

A Multi-Criteria Decision Making Tool to Prioritize Network Component for Recovery based on Importance Measures

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Abstract

As demands are growing and networks are becoming more complex every day, the cost and consequences of even a small disruption in the network is going to be huge. Analyzing and prioritizing edges for the recovery of the complete network is essential for a successful continuity of business operation. Measuring the importance of network components is of significant value in prioritizing improvement efforts and planning recoverability. Prioritizing transportation network components for recovery based on topology based and flow based importance measures (IMs) will be useful for decision makers. In this project, we aim to create a multi-criteria decision making (MCDM) tool, i.e., PROMETHEE, to analyze and prioritize transportation network components for recovery based on multiple IMs: all pairs max flow edge count, min cutset count, edge flow centrality, flow capacity rate and damage impact, betweenness centrality, and information centrality. The proposed approach is validated through simulation and illustrated with an example. Also, a sensitivity analysis has been carried out for the proposed approach. Conclusion remarks are presented with some suggested future work too.

Keywords

Network, importance measure, PROMETHEE, simulation.

1. Introduction

The role of networks and their performance is patent to many industries, such as transportation, supply chain management, and healthcare systems. Consequently, the disruption in the system and any bottleneck that stops the system can have adverse effects of the whole system. The planning and maintenance teams put all their effort and contemplation to find the criticality in the network and enhance the robustness of the network and moreover find an optimal way to recover the system after the presence of the disaster. Managers are seeking ways whereby they can identify criticality in networks specially the links. Consequently, the most efforts are put to examine links significance and prioritize them. Our goal in this project is to identify importance measures which can be used to rank the network edges. Of all the techniques available to rank the edges, a multi criteria decision making tool called PROMETHEE is used in this project. In reality there are some situation that a network cannot be recovered thoroughly as a result of budget limit, resource availability and other economic and environmental limitation.

1.1 Previous Work

Several importance measures (IMs) have been suggested in the literature to identify important components in a network. Najir and Gaudiot (1990) introduce a network fault tolerance measure which calculates the maximum number of failure the network can sustain before the disruption of the whole system occurs. Freeman et al. (1991) propose a new centrality measure for network edges, using the concept of network flows, which is defined for both valued and non-valued graphs and the computation of this measure is based on all independent paths between all pairs of nodes in the networks rather than geodesic paths. Cho and Yum (1997) suggest the uncertainty as an IM of an activity or a pair of them which utilizes Taguchi tolerance design techniques; hence it is applicable for large PERT networks. A new IM for multi-state with multi-components based on reliability is introduced by Ramirez-Marquez et al. (2006) and Zio et al. (2007) which enhance the networks reliability from two points: (i) it measures how a specific component effects multi-state system reliability and (ii) it measures how a specified component state or set of states affects multi-state system reliability. Dall'Asta et al. (2006) consider topology, weight of the links, and geography of network components to study the vulnerability of the networks to intentional attack. Chen et al. (2007) use network-based accessibility as a measure for network component with the variation of travel time and

travel cost during the occurrence of the disaster. Nagurney and Qiang (2007), and Nagurney and Qiang (2008) provide a new network performance measure which uses demands, flows, and behavior of networks to evaluate the importance of the links in the network. Whitson and Ramirez-Marquez (2009) consider the resiliency as an IM related network reliability. Cadini et al. (2009) use the concept of centrality introduced in the complexity of the network to measure the criticality of the network components. Agryzkov et al. (2014) proposed a new centrality index called ranking-betweenness centrality which helps in classifying the nodes. This idea is very similar to the ranking of webpages by Google website. Dwivedi et al. (2010) study the power networks and consider the maximum flow of the power instead of shortest path. Jenelius (2010) studies the importance of links in the absence of other links and propose an IM which considers the network robustness and network redundancy based on traffic flow and impact base flow. Rocha et al. (2014) proposed a centrality measure for temporal networks based on random walks under periodic boundary conditions and named it TempoRank. Claudio et al. (2012) analyze the importance of network components through the evaluation of classical metrics and by using sensitivity methods the effect of components are quantified. This quantification helps decision makers to find the critical components. Sun and Yang (2013) and Sun (2014) study different edge centralities based on the network topology, walks and paths and then present a divisive algorithm based on a rational algorithm to weight the component links and find communities. Barker et al. (2013) and Baroud et al. (2014) propose some resilience-based IMs which quantify the potential impact of a disaster on systems resiliency and the positive impact of a link on the networks when it remains undisrupted in the presence of the disaster. Nicholson et al. (2014) develop and evaluate multiple flow-based importance measures to rank network edges for the phase of network preparedness which means the comparison of different importance measures for network vulnerability as well as recovery. Du et al. (2014) propose a new method of evaluation based on the node importance and the technique for order performance by similarly the ideal solution (TOPSIS) in complex networks considering multiple node centrality measures to find the best order nodes for the recovery in complex networks.

So, as it can be noticed from the literature review, there have been numerous studies conducted about importance measures of networks with different bases such as: flow, topology, centrality, resiliency, etc. Although many IMs have been considered to find the importance of network component, the importance of components was obtained by individual measures only except one study (Du et al. 2014) in which they propose TOPSIS to rank the nodes of a network considering multiple nodes centrality measures. Moreover, a validation process has not been carried out by any of the proposed methods.

1.2 Problem Statement

As demands are growing and networks are becoming more complex every day, the cost and consequences of even a small disruption in the network are going to be huge. Analyzing and prioritizing edges for the recovery of the complete network is essential for a successful continuity of business operation. Measuring the importance of network components is of significant value in prioritizing improvement efforts and planning recoverability. Prioritizing transportation network components for recovery according to topology-based and flow-based IMs will be useful for decision makers.

In this project, we propose a multi-criteria decision making (MCDM) tool to prioritize network components for recovery based on multiple IMs with different basis, i.e., flow and topology. The proposed tool considers not only one IM but a combination of IM heuristics to analyze the ranking problem for network components from different aspects. This problem is applicable in many fields including infrastructure network, traffic in a road system, circulation with demands, fluids in pipes, currents in an electrical circuit, or anything similar in which something travels through a network of nodes. After an extensive literary review we have decided to consider a total of seven IMs for prioritizing network edges. Five IMs are flow-based and two are topology-based. The flow-based measures are all pairs max flow edge count, min cutset count, flow centrality, flow capacity rate and damage impact (i.e. network efficiency). The topology-based IMs are betweenness centrality and information centrality (i.e. network efficiency). The MCDM technique that we are planning to use is the preference ranking organization method for enrichment evaluation technique commonly called PROMETHEE (Tzeng et al. 2011). The output will be ranked network edges based on different suggested IMs using PROMETHEE. This project is different from others because no one has considered both topology based and flow based importance measures together and introduction of PROMETHEE as a ranking technique is new too. This paper takes one more step ahead by validating the work that has been done.

2. Preliminaries

In this project, we consider a network that can be classified as a directed graph which is denoted by $G = (V, E)$ where V is a set of n vertices or nodes and $E \subset \{(i, j) : i, j \in V, i \neq j\}$ is a set of directed edges or links. Let the flow and capacity on edge $(i, j) \in E$ denoted by f_{ij} and Cap_{ij} respectively. Let P represent a finite directed path from a source node s to a target node t , which are connected through a set of nodes in V and one or more direct edges in E . The nodes between the source node s and the target node t are called internal nodes. The maximum capacity of a path is equal to the minimum capacity of all edges within the path, i.e. $\text{Min}_{i \in P} \text{Cap}_{ij}$.

2.1. Definitions

2.1.1 Max-Flow Problem

The $s - t$ max flow problem utilizes a subset of all possible paths between s and t to route a maximum flow from s to t taking into consideration the capacity of edges. So, the $s - t$ max flow problem can be formulated as shown in Eqs. (1) – (3) (Bazaraa 2011).

$$\max x_{st} \quad (1)$$

$$\text{s.t.} \quad \sum_{(i,j) \in E} f_{ij} - \sum_{(i,j) \in E} f_{ji} = \begin{cases} x_{st} & \text{if } i = s \\ 0 & \text{if } i \in V \setminus \{s, t\} \\ -x_{st} & \text{if } i = t \end{cases} \quad (2)$$

$$0 \leq f_{ij} \leq \text{Cap}_{ij}, \quad i, j \in V \quad (3)$$

where x_{st} in the objective function, Eq. (1), denotes the maximum flow from node s to node t for any source and target node pair such that $s, t \in V$ where $s \neq t$, otherwise $x_{st} = 0$ if $s = t$. Eq. (2) represents the flow-conservation constraints which assure that the flow into and out of any internal node must be equal and the flow out of the source node s and into the target node t must equal x_{st} . The capacity constraints in Eq. (3) ensure that there is no negative flow as well as flow through any edge does not exceed its capacity.

2.1.2 Shortest-Path Problem

A shortest path problem is to find the shortest path between any two arbitrary vertices, s and t . The weight of a directed path P from a source node s to a target node t is the sum of the weights of the edges in between s and t , i.e.

$$w(P) = \sum_{i=1}^k w(v_{i-1}, v_i) \quad (4)$$

The shortest-path weight from s to t is defined as:

$$sp(s, t) = \begin{cases} \min\{w(p)\} & \text{if there is a path from } s \text{ to } t \\ \infty & \text{otherwise} \end{cases}$$

Hence, a shortest path from s to t is any path P with $w(p) = sp(s, t)$.

2.2. Importance Measures

In this project, seven IMs are considered identify the most important edges for networks, five of which are flow-based IMs (Nicholson et al. 2014, Rocco et al. 2010) and two are topology-based IMs (Cadini et al. 2009, Sun and Yang 2013, Sun 2014). Hence, the five flow-based IMs are All Pairs Max Flow Count, Min Cutset Count, Edge Flow Centrality, Flow Capacity Rate, and One-at-a-Time Damage Impact. The two topology-based IMs are Betweenness Centrality and Information Centrality. Most important edges proposed by these IMs are subjected to be reinforced, protected prior to any disruption, or expedited during recovery stage. The seven considered IMs are explained as follows.

2.2.1 All Pair Max Flow Count

The *all pairs max flow edge count* (*MF count*) IM measures the utilization of an edge in all $s - t$ pairs max flow problems. Accordingly, if an edge is contributing more than others, then it could cause a significant impact on network performance when its capacity is affected by any disruptive event. The *MF count* of an edge (i, j) , denoted as $I_{(i,j)}^{MF\ count}$, is defined as shown in Eq. (5).

$$I_{(i,j)}^{MF\ count} = \frac{1}{n(n-1)} \sum_{s,t \in V} \mu_{st}(i,j) \quad (5)$$

Where the value of $\mu_{st}(i,j)$ is defined as 1 if edge (i,j) is used in a given $s - t$ max flow problem, and 0 otherwise.

2.2.2. Min Cutset Count

The *min cutset count* (*MC count*) IM quantifies the involvement of an edge to the min cutset for all $s - t$ pairs. An $s - t$ *cut* on a graph is a segregating of the nodes into two disjoint sets S and T such that $s \in S$ and $t \in T$ and the $s - t$ *cutset* is the set of edges which starts in S but ends in T . Hence, the *min cut* of a graph is the $s - t$ *cut* with minimal capacity. So, if an edge (i,j) is involved in the *min cutset* for an $s - t$ pair, then it is considered as a bottleneck for the corresponding max flow problem. The MC count of an edge (i,j) , denoted as $I_{(i,j)}^{MC\ count}$, is defined as shown in Eq. (6).

$$I_{(i,j)}^{MC\ count} = \frac{1}{n(n-1)} \sum_{s,t \in V} \delta_{st}(i,j) \quad (6)$$

where the value of $\delta_{st}(i,j)$ is defined as 1 if edge (i,j) is a member in the $s - t$ cutset, and 0 otherwise.

2.2.3 Flow Centrality

The *edge flow centrality* (*FC*) IM measures the contribution of a given edge to the max flow of all $s - t$ pairs. Hence, it provides an importance based on the ratio of the total volume of flow through a given edge for all possible $s - t$ pair max flow problems to the flow of all $s - t$ pairs max flows. The FC of an edge (i,j) , denoted as $I_{(i,j)}^{FC}$, is defined as shown in Eq. (7).

$$I_{(i,j)}^{FC} = \frac{\sum_{s,t \in V} x_{st}(i,j)}{\sum_{s,t \in V} x_{st}} \quad (7)$$

where $x_{st}(i,j)$ is the flow on (i,j) when the max flow is routed from s to t for all $s - t$ pair max flow problems.

2.2.4 Flow Capacity Rate

The *flow capacity rate* (*FCR*) IM measure which edge has the potential to be the bottleneck, based on the amount of flow and capacity. It calculates the percentage of the edge flows to the capacity of it. As a result, if the flow through an edge is close to its capacity, then network performance could be affected by any disruption occurred on that edge. The FCR of an edge (i,j) , denoted as $I_{(i,j)}^{FCR}$, is defined as shown in Eq. (8).

$$I_{(i,j)}^{FCR} = \frac{1}{n(n-1)} \sum_{s,t \in V} \frac{x_{st}(i,j)}{c_{ij}} \quad (8)$$

2.2.5 Damage Impact

The *one-at-a-time damage impact* (*Impact*) IM examines how the network efficiency changes when an edge is completely damaged due to a disruptive event. It provides the average percent change through all $s - t$ max flow problems when an edge (i,j) is completely incapacitated. The Impact of an edge (i,j) , denoted as $I_{(i,j)}^{Impact}$, is defined as shown in Eq. (9).

$$I_{(i,j)}^{Impact} = \frac{1}{n(n-1)} \sum_{s,t \in V} \frac{x_{st} - x'_{st,ij}}{c_{ij}} \quad (9)$$

Where $x'_{st,ij}$ is the max flow from node s to node t when the capacity of edge (i,j) is completely damaged, it is not considered in the network as its capacity equals to 0.

2.2.6. Betweenness Centrality

The *betweenness centrality* (BC) IM is defined as the number of the shortest paths that go through an edge in a network. An edge with a high BC score represents could affect the communication between many $s - t$ pairs of nodes through the shortest paths between them in case if it is disrupted or damaged. The BC of an edge (i, j) , denoted as $I_{(i,j)}^{BC}$, is defined as shown in Eq. (10).

$$I_{(i,j)}^{BC} = \sum_{s,t \in V} \frac{\sigma_{st}^e}{\sigma_{st}} \quad (10)$$

where σ_{st} is the number of shortest paths from s to t . σ_{st}^e is the number of shortest paths from s to t that pass through edge e .

2.2.7 Information Centrality

The *information centrality* (IC) IM quantifies the importance of an edge on the network. It is defined as the relative drop in the network efficiency resulted by the removal of an edge from the network. The IC of an edge (i, j) , denoted as $I_{(i,j)}^{IC}$, is defined as shown in Eq. (11).

$$I_{(i,j)}^{IC} = \frac{NE(G) - NE(G'_e)}{NE(G)} \quad (11)$$

Where G'_e is the network G but with the removal of edge e . $NE(G)$ is the network efficiency of network G and can be calculated as shown in Eq. (12).

$$NE(G) = \frac{1}{n(n-1)} \sum_{s \neq t \in G} \frac{1}{d_{st}} \quad (12)$$

Where d_{st} is the length of the shortest path from s to t .

3. Proposed Approach

The PROMETHEE method (Preference Ranking Organization Method for Enrichment Evaluations) is one of the most recent MCDM methods that was developed by Brans (1982) and further extended by Vincke and Brans (1985). PROMETHEE I method indicates the best alternative among the ones in question; PROMETHEE II gives a complete ranking of the alternatives. When a subset of alternatives must be identified by the decision maker for given a set of constraints PROMETHEE V can be used (Fontana et al 2013). PROMETHEE can be applied in many fields including environment management, hydrology, business and financial management, chemistry, and other topics (Behzadian et al. 2010). PROMETHEE II is the method of interest for our paper. It assumes that the decision-maker is able to weigh the criteria appropriately, at least when the number of criteria is not too large (Macharis et al. 2004). The method is based on comparison of each alternative pair with respect to each of the selected criteria. In order to perform alternative ranking by the PROMETHEE method, it is necessary to define preference function $P(a, b)$ for alternatives a and b which converts the pairwise difference into a preference degree ranging from zero to one.

In this paper, V-shaped analytical expression is chosen for the shape of preference function which is as shown in Eq. (15). In the V-shaped preference index an upper boundary parameter s is set, this helps in setting a strict preference of one alternative over another. Once the preference function is set we can aggregate the preference degree of all criteria for each pair of possible and create a global preference index. For each possible decision a , we compute the positive outranking flow $\emptyset^+(a)$ and the negative outranking flow $\emptyset^-(a)$. The difference between the positive outranking flow and the negative outranking flow will give you the net flow. This net flow can be ordered and used for ranking the alternatives. Steps for the proposed method are as given below.

Step 1: Collect the IMs values for each alternative and create an $m \times n$ matrix. Where m equals the number of IMs and n equals the number of alternatives.

Step 2: Set the weights for each IM. Let w_j be the weight for each criterion.

Step 3: Find the deviation by pairwise comparison between the alternatives of each criteria like Eq. (14)

$$d_m(a, b) = F_m(a) - F_m(b) \quad (14)$$

where $d_m(a, b)$ means the difference between the values of alternatives a and b for a criteria.

Step 4: We apply the preference function of our choice. We have chosen a V-shaped analytical equation as below:

$$p_m(d_m) = \begin{cases} 0 & \text{when } d \leq 0 \\ \frac{d_m}{s} & \text{when } 0 < d_m < s \\ 1 & \text{when } d_m > s \end{cases} \quad (15)$$

where $p_m(d)$ represents the preference of alternative a with regards to alternative b on each IM.

Step 5: Aggregate the preference degree of all criteria for each pairwise combination and create a global preference index using Eq. (16).

$$\pi(a, b) = \sum_{j=1}^m w_j \times P_j(d_j) \quad (16)$$

$\pi(a, b)$ is the weighted sum of each $p(d)$ for all IMs. w_j is the weight of each IM.

Step 6: For each possible decision a , we compute the positive outranking flow $\phi^+(a)$ and the negative outranking flow $\phi^-(a)$ by Eq.s (17) and (18), respectively where n represents all the alternatives.

$$\phi^+(a) = \sum_{i \in n} \pi(a, i) \quad (17)$$

$$\phi^-(a) = \sum_{i \in n} \pi(i, a) \quad (18)$$

Step 7: Establish a complete ranking between the possible alternatives. The ranking is based on the net outranking flow which is obtained by Eq. (19).

$$\phi(a) = \phi^+(a) - \phi^-(a) \quad (19)$$

4. Illustrative Example

In this section, a numerical example is introduced to illustrate the proposed method in this paper. Hence, we consider a small network with seven nodes and twelve edges (Hillier and Lieberman, 2009), as shown in Figure 1 below, where edges are labeled by their number and capacities, respectively.

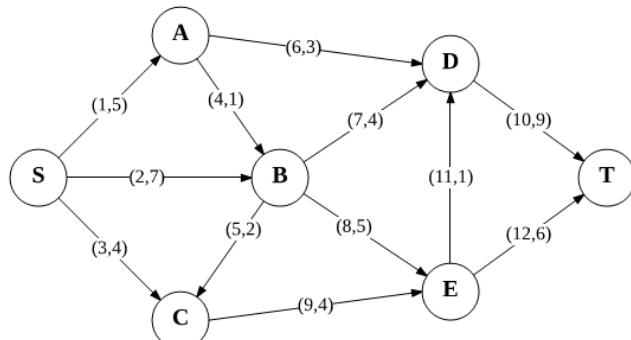


Figure 1: A network example with labeled edges by (number, capacity)

Table 1: Alternative (edges) scores based on different IMs

Edge #	MF count	MC count	FC	FCR	Impact	BC	IC
1	0.1429	0.0238	0.1429	0.0762	0.0408	1.7500	0.0612
2	0.1190	0.0238	0.2054	0.0782	0.0545	2.5000	0.0306
3	0.0952	0.0238	0.0982	0.0655	0.0263	1.7500	0.0306
4	0.2381	0.1429	0.0893	0.2381	0.0863	3.0000	0.1224
5	0.1429	0.0714	0.0982	0.1310	0.0667	2.0000	0.0918
6	0.0952	0.0952	0.1071	0.0952	0.0497	2.7500	0.0408
7	0.1190	0.0952	0.1518	0.1012	0.0464	2.2500	0.0306
8	0.1667	0.0476	0.1875	0.1000	0.0462	3.2500	0.0408
9	0.1905	0.0714	0.1964	0.1310	0.0982	3.7500	0.1224
10	0.1190	0.0238	0.2321	0.0688	0.0695	3.0000	0.0714
11	0.1667	0.1667	0.0625	0.1667	0.0626	2.0000	0.0918
12	0.1190	0.0714	0.2054	0.0913	0.0615	3.0000	0.0408

The IMs MF count, MC count, FC, FCR, Impact, BC and IC will be addressed in the same order for all purposes from here after. The weights for every IM are assumed to be equal. Thus every IMs will have a weight of 1/7. The next step is to do a pairwise comparison of every alternative for each IM. Comparison of alternatives 1 and 2 for all the IMs are {0.0238, 0, -0.0625, -0.0020, -0.0137, -0.75, 0.0306}. The upper boundary parameter s has been chosen as the individual mean of each criterion and the value of it is as follows {0.1429, 0.0714, 0.1481, 0.1119, 0.0591, 2.5833, 0.0646}. The next step is to find the preference function values. After applying the preference function for the deviation of Alternative 1 and 2 for all IMs the result is {0.02381, 0, 0, 0, 0, 0.067}. The entering flow for alternatives 1 to 12 are {0.00, 0.09, 0.21, 0.05, 0.04, 0.13, 0.09, 0.05, 0.00, 0.03, 0.08, 0.07}, respectively. The aggregated preference degree is shown in Table 2.

Table 2: Flow into and out of all edges

		In Flow											
edge		1	2	3	4	5	6	7	8	9	10	11	12
Out Flow	1	0.00	0.09	0.21	0.05	0.04	0.13	0.09	0.05	0.00	0.03	0.08	0.07
	2	0.14	0.00	0.25	0.11	0.13	0.13	0.09	0.04	0.01	0.01	0.17	0.00
	3	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.03	0.00
	4	0.70	0.65	0.78	0.00	0.55	0.63	0.64	0.60	0.33	0.56	0.37	0.61
	5	0.31	0.35	0.47	0.01	0.00	0.25	0.25	0.25	0.00	0.24	0.04	0.20
	6	0.24	0.20	0.32	0.02	0.10	0.00	0.06	0.10	0.05	0.18	0.08	0.05
	7	0.22	0.17	0.34	0.06	0.11	0.07	0.00	0.10	0.05	0.18	0.10	0.06
	8	0.24	0.19	0.40	0.11	0.18	0.18	0.16	0.00	0.00	0.15	0.19	0.07
	9	0.65	0.55	0.77	0.17	0.38	0.54	0.50	0.42	0.00	0.47	0.40	0.40
	10	0.25	0.18	0.42	0.14	0.19	0.27	0.27	0.17	0.03	0.00	0.21	0.11
	11	0.42	0.46	0.58	0.05	0.21	0.45	0.45	0.38	0.19	0.36	0.00	0.40
	12	0.29	0.18	0.43	0.11	0.16	0.16	0.15	0.10	0.01	0.12	0.19	0.00

Leaving flow for alternative 1 to 12 are {0.00, 0.14, 0.00, 0.70, 0.31, 0.24, 0.22, 0.24, 0.65, 0.25, 0.42, 0.29}, respectively. So, the final flow values of the alternatives are {0.505564, 0.417397, 0.188065, 0.028911, -0.00517, -0.00596, -0.02945, -0.10672, -0.12782, -0.17738, -0.2383, -0.44914}.

Once we tested the model on all seven chosen IMs. We used PROMETHEE to rank the alternatives considering different combination of IMs: flow-based (MF count, MC count, FC, FCR, and Impact), topology-based IMs (BC and IC), network efficiency (NE) (Impact and IC), and centrality (FC, BC, and IC). The ranking of the alternative considering different IMs combination is shown in Table 3.

Table 3: PROMETHEE ranks for edges considering different IMs combinations

Rank	All IMs	Flow	Topology	NE	Centrality
1	4	4	9	9	9
2	9	9	4	4	10
3	11	11	10	5	12
4	5	12	5	11	8
5	12	5	11	10	2
6	10	7	8	12	7
7	8	8	12	1	6
8	7	10	6	6	4
9	6	6	1	2	1
10	2	2	2	8	5
11	1	1	7	7	3
12	3	3	3	3	11

For PROMETHEE there is no particular rule to choose the weight for the criteria and it is dependent on the decision maker. We have performed a sensitivity analysis on the ranking of PROMETHEE by considering different weights criteria, i.e., equal weights, weights generated using the concept of Analytic Hierarchy Process (AHP), and random weights. Results are given in Table 4.

Table 4: PROMETHEE ranks for edges considering different weight criteria

Equal weight	AHP	Random weight
4	4	9
9	9	4
11	11	12
5	5	8
12	12	10
10	10	11
8	7	6
7	8	7
6	6	5
2	2	2
1	1	1
3	3	3

5. Validation

To start the validation we consider a small network with 7 nodes and 12 links, see Figure 1. The capacity of each link is shown in Table 1.

5.1. Methodology

As it is mentioned, the simulated network which is coded by Awesim software is an example of the real system which is to be compared with the model which is represented to rank the importance of each links. To start, we inject the flow of entities to the network. The time between the entities are as small as possible so as we can have flow in each link before a disaster happens. In the second step, we consider a mechanism for each links whereby the effect of disaster and the recovery process are simulated as a real case. To consider the effect of the disaster we put a Gate for each link. The responsibility of a Gate in a link is to be open, let entities transfer through the link, before a

disaster happens, and then in a post disaster situation the Gate becomes close and all entities are sent to the waiting block. The waiting block keeps the entities coming to the link when the Gate is closed and counts the number of entities, the average waiting time and the maximum length of entities waiting in the waiting block until the link is recovered. In other words, the waiting block gives information whereby the decision makers can track what happens for the entities; meanwhile, the link is under restoration. Depending on the time and order of recovery, the Gate can be opened in various times. For example, if the time of recovery for link 4 is 100 and the time of recovery for link 5 is 189 and link 4 is to be recovered after link 5, so it takes 289 (100+189) units of time for Gate related to link 4 to be opened. The third step is concerned with the capacity of the links and for this purpose the queue blocks in are applied to each link which does not permit to link to get more entities than its capacity. Apart from that, the activity after the link with the server and service duration can be an introduction of the characteristics of that link such as the traffic flow the bottle necks it has.

5.2. Example

The simulated network which is coded by Awesim software evaluate the average and maximum length of queue in the presence of disaster, the average idle and busy time of server, and the utilization of activities. Awesim considers three links (9,8,7) to be disrupted and then examine all types of orders of recovery to find the one with the optimal values.

Table 5: Results of all types of recovery order

Links	Gate		Activity utility		Servers	
	Order	Max Length	Ave Length	Utilization	Count	Max Idle Time
8,7,9	146	55	1.29	119	6830	5169.9
7,8,9	163	59.5	1.12	102	7510	4489.9
8,9,7	133	60	1.46	133	6130	5869
7,9,8	166	61	1.11	98	7530	5169.9
9,7,8	151	57	1.29	112	6830	5169.9
9,8,7	131	49	1.47	133	6110	5889.9

As is shown in Table 5, the maximum length and the average length of the entities waiting for gates to be open is has its least value when the order of 9,8,7 in the recovery. And the utilization and the number of entities which receiving services are maximum when we choose the recovery order 9,8,7. The idle time, which include the time of server waiting for the links to be recovered, has its least value for 9,8,7 and the Maximum busy time, which means links are recovered, is maximum for the order. This prove that the model and simulation are compatible and emphasize that the model works right.

6. Conclusion

In this project we were able to provide seven IMs which prioritize the configuration of the edges needed to be recovered. We were successfully able to code these seven IMs with R in order to obtain an applicable framework measuring the edge's importance of any networks. On applying PROMETHEE, the MCDM tool, we went on to prioritize and rank the edges. Coding PROMETHEE in R gave a framework with which we could rank the edges based on the flow and topology within a network. Validating the produced model with simulated existed systems proved that the suggested model works. As an extension of this project different MCDM tools can be applied. The results of which can be analysed to compare the performance of recovery.

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