation

$$E[\alpha | \underline{x}] = \frac{\int_0^\infty \alpha \times \frac{1}{\alpha^{n-\kappa+1}} e^{-\left(\frac{1}{\alpha} \sum_{i=1}^n x_i^\beta + \frac{\alpha}{\gamma}\right)} d\alpha}{\int_0^\infty \frac{1}{\alpha^{n-\kappa+1}} e^{-\left(\frac{1}{\alpha} \sum_{i=1}^n x_i^\beta + \frac{\alpha}{\gamma}\right)} d\alpha} \quad (8)$$

If equation (9) is solved, a unique function for α can be derived

$$\hat{\alpha} = \frac{\sqrt{\gamma} \sqrt{\sum_{i=1}^n x_i^\beta} \text{Besselk} \left[-1 - \kappa + n, \frac{2\sqrt{\sum_{i=1}^n x_i^\beta}}{\sqrt{\gamma}} \right]}{\text{Besselk} \left[-\kappa + n, \frac{2\sqrt{\sum_{i=1}^n x_i^\beta}}{\sqrt{\gamma}} \right]} \quad (9)$$

and here *Besselk* is Bessel function of the second kind. For all distribution mentioned earlier estimation for parameter α shows in Table 1. Finally reliability of the new product can be estimated with following equation

$$R(t) = e^{-\frac{t^\beta}{\hat{\alpha}}} \quad (10)$$

3.2 Parameter estimation for prior distribution of α

This section discusses parameter estimation for prior distribution of α that described earlier. As mentioned, α parameter is estimated by the average of α for all past production. Based on the prior distributions and the expert opinion, α parameters is estimated. If the mean value of α , shown as $\bar{\alpha}$, and expected value of prior α , shown as $E(\alpha)$, then the following equation is satisfied

$$\hat{E}(\alpha) = \bar{\alpha} \quad (11)$$

Estimation of λ , the only parameter of exponential distribution is very simple. So exponential distribution parameter is calculated using the relationship $\lambda = \bar{\alpha}$.

For distributions with more than one parameter, the estimate should be made with the help of experts. Here, the method proposed by Yadav et al. (2003) are used. The procedure is to find one of the parameter based on another one and obtain $\bar{\alpha}$. Then ask experts to give their opinion about minimum and maximum of other parameters and consider this as lower and upper for the limit equation. Here β is considered as known parameter so we may try to find the scale parameter of Weibull distribution. So the equation is:

$$P(\alpha_{LL} \leq \alpha \leq \alpha_{UL}) = p_0 \quad (12)$$

that can be transfer to:

Table 1. Estimation of α parameter with different prior function

prior distribution	prior distribution function	α parameter estimation
Gamma	$g(\alpha) = \frac{1}{\gamma^\kappa \Gamma(\kappa)} \alpha^{\kappa-1} e^{-\frac{\alpha}{\gamma}}$	$\hat{\alpha} = \frac{\sqrt{\gamma} \sqrt{\sum_{i=1}^n x_i^\beta} \text{Besselk} \left[-1-\kappa+n, \frac{2\sqrt{\sum_{i=1}^n x_i^\beta}}{\sqrt{\gamma}} \right]}{\text{Besselk} \left[-\kappa+n, \frac{2\sqrt{\sum_{i=1}^n x_i^\beta}}{\sqrt{\gamma}} \right]}$
Exponential	$g(\alpha) = \frac{1}{\lambda} e^{-\frac{\alpha}{\lambda}}$	$\hat{\alpha} = \frac{\sqrt{\lambda} \sqrt{\sum_{i=1}^n x_i^\beta} \text{Besselk} \left[-2+n, \frac{2\sqrt{\sum_{i=1}^n x_i^\beta}}{\sqrt{\lambda}} \right]}{\text{Besselk} \left[-1+n, \frac{2\sqrt{\sum_{i=1}^n x_i^\beta}}{\sqrt{\lambda}} \right]}$
Inverted Gamma	$g(\alpha) = \frac{1}{\rho^\nu \Gamma(\nu)} \left(\frac{\rho}{\alpha}\right)^{\nu+1} e^{-\frac{\rho}{\alpha}}$	$\hat{\alpha} = \frac{(\rho + \sum_{i=1}^n x_i^\beta) \Gamma[-1+n+\nu]}{\Gamma[n+\nu]}$
Truncated Normal	$g(\alpha) = \frac{\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\alpha-\mu}{\sigma}\right)^2}}{\int_{\frac{\mu}{\sigma}}^{\frac{\mu}{\sigma}} e^{-\frac{z^2}{2}} dz}$	$\hat{\alpha} = \frac{\int_0^\infty \frac{1}{\alpha^{n-1}} e^{-\left(\frac{1}{\alpha} \sum_{i=1}^n x_i^\beta + \frac{1}{2}\left(\frac{\alpha-\mu}{\sigma}\right)^2\right)} d\alpha}{\int_0^\infty \frac{1}{\alpha^n} e^{-\left(\frac{1}{\alpha} \sum_{i=1}^n x_i^\beta + \frac{1}{2}\left(\frac{\alpha-\mu}{\sigma}\right)^2\right)} d\alpha}$

$$\int_{\alpha_{LL}}^{\alpha_{UL}} g(\alpha) d\alpha = p_0 \tag{13}$$

In this equation p_0 is an analyst confidence in the accuracy of α parameter and generally equal to 0.90, 0.95 or 0.99. So, according to these equations, one parameter is calculated from the direct relationship of parameters and another one derived using equation (13). For better understanding, method presented here, is applied to inverted gamma prior distribution. Similarly, gamma and normal distribution parameters can be interrupted or otherwise distributed with the help of this method. In inverted gamma we have

$$\frac{\hat{\rho}}{\nu - 1} = \bar{\alpha} \tag{14}$$

Based on experts opinion

$$P(2500 \leq \alpha \leq \infty) = 0.99 \tag{15}$$

Then by solving below integral we could have inverted gamma distribution parameters.

$$\int_{2500}^{\infty} \frac{1}{\bar{\alpha}(\nu - 1)\Gamma(\nu)} \left(\frac{\bar{\alpha}(\nu - 1)}{\alpha}\right)^{\nu+1} e^{-\frac{\bar{\alpha}(\nu - 1)}{\alpha}} d\alpha = 0.99 \tag{16}$$

3.2 Behavior of prior distributions on scale parameter

In this section the behavior of the scale parameter and reliability function for different prior distributions is discussed. For three prior distributions, exponential, gamma and inverse gamma, solving the integrals give us an exact equation for α . Figure 1 shows how reliability change for different values of α . Here β is 2, n define as number of new product available and t as unit of time when we want to measure reliability, are considered to be 10 and 100 and R is calculated accordingly for each four cases. Results are shown in Figure 1.a and 1.b.

We can estimate reliability of new introduced product base on only historical information available from past generation of product. The blue curve in figure 1 represents this scenario and by increasing α , reliability will also increase. The following three curves are for the case that α is estimated using Bayes estimator based on the past product's information and several months new product's failure data. In another words, the combination of past and current information is intended.

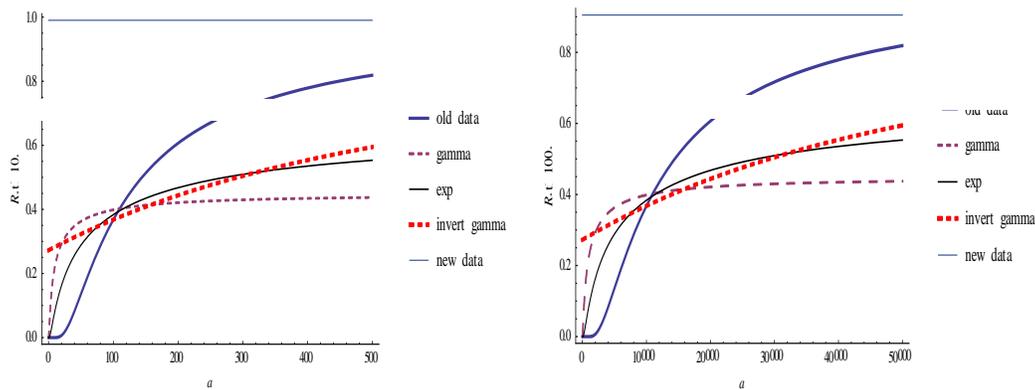


Figure 1.a. reliability change over different prior distribution for scale parameter when $n=10$ (left $t=10$ and right $t=100$).

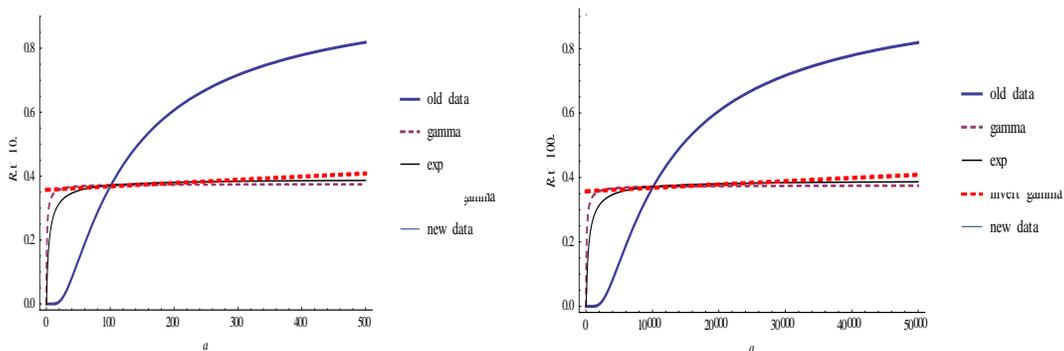


Figure 1.b. reliability change over different prior distribution for scale parameter when $n=100$ (left $t=10$ and right $t=100$).

According to Figure 1, it can be seen that when a fewer data is available, in the case $n=10$, the difference between prior distribution is more significant compared to $n=100$, especially when α is increasing. Also for lower α , which depends on the magnitude of t , prior distribution graph stands between old and new data graph. It means that in that case the Bayesian model comes with more reliable results. Thus, it can be concluded that the lack of failure information, leading to an inaccurate estimation of new product's reliability because it only relies on limited available new data. Using Bayesian method, scale parameter of Weibull distribution is estimated based on past product failure data. The reason we consider Bayesian results more reasonable is that it uses both past and few current available data to estimate. Furthermore reliability estimation only base on new data regardless of past generation information could not be accurate.

4. Application of the proposed Bayesian estimation model

For the proposed model, two examples are discussed in this section. As stated above, the proposed Bayesian methodology is applied to a novel Weibull distribution for reliability estimation. The Bayesian estimation can be applied to any manufacture in order to come with more accurate reliability prediction for a new product.

4.1 Numeric Example

An example for the results obtained in the previous sections can be studied as follows. First it should be mentioned that in reliability studies we always have two cases for failure time, censored or uncensored. Censoring occurs when the value of a measurement or observation is only partially known. Assume that failure data for a product that recently introduced to market is available. For the uncensored data, failure information of 10 products is available and for censored case we have that information for 15 products which 5 out of 15 have time censored and it means that as an example at $t=180$ unit of time the data collection stopped and we observed that for only 5 out of 15 product failure happen. Again assumed that data has Weibull distribution with $\beta=2$ and scale parameter is unknown. According to maximum likelihood method, α estimation for uncensored and censored data is as follow,

$$\hat{\alpha}_{MLE} = \frac{\sum_{i=1}^n x_i^\beta}{n} \quad (18)$$

$$\hat{\alpha}_{MLE} = \frac{\sum_{i=1}^r x_i^\beta + (n-r)x_r^\beta}{r} \quad (19)$$

Due to the limited number of available data, this estimation cannot have a high degree of accuracy. What has been suggested in this study is using previous product failure information with Bayes method in order to achieve more accurate estimation for the scale parameter of Weibull distribution that lead to more accurate new product reliability prediction. For this example, failure data for 10 past generations of the product is available. Scale parameter can be estimated based on this information. In other words there are 10 different prior distributions and in this case it means 10 different α parameters. The most important part is the parameter estimation of prior distribution that discussed in section 3.2. After estimating the parameters of the prior distributions, Bayes method is used to estimate α of failure distribution. Table 2 shows α estimation based on the new product failure rate data and Table 1.

Table 2. Estimation of Reliability at $t=100$ ($R(t)$) with different scale prior distribution when $\beta=2$

Method of data gathering	Previous product failure data	Exponential	Gamma	Inverted Gamma	Truncated Normal	new product failure data
Complete data	0.28	0.40	0.39	0.41	0.36	0.45
Censored data	0.28	0.67	0.55	0.65	0.52	0.71

According to Figure 2 there is a significant difference between each method of α estimation. It suggests that reliability prediction for new product based on few sparse available data might not be a good choice, as also indicated that only the past information couldn't contain improvement made to decrease failure rate of a new product. In between we can conclude that proposed Bayesian model could result in more logical reliability prediction.

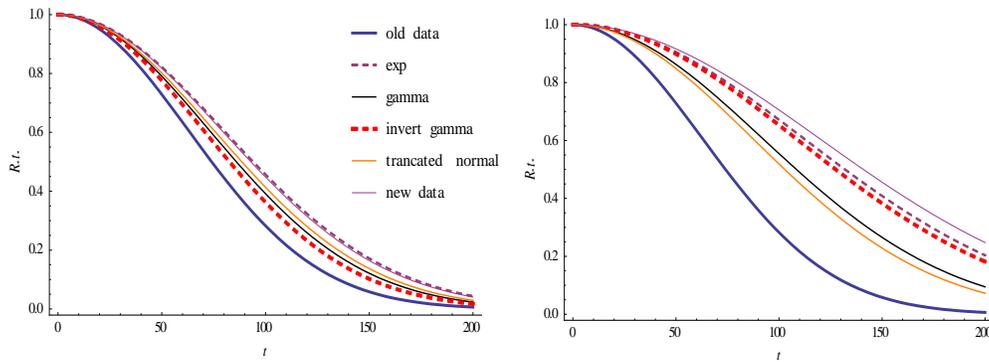


Figure 2. Reliability prediction for new product based on different method both for uncensored (left graph) and censored (right graph) data.

4.2 Case Study

This real case study from automotive industry shows a battery failure data for a specific vehicle's model. The data is obtained for life times of three previous generations of this battery. Life time shows that the Weibull distribution is a proper choice for the time to the first failure of these batteries. The results of fitting distribution to the life times are as follows:

$$f_1(t) = \frac{2.7}{4.8 \times 10^6} t^{2.7-1} e^{-\frac{t^{2.7}}{4.8 \times 10^6}} \quad \text{Battery series 1}$$

$$f_2(t) = \frac{2.85}{1.6 \times 10^7} t^{2.85-1} e^{-\frac{t^{2.85}}{1.6 \times 10^7}} \quad \text{Battery series 2}$$

$$f_3(t) = \frac{2.8}{3.6 \times 10^7} t^{2.8-1} e^{-\frac{t^{2.8}}{3.6 \times 10^7}} \quad \text{Battery series 3}$$

MTTF for battery series 1 to 3 is 266.78 day, 302.96 and 445.25 accordingly and it means that company did a good job improving its products. Now this company produces a new battery and tries to increase the MTTF for this evolutionary product. 20 batteries of the new product are used and failure data for 5 of them is available, but the rest are censored. According to the past information $\beta=2.8$. We assumed that α is a random variable and due to expert opinion it has gamma distribution with $\kappa=1/846$ and $\gamma=10354973$ according to pervious data. Likelihood estimation of α based on only new data is $9/2 \times 10^7$. It also can be calculated based on only previous data available from past product and for this case study it is equal to $1/9 \times 10^7$. If Bayesian model develop, $5/7 \times 10^7$ is obtain for α . The result for different methods is shown in Table 3 and Figure 3.

The importance of the proposed model is for managers who wish to have an accurate warranty prediction for their evolutionary products. For this example if manager wishes to assign a warranty period for its product, using past information results in underestimation and using only few available data cause overestimation for reliability of the new product. In this case Bayesian approach gives more logical and accurate estimation. So manager can assign 10 months with probability of 0.85. In another words, the manager can be more confident that for the new product, 85% of batteries do not fail during the first 10 month of their production.

Table 3- estimation of α parameter, MTTF and warranty period (based on 85%)

	Past Information	Bayesian Approach	New Data
α estimation	$1/9 \times 10^7$	$5/7 \times 10^7$	$9/2 \times 10^7$
MTTF	353 days	580 days	625 days
Warranty period (based on 85%)	209 days 7 months	307 days 10 months	366 days 12 months

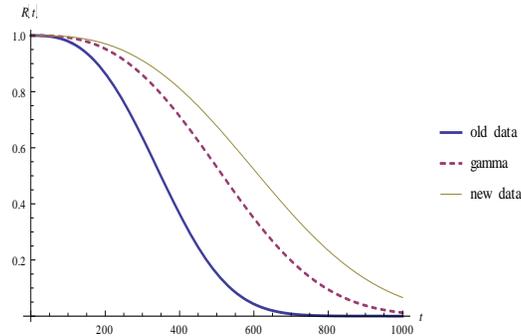


Figure 3, Reliability prediction for new battery over time. Thick Curve refers to reliability prediction based on only past information, thin curve refers to using only the new set of data and finally dashed curve, in middle, shows the Bayesian approach introduced in this paper.

5. Conclusions

For the evolutionary product accurate reliability estimation early after production is crucial. Lack of data is the main problem that arises when trying to predict a product's reliability in its early life cycle. When enough data is not available, conventional methods of prediction, as described in sections 1 and 2, are less useful and provide inaccurate results. The Bayesian approach on the other hand, models probability distribution based on a limited set of data and updates the information every time a new data becomes available. In this paper, a simple Bayesian estimation procedure has been proposed for reliability prediction of a new product. A combination of past information with new failure data through the Bayesian approach proved to be a successful answer to all the limitation and problems of current reliability prediction methods. The proposed estimation procedure only requires statistical data from the previous generation of the new introduced product. Indeed, available knowledge is directly and efficiently incorporated by Bayes methods into the estimation process. The procedure, which may be easily performed by analytical methods, has been illustrated in the paper by means of a numerical application. In this study we focused on non-repairable products. This means that we look at a product first failure. Future repair is not considered since a second product failure of the same item is unlikely to happen within the warranty period. So it can be suggested for future research to investigate the models for repairable and more complex products. It is suggested to extend the procedure to the case of Weibull model with shape parameter as a random variable. It is also interesting to use fuzzy logic for the Bayesian model to solve the uncertainty in parameters of prior distribution.

References

- Alzbutas, Robertas and Tomas Iešmantas. 2014. "Application of Bayesian Methods for Age-Dependent Reliability Analysis." *Quality and Reliability Engineering International* 30(1):121–32.
- Blanks, Henry S. 1998. "The Challenge of Quantitative Reliability." *Quality and Reliability Engineering International* 176(November 1997):167–76.

- Canavos, George C., Chris P. Tsokos, and Hampton Virginia. 1973. "Bayesian Estimation of Life Parameters in the Weibull Distribution Bayesian Estimation of Life Parameters in the Weibull Distribution." *Operations Research* 21(3):755–63.
- Coit, David W. 2000. "System Reliability Prediction Prioritization Strategy." Pp. 175–80 in *Reliability and Maintainability Symposium*.
- Denson, W. 1998. "The History of Reliability Prediction." *IEEE Transactions on Reliability* 47(3):SP321–28.
- Dupow, Heiner and Gordon Blount. 1997. "A Review of Reliability Prediction." *Aircraft Engineering and Aerospace Technology* 69(4):356–62.
- Fajdiga, M. and T. Jurejevic. 1996. "Reliability Prediction in Early Phases of Product Design." *Engineering Design* 7(2):107–29.
- Houben, Maurits J. H. A. et al. 2009. "Reliability Prediction Through Bayesian Inference Based On Product Change." Pp. 292–97 in *Annual Reliability and Maintainability Symposium*.
- Ion, R. a. and P. C. Sander. 2005. "Early Reliability Prediction in Consumer Electronics Using Weibull Distribution Functions." Pp. 43–47 in *Annual Reliability and Maintainability Symposium IEEE*.
- Ion, Roxana A., H. Kalishoek, D. M. Karydas, and I. Nitescu. 2006. "Bayesian Model for Early Reliability Prediction." Pp. 260–65 in *Reliability and Maintainability Symposium*, vol. 00.
- Johannesson, Pär et al. 2013. "A Robustness Approach to Reliability." *Quality and Reliability Engineering International* 29(1):17–32. Retrieved June 4, 2014 (<http://doi.wiley.com/10.1002/qre.1294>).
- Liang, Yi-Hui. 2008. "Combining Neural Networks and Genetic Algorithms for Predicting the Reliability of Repairable Systems." *International Journal of Quality & Reliability Management* 25(2):201–10.
- Lins, Isis Didier, Márcio Das Chagas Moura, Enrico Zio, and Enrique López Droguett. 2012. "A Particle Swarm-Optimized Support Vector Machine for Reliability Prediction." *Quality and Reliability Engineering International* 28(2):141–58.
- Lolas, S. and O. Olatunbosun. 2008. "Prediction of Vehicle Reliability Performance Using Artificial Neural Networks." *Expert Systems with Applications* 34(4):2360–69.
- Molenaar, P. A., A. J. M. Huijben, D. Bouwhuis, and A. C. Brombacher. 2002. "Why Do Quality and Reliability Feedback Loops Not Always Work in Practice: A Case Study." *Reliability Engineering & System Safety* 75(3):295–302.
- Nelson, Wayne. 1980. "Accelerated Life Testing Step-Stress Models and Data Analyses." *IEEE Transactions on Reliability* (2):103–8.
- Ormon, Stephen W., C. Richard Cassady, and Allen G. Greenwood. 2002. "Reliability Prediction Models to Support Conceptual Design." *IEEE Transactions on Reliability* 51(2):151–57.
- Pan, Rong. 2009. "A Bayes Approach to Reliability Prediction Utilizing Data from Accelerated Life Tests and Field." *Quality and Reliability Engineering International* (July 2008):229–40.
- Singh, H., V. Cortellessa, B. Cukic, E. Gunel, and V. Bharadwaj. 2001. "A Bayesian Approach to Reliability Prediction and Assessment of Component Based Systems." in *Software Reliability Engineering*.
- Siu, Nathan O. and Dana L. Kelly. 1998. "Bayesian Parameter Estimation in Probabilistic Risk Assessment." *Reliability Engineering & System Safety* 62(1-2):89–116.
- Tong, LI and YH Liang. 2005. "Forecasting Field Failure Data for Repairable Systems Using Neural Networks and SARIMA Model." *International Journal of Quality & Reliability Management* 22(4):410–20.
- Usher, John S., Suraj M. Alexander, and John D. Thompson. 1990. "SYSTEM RELIABILITY PREDICTION BASED ON HISTORICAL DATA." *Quality and Reliability Engineering International* 6(January):209–18.
- Wang, Lizhi, Rong Pan, Xiaoyang Li, and Tongmin Jiang. 2013. "A Bayesian Reliability Evaluation Method with Integrated Accelerated Degradation Testing and Field Information." *Reliability Engineering & System Safety* 112:38–47.

- Yadav, Om Prakash, Nanua Singh, Parveen S. Goel, and Rachel Itabashi-Campbell. 2003. "A Framework for Reliability Prediction During Product Development Process Incorporating Engineering Judgments." *Quality Engineering* 15(4):649–62. Retrieved June 5, 2014.
- Ye, Zhisheng, Nan Chen, and Kwok-Leung Tsui. 2013. "A Bayesian Approach to Condition Monitoring with Imperfect Inspections." *Quality and Reliability Engineering International* n/a – n/a. Retrieved June 4, 2014 (<http://doi.wiley.com/10.1002/qre.1609>).
- Ye, Zhi-Sheng and Min Xie. 2015. "Stochastic Modelling and Analysis of Degradation for Highly Reliable Products." *Applied Stochastic Models in Business and Industry* 31(1):16–32.
- Zacks, Shelemyahu. 1992. *Introduction to Reliability Analysis : Probability Models and Statistical Methods*. New York: Springer.

Biography

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