

A Control Policy for a Multi-Stage Remanufacturing System

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Abstract

In this paper, we study a single-item multi-stage remanufacturing inventory system using stochastic decomposition process. We extend the classic continuous inventory control policy to develop a control policy for this remanufacturing system. We use queuing theory techniques to derive steady-state probabilities and, consequently, the closed-form cost function of the remanufacturing system is estimated. A main contribution of this paper is providing a closed-form cost function for a multi-stage remanufacturing system which enables developing solution algorithm to find the optimal policy. Furthermore, a simulated annealing algorithm is proposed to find near optimal values of the inventory control policy parameters. Finally, sensitivity of the derived model to some parameters is analyzed.

Keywords: Reverse Logistics, Remanufacturing System, Inventory Control, Queuing System, Simulated Annealing.

1. Introduction

In recent years, manufacturers have paid growing attention to reuse activities that provide material waste reduction via the recovery of some content of used products. Motivation behind these product recovery activities is two-fold: growing environmental concerns and potential economic benefits (Ahiska, 2008). These reuse processes and related activities are studied in terms of reverse logistics.

Reverse logistics is the process of efficiently planning, implementing, and controlling inbound flow and storage of secondary items and related information opposite to the traditional supply chain direction for the purpose of recovering value or suitable disposal (Beltran, 2002; Fleischmann, 1997). Remanufacturing is a typical example for economically attractive reuse activities. Remanufacturing transforms used products into like new products. After disassembly, modules and parts are extensively inspected and problematic parts are repaired, or if not possible, replaced with new parts (Ahiska, 2008).

The literature on inventory control and production planning in reverse logistics is reviewed by Fleischmann *et al.* (1997). They stated that recovery process effect three main areas, namely distribution planning, inventory control, and production planning. This paper belongs to the inventory control area. Due to uncertainty of product returns and coordination of product recovery option with regular procurement, inventory control for recoverable manufacturing systems is more complex than traditional systems (Inderfurth and van der Laan, 2001).

Papers such as Dobos and Richter (2006), Heyman (1977), Inderfurth and van der Laan (2001), Kiesmuller and van der Laan (2001), van der Laan *et al.* (1996), van der Laan and Salomon (1997), van der Laan and Teunter (2006), Muckstadt and Isaac (1981), Ouyang and Zhu (2006), Roy *et al.* (2009), Tang and Grubbstrom (2005), and Bagherpour *et al.* (2009). Each model in the literature differs from other models with respect to its assumptions. Assumptions must be made on the demand and return processes, existence of a disposal option for returned items, relevant cost parameters, lead times of production and remanufacturing activities, and the length of planning horizon.

The physical design of the remanufacturing system we study in this paper is based Takahashi *et al.* (2007) who studied a remanufacturing system with a stochastic decomposition process (see Fig. 1). In the decomposition process, a returned item is disposed or decomposed into parts or materials. The recovered materials and parts are stored to be used in the production process. For this remanufacturing system, Takahashi *et al.* (2007) developed a Markov chain

model and flow balance equations. Takahashi *et al.* (2007) stated that they used the numerical methods presented in Bolch *et al.* (1998) in order to solve the flow balance equations. To the best of our knowledge, the average cost of a remanufacturing system with stochastic decomposition process is not yet derived in the form of a closed-form function.

To determine the steady-state probabilities of finite Markov chains, three different approaches are commonly used: direct or iterative numerical methods and techniques that yield closed-form results. Direct methods operate and modify the parameter matrix. They use a fixed amount of computation time independent of the parameter values and there is no issue of convergence (Bolch *et al.*, 1998). Among the techniques most commonly applied, Gaussian Elimination (GE) and Grassmann Algorithm (GA) (Kumar *et al.*, 1987) can be named. Whereas direct methods yield exact results, iterative methods are generally more efficient, both in time and space. So, they are commonly used for larger models (Bolch *et al.*, 1998). Power Method, Jacobi's Method, and Gauss-Seidel Method are some examples of iterative methods (for more details see Bolch *et al.*, 1998 and Stewart, 1994). Though closed-form of steady-state probabilities are highly advantageous, they can be obtained for only a small class of models and normally the steady-state probabilities are not interpreted as closed-form functions for Markov chains with a more general structure (see for example Pourghannad, 2013, and Bagherpour *et al.*, 2009).

To the best of our knowledge, for the first time in the literature, this paper develops the closed-form steady-state probabilities for a single-item remanufacturing system with stochastic decomposition process. Afterwards, the closed-form steady-state probabilities are used to estimate the system cost function. The closed-form cost function has some advantages over numerical methods. Firstly, numerical algorithms have round-off errors resulting from finite precision arithmetic (Bolch *et al.*, 1998). Secondly, the closed-form version of cost function is valuable because it provides a base to analyze the cost function more efficiently. In addition, the closed-form cost function enhances time efficiency of possible optimization algorithms.

In our paper, the traditional continuous review policy is developed and a $(Q, \alpha, \beta, \gamma)$ policy is proposed. Then, queuing system analysis is implemented to derive the closed-form cost function of the understudy remanufacturing system under proposed policy. Simulation experiments are used in order to evaluate the performance of estimated closed-form cost function. Statistical hypothesis test with 95% confidence level reveals that, statistically, there is no significant difference between the proposed cost function and simulation results.

Main contribution of this paper is to present the closed-form cost function. Furthermore, to determine the values of inventory control policy parameters in derived optimization problem, we apply a widely available and easily programmed metaheuristic procedure, Simulated Annealing (SA). We compared results obtained by our SA with Genetic Algorithm which is used extensively in the literature for remanufacturing problems and other supply chain applications (see for example Roy *et al.* (2008), Pourghannad *et al.* (2015) and Aliabadi *et al.* (2013)). This comparison shows that our proposed SA provides the solutions that are reliable. Consequently, via extensive simulation experiments, we recognized that our proposed estimated cost function is accurate, and the proposed SA is a convenient, reliable, and effective approach for practical application.

The rest of this paper is organized as follows. In Section 2 notations are introduced and problem definition is presented. The proposed queuing system analysis which is used to derive cost function of inventory remanufacturing model is presented in Section 3. In addition, in Section 4, the SA method to find near optimal value of estimated cost function is explained. The accuracy of estimated cost function and performance of the proposed solution method are evaluated by using simulation in Section 5. Sensitivity of model to some parameters is surveyed in section 6. Finally, in section 7, conclusion and some suggestions for future research are presented.

2. Problem Description and Notations

We study a remanufacturing system which consists of three processes: decomposing, producing parts, and producing products. Also, there are three storages for storage of materials, parts, and products, as shown in Fig. 1. We assume (1) There is one single finished product, (2) by decomposing a returned item, either a part or a material is obtained, or the returned item is disposed, (3) unsatisfied demands are lost, (4) for raw material, there is a purchasing process with lot size equal to Q and zero lead time, and (5) the storages have limited capacities. The capacities of material, part, and product storages are α , β , and γ , respectively. These assumptions are standard assumptions which are used in most of inventory problems (see Frenk *et al.* (2014) for general discussions)

The vector $(Q, \alpha, \beta, \gamma)$ represents inventory control policy and is defined as follows. Materials are purchased with lot size Q when the system runs out of raw material. Parts are produced, unless the stock of parts reaches the upper limit β , or the stock of materials

runs out. Products are produced, unless the stock of products reaches the upper limit γ , or the stock of parts runs out. When the stock of materials reaches the upper limit α , the recovered materials will be disposed; also, when the stock of parts reaches the upper limit β , the recovered parts will be disposed.

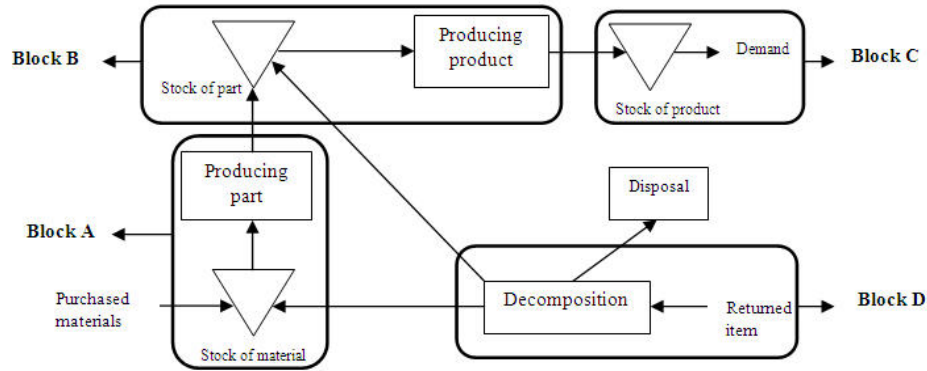


Fig. 1. Schematic model of remanufacturing

Our mathematical model is developed based on the following notations.

λ_d : Demand rate per unit time, according to a Poisson process

λ_r : Recovery rate per unit time, according to a Poisson process

M_p : Mean time for producing each product, according to an Exponential process

M_{pa} : Mean time for producing each part, according to an Exponential process

λ_{dis} : Rate of disposal of decomposed recovered items per unit time, according to a Poisson process

λ_m : Rate of reuse of decomposed recovered items as material per unit time, according to a Poisson process

λ_{pa} : Rate of reuse of decomposed recovered items as part per unit time, according to a Poisson process

α : Capacity of material storage

β : Capacity of part storage

γ : Capacity of product storage

$\pi^p(x)$: Steady-state probability of having x units of products on hand

$\pi^{pa}(x)$: Steady-state probability of having x units of parts on hand

$\pi^m(x)$: Steady-state probability of having x units of materials on hand

$L_{q|M/M/y/z}$: Average number of q (product/part/material in storage) when queue system is $M/M/y/z$

h_{pa} : Part holding cost per unit per unit time

h_p : Product holding cost per unit per unit time

h_m : Material holding cost per unit per unit time

S : Shortage cost per unit at the product storage

D : Disposal cost per unit

c_i : Expected cost related to block i ; $i = A, B, C$, and D

C_{total} : Expected total cost

r : Utilization factor that expresses the proportion of time the system is busy and is equal to the ratio of system output rate to its input rate.

3. Cost Function Derivation

The total cost equals to the summation of costs in blocks A , B , C , and D , i.e. $C_{total} = c_A + c_B + c_C + c_D$. The total cost includes holding costs in all the three mentioned storages, shortage cost in block D , and disposal cost in block A .

Lemma 1: c_A that represents the expected inventory cost in block A can be presented as $c_A = E(x) \times h_m$ with $E(x) = \sum_{x=1}^{\alpha} x \cdot \pi^m(x)$ and $\pi^m(x) = (1 - \pi^{pa}(\beta)) \times \pi^m(x|\text{part storage is not full}) + \pi^{pa}(\beta) \times \pi^m(x|\text{part storage is full})$. Where, $\pi^{pa}(\beta)$ is the steady-state probability of fullness of part storage and is calculated according to Eq. A.3 (see Appendix). We have

$$\pi^m(x|\text{part storage is not full}) = \begin{cases} \frac{1-r}{Q-r^{\alpha-Q+1} \sum_{i=0}^{Q-1} r^i} & x=1 \\ \sum_{i=0}^{x-1} r^i \pi^m(1) & 2 \leq x \leq Q \\ r^{x-Q} \sum_{i=0}^{Q-1} r^i \pi^m(1) & Q+1 \leq x \leq \alpha \end{cases}$$

where r is the utilization factor and is equal to $\lambda_m M_{pa}$ and, in the steady-state, if part storage is full, r tends to infinity. Therefore,

$$\pi^m(x|\text{part storage is full}) = \begin{cases} 0 & 1 \leq x \leq \alpha - 1 \\ 1 & x = \alpha \end{cases}$$

Proof: For brevity, we only outline the main steps which should follow to establish the results in this lemma. First, the steady-state probability of occurrence of this state is represented by $\pi^m(x|\text{part storage is not full})$. Second partition refers to the case in which part storage is full. So, the output rate of block A is zero, and consequently r tends to infinity. The steady-state probability of occurrence of this state is represented by $\pi^m(x|\text{part storage is full})$. The inventory on hand is an irreducible continuous time Markov chain, the proof of which is presented by Ouyang and Zhu (2006) and also Bagherpour *et al.*, (2009). This continuous time Markov chain has a unique steady-state probability $\pi^m(x)f_x = \sum_{y=1}^{\alpha} \pi^m(y)f_{yx}$. The steady-state probabilities can be obtained as follow.

$$\pi^m(x) = \begin{cases} \frac{1-r}{Q-r^{\alpha-Q+1} \sum_{i=0}^{Q-1} r^i} & x=1 \\ \sum_{i=0}^{x-1} r^i \cdot \pi^m(1) & 2 \leq x \leq Q \\ r^{x-Q} \sum_{i=0}^{Q-1} r^i \cdot \pi^m(1) & Q+1 \leq x \leq \alpha \end{cases}$$

Lemma 2: c_B can be written as follows.

$$c_B = \pi^p(\gamma) \times \beta \times h_{pa} + (1 - \pi^p(\gamma)) \times L_{q|M/M/1/(\beta+1)} \times h_{pa}.$$

where $\pi^p(\gamma)$ that is the steady-state probability of fullness of the product storage is calculated according to Eq. A.1 (for more details see Appendix).

Proof: We use $M/M/1/(\beta+1)$ queuing system with proper input and output rates and solve for the limiting probabilities. By using conditional probability for these two partitions, the expected cost for block B can be expressed as follows

$$\begin{aligned} c_B &= [P(\text{Case 1}) \times E[\text{Inventory on hand in part storage} | \text{Case 1}] \\ &+ P(\text{Case 2}) \\ &\times E[\text{Inventory on hand in part storage} | \text{Case 2}]] \times h_{pa}. \end{aligned}$$

If the product storage is full, i.e. the first case, then,

$$L_q = \lim_{r \rightarrow \infty} \left(\frac{r}{1-r} - \frac{(\beta+2)r^{\beta+2}}{1-r^{\beta+2}} - \frac{r(1-r^{\beta+1})}{1-r^{\beta+2}} \right) = \beta.$$

By substituting the corresponding values, lemma 2 can be easily obtained.

Lemma 3: c_C can be written as follows.

$$c_C = (1 - \pi^{pa}(0)) \times L_{q|M/M/1/(\gamma+1)} \times h_p + S \times \pi^p(0) \times \lambda_d.$$

where $\pi^{pa}(0)$ and $\pi^p(0)$ are calculated according to Eq. A.2 and Eq. A.4 respectively (see Appendix).

Proof: Similar to lemma 2 for calculating c_B , all possible states can be divided into two partitions, based on the fact that part storage is empty, with the probability of $\pi^{pa}(0)$, or not empty, with the probability of $1 - \pi^{pa}(0)$. Completing the steps lead to lemma 3.

Lemma 4: c_D can be written as follows:

$$c_D = D \times (\lambda_{dis} + \pi^m(\alpha) \times \lambda_m + \pi^{pa}(\beta) \times \lambda_{pa}).$$

where $\pi^m(\alpha)$ and $\pi^{pa}(\beta)$ are calculated according to lemma 1 and Eq. A.3, respectively.

Proof: Since the mean time for decomposing returned items is assumed to be negligible, cost of block D only includes disposal costs. Therefore, there is no inventory on hand in this block. As mentioned in section 2, the returned items can be disposed under three circumstances: first, when the returned items are not recoverable and they will be disposed with the rate of λ_{dis} ; second, when the material storage is full with probability of $\pi^m(\alpha)$, the recovered item is disposed with the rate of $\pi^m(\alpha) \times \lambda_m$, and thirdly, when the part storage is full with probability of $\pi^{pa}(\beta)$, the recovered item is disposed with the rate of $\pi^{pa}(\beta) \times \lambda_{pa}$. Therefore, the expected total cost of this block can be calculated by multiplying the summation of these three disposal rates by disposal cost per unit (D).

Lemma 5: The total cost of the remanufacturing system under proposed $(Q, \alpha, \beta, \gamma)$ inventory control policy can be expressed as the summation of the related costs of four blocks and is formulated as follows:

$$Z = E(x) \times h_m + \pi^p(\gamma) \times \beta \times h_{pa} + (1 - \pi^p(\gamma)) \times L_{q|M/M/1/(\beta+1)} \times h_{pa} + (1 - \pi^{pa}(0)) \times L_{q|M/M/1/(\gamma+1)} \times h_p + S \times \pi^p(0) \times \lambda_d + D \times (\lambda_{dis} + \pi^m(\alpha) \times \lambda_m + \pi^{pa}(\beta) \times \lambda_{pa}).$$

Proof: Putting all cost function for blocks we have calculated we obtained the total cost function above.

4. Solution Procedure

In this section, we develop a Simulated Annealing (SA) based heuristic to obtain near optimal parameters of inventory control policy of proposed cost function. A solution of our problem in the proposed SA is a vector consisted of inventory control policy parameters, namely α , β , γ , and Q . The scheme of proposed SA algorithm to find values of inventory control policy parameters for our remanufacturing problem is depicted in Table 1. In steps 1 and 2, the initial solution and initial temperature are determined. Then, the new solutions are created at each temperature with the defined neighborhood function explained in step 5. Number of new solutions in each temperature is equal to the number of iterations defined by $T/(1 + 0.2 * T)$ (for more details see Lundy and Mees, 1986). If the new solution satisfies the conditions of step 7, it is considered as the input of neighborhood function in step 5-a. Furthermore, according to step 9, the temperature decreases with the rate c and when the condition in step 10 is satisfied, the algorithm terminates.

5. Computational Results

In subsection 5.1, validation of proposed estimated cost function is investigated. Then, in subsection 5.2, performance of the proposed SA is evaluated by comparing its results with near optimal values obtained by a Genetic Algorithm (GA) proposed by Roy *et al.* (2008) for a remanufacturing problem.

The real GA is used with following parameters: population size = 30, probability of crossover = 0.2, probability of mutation = 0.1, and maximum number of generation = 300. Similar to our proposed SA, these parameters are selected by trial and error to obtain good performance. The SA procedure was run with the following set of parameters: initial temperature: 0.5, cooling schedule rule: exponential, $T_{K+1} = 0.9T_K$, the values of decreasing and increasing: 1, initial inventory policy parameters are values considered to be $\alpha = \beta = \gamma = Q = 1$. The values of the mentioned parameters and cooling schedule rule are selected by trial and error through experiments to tune the algorithm for a good performance.

To construct scenarios, we consider five parameters, namely, material inventory cost (h_m), part inventory cost (h_{pa}), product inventory cost (h_p), shortage cost (S), and disposal cost (D), to be changeable. Other parameters are assumed to be constant and are set as follows $\lambda_m=2.5$; $\lambda_{pa}=2.5$; $\lambda_{dis}=1$; $\lambda_d=6$; $M_p=5$; and $M_{pa}=4$. We assign values randomly

selected from integers between 1 and 20 to the five changeable parameters. Using this method, 30 different scenarios are randomly obtained and each scenario is labeled by (S, D, h_m, h_p, h_{pa}) .

Table 1. Simulated annealing algorithm for the inventory control policy parameters optimization.

1.	[Set initial control policy parameters configuration.] Set $\alpha \leftarrow l$, $\beta \leftarrow l$, $\gamma \leftarrow l$, and $Q \leftarrow l$.
2.	[Set initial temperature T_0 .] Set $T \leftarrow 0.5$.
3.	[Initialize step for number of iterations for terminating algorithm.] Set $S \leftarrow 0$.
4.	[Initialize step for number of iterations in each temperature.] Set $I \leftarrow 0$.
5.	[Create new configurations of control policy parameters.]
a)	[Create a copy of control policy parameters configuration.] Set $\alpha' \leftarrow \alpha$, $\beta' \leftarrow \beta$, $\gamma' \leftarrow \gamma$, and $Q' \leftarrow Q$.
b)	[Create 2 numbers: increase value and decrease value.] Set increase value $\leftarrow u$, and decrease value $\leftarrow l$.
c)	[Determine a parameter, among four control policy parameters (α , β , γ , and Q), to modify.] Set $Z_v \leftarrow [\text{rand}(1 \dots 4)]$.
d)	[Determine whether the selected parameter will increase or decrease.] Set $b \leftarrow [\text{rand}(0,1)]$.
e)	If $Z_v = 1$, then if $b = 1$; set $\alpha \leftarrow \alpha + u$, if $b = 0$; set $\alpha \leftarrow \alpha - l$, In addition, if $\alpha \leq 0$; set $\alpha \leftarrow 1$.
f)	If $Z_v = 2$, then if $b = 1$; set $\beta \leftarrow \beta + u$, if $b = 0$; set $\beta \leftarrow \beta - l$, In addition, if $\beta \leq 0$; set $\beta \leftarrow 1$.
g)	If $Z_v = 3$, then if $b = 1$; set $\gamma \leftarrow \gamma + u$, if $b = 0$; set $\gamma \leftarrow \gamma - l$, In addition, if $\gamma \leq 0$; set $\gamma \leftarrow 1$.
h)	If $Z_v = 4$, then if $b = 1$; set $Q \leftarrow Q + u$, if $b = 0$; set $Q \leftarrow Q - l$, In addition, if $Q > \alpha$; set $Q \leftarrow \alpha$.
6.	[Calculate energy differential.] Set $\Delta E \leftarrow Z(\alpha, \beta, \gamma, Q) - Z(\alpha', \beta', \gamma', Q')$.
7.	[Decide upon acceptance of new configuration.] Accept new configuration, if $\Delta E > 0$, or, following the Boltzmann probability distribution, if $\Delta E < 0$ and $\exp\left(\frac{-\Delta E}{T}\right) > \text{rand}(0 \dots 1)$, set $\alpha \leftarrow \alpha'$, $\beta \leftarrow \beta'$, $\gamma \leftarrow \gamma'$, and $Q \leftarrow Q'$.
8.	[Repeat for current temperature.] Set $I \leftarrow I + 1$. If $I < \text{maximum number of iterations in each temperature}$, go to step 4.
9.	[Lower the annealing temperature.] Set $T \leftarrow c \times T$ ($0 < c < 1$).
10.	[Check if progress has been made.] Set $S \leftarrow S + 1$. If $S < \text{maximum number of iterations}$, go to step 4; otherwise, stop.

We express our finding in Table 2 using %deviation between two methods' cost function in which % deviation $= \frac{|Z_{SA} - Z_{GA}|}{Z_{GA}}$, where the cost function obtained from SA is denoted by Z_{SA} and the cost function obtained from genetic algorithm is denoted by Z_{GA} . We use the following statistical hypothesis test to compare our proposed SA with the real GA.

$$\begin{cases} H_0: \mu_{Z_{SA}} \leq \mu_{Z_{GA}} \\ H_1: \mu_{Z_{SA}} > \mu_{Z_{GA}} \end{cases}$$

The result of proposed statistical hypothesis test indicates a failure to reject the null hypothesis at 95% confidence level. Therefore, it is inferred that the proposed SA works better than GA. Also, based on the above statistical

hypothesis test, there is no significant difference between the results of two mentioned methods. Consequently, SA is a convenient, reliable, and effective approach for our problem.

Table 2. The results obtained by SA and GA

Scenario Number	Scenario	SA			GA			% deviation
		Z_{SA}	$(\alpha_{SA}, \beta_{SA}, \gamma_{SA}, Q_{SA})$	time	Z_{GA}	$(\alpha_{GA}, \beta_{GA}, \gamma_{GA}, Q_{GA})$	time	
1	(14,3,7,11,10)	70.7035	(2,2,3,1)	38.08	74.1904	(2,3,3,2)	37.08	4.699934
2	(13,9,5,12,14)	85.7106	(2,2,3,1)	36.36	88.552	(3,2,4,2)	36.94	3.208736
3	(6,19,13,7,5)	93.9671	(2,2,4,1)	37.83	94.0022	(2,2,3,1)	36.75	0.03734
4	(4,17,14,11,7)	90.9575	(2,2,2,1)	37.73	101.6422	(3,2,5,2)	37.28	10.51207
5	(2,2,8,7,4)	29.8276	(1,2,1,1)	40.84	31.4553	(1,2,3,1)	38.02	5.174645
6	(3,14,6,1,7)	61.483	(3,3,4,1)	36.00	64.4805	(6,2,5,2)	34.67	4.648692
7	(3,7,6,11,14)	50.6801	(2,1,2,1)	34.65	53.8952	(3,1,2,2)	35.51	5.965466
8	(16,9,5,5,3)	65.9818	(3,3,7,1)	36.14	67.0597	(4,4,6,1)	36.17	1.607374
9	(4,9,3,3,11)	49.5954	(3,2,6,1)	35.68	51.9829	(3,2,2,1)	36.07	4.592856
10	(18,17,15,14,6)	122.5864	(2,3,4,1)	30.70	132.904	(2,3,5,2)	36.38	7.763197
11	(10,4,6,11,13)	65.9017	(2,2,3,1)	33.64	68.3731	(3,2,2,1)	36.13	3.614579
12	(10,9,2,3,12)	60.0421	(4,2,8,1)	34.70	62.8742	(5,3,6,1)	36.94	4.504391
13	(1,14,13,2,14)	67.0154	(2,1,1,1)	33.86	74.6019	(2,1,5,2)	36.12	10.16931
14	(13,7,8,13,11)	82.8988	(2,2,3,1)	37.41	83.0391	(2,2,2,1)	35.82	0.168957
15	(12,1,4,13,4)	48.1349	(1,3,2,1)	37.12	50.32	(1,4,2,1)	35.81	4.342409
16	(18,2,7,5,11)	68.8674	(1,3,7,1)	37.90	72.6353	(1,3,3,1)	35.33	5.187423
17	(14,2,13,11,14)	78.157	(1,2,3,1)	36.70	86.4889	(2,3,4,1)	36.30	9.633491
18	(10,2,13,2,12)	56.99	(1,2,9,1)	37.09	73.4604	(3,2,3,2)	36.18	22.42079
19	(20,14,6,4,3)	80.5711	(3,4,10,1)	34.88	88.5711	(4,4,4,2)	35.72	9.032292
20	(5,12,4,10,13)	69.5019	(3,2,3,1)	38.30	69.6883	(3,1,4,1)	35.73	0.267477
21	(7,19,14,11,1)	94.3215	(2,4,3,1)	36.90	94.6442	(2,5,3,1)	35.46	0.340961
22	(13,19,13,12,3)	108.2062	(2,3,3,1)	37.32	113.2094	(4,3,4,1)	32.84	4.419421
23	(4,12,11,2,15)	73.311	(2,2,9,1)	37.58	80.0305	(3,2,5,2)	36.39	8.396174
24	(7,4,1,9,5)	39.317	(3,2,2,1)	38.95	41.231	(6,3,3,1)	36.71	4.642138
25	(19,10,2,13,3)	77.1694	(4,3,4,1)	39.28	79.4728	(2,3,3,1)	36.47	2.89835
26	(6,1,9,7,4)	36.657	(1,2,2,1)	37.40	37.4607	(1,3,2,1)	36.56	2.145448
27	(17,20,4,6,7)	95.5097	(4,3,7,1)	38.82	103.4197	(4,2,5,2)	37.08	7.648446
28	(17,4,15,11,3)	78.333	(1,3,3,1)	40.60	83.1584	(1,5,6,1)	36.88	5.802661
29	(7,3,1,12,4)	37.2784	(3,2,1,1)	40.32	38.2096	(5,2,2,2)	36.70	2.437084
30	(15,12,2,10,13)	86.8729	(4,2,4,1)	40.91	90.3022	(4,3,5,2)	39.26	3.797582

6. Sensitivity Analysis

In this section, sensitivity of the model to shortage cost, holding costs, disposal cost and demand rate is analyzed. To do these analyses, we assume the following parameters to be fixed as follows: $h_m=2$, $h_{pa}=4$, $h_p=5$, $S=5$, $D=5$, $\lambda_m=2.5$, $\lambda_{pa}=2.5$, $\lambda_{dis}=1$, $\lambda_d=6$, $M_p=5$, and $M_{pa}=4$. Parameters S and D are two other important parameters to analyze. The results of increasing in these two parameters are depicted in Fig. 2.

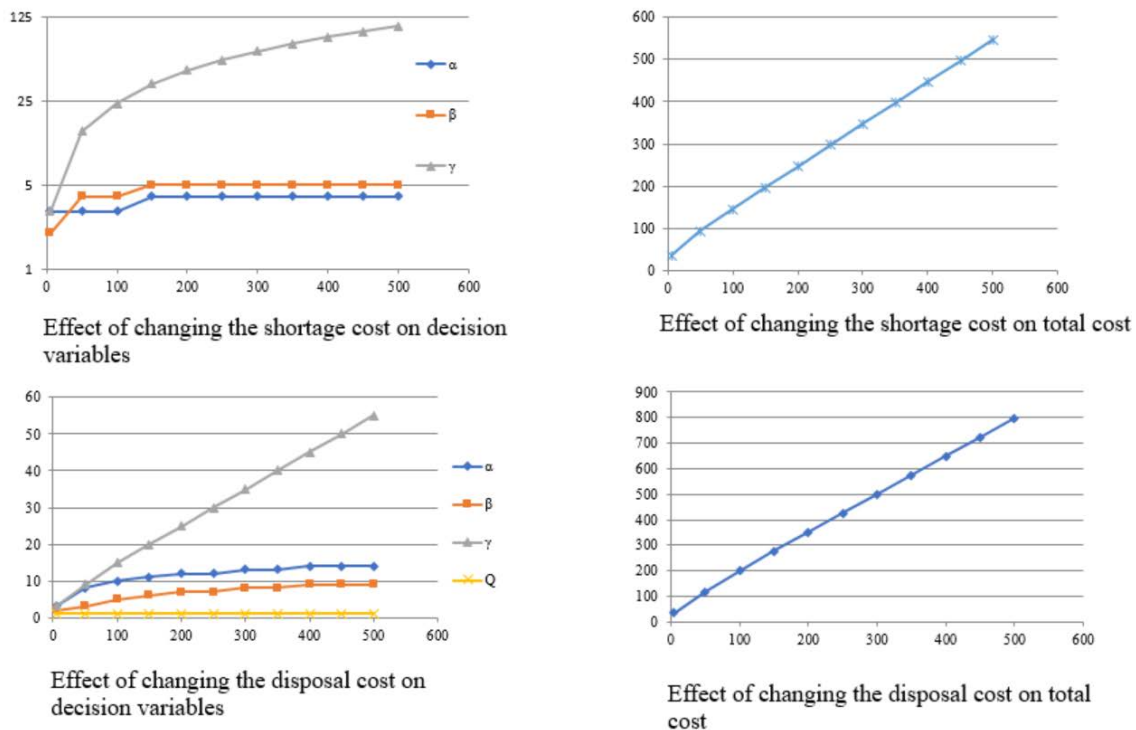


Fig 2. Effect of changing parameters S and D on decision variables (α , β and γ) and total cost

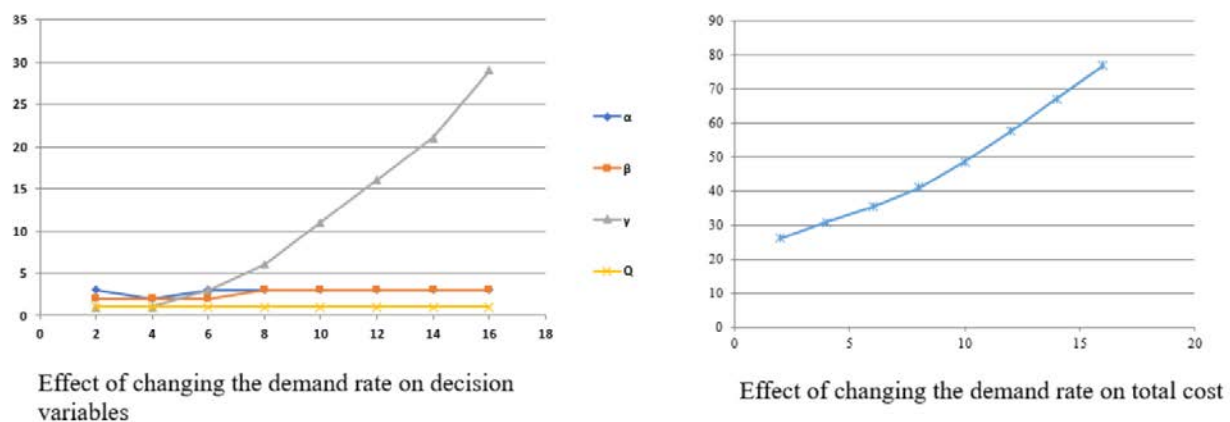


Fig 3. Effect of changing parameters λ_d on decision variables (α , β and γ) and total cost

When S increases the remanufacturing system needs to hold more inventory at the product storage, so the optimum value of γ increases to overcome the high shortage cost. When disposal cost per item increases the optimum values of α , β , and γ increase. Obviously, when D increases the remanufacturing system tends to decrease the number of disposed items. To aim this goal, the system tries to decrease the probability of fullness in the martial storage and part storage by increasing the capacities of these storages. Consequently, the optimum values of all three storages increase. The

increase in γ is also decreases the probability of fullness in the martial storage and part storage. It is because when γ increases the portion of time that the part storage is full due to the fullness of product storage decrees.

Fig. 3 depict the effect of changing in λ_d . When demand intensity increases, the remanufacturing system tends to hold more inventory in the product storage, similar to the case that S increases. So, it mostly affects γ . It is worth mentioning that the increase in λ_d has no effect on α , because lead time is assumed to be negligible.

7. Conclusion

In this paper, we investigated a remanufacturing inventory system with stochastic decomposition process. The understudy remanufacturing system is a queuing network system that its close-form steady-state probabilities are unknown in the literature. The main contributions of this paper to the literature are developing the closed-form steady-state probabilities and as a result the closed-form cost function of this remanufacturing system. In stochastic inventory models it is always a challenge to derive the closed-form cost function (Sajadifar and Pourghannad, 2012). Having the closed-form cost function on-hand not only overcomes the round-off errors of numerical examples, but also it facilitates the analysis of cost function and it also increases the efficiency of possible algorithms to optimize the cost function by reducing the run time. To formulate the closed-form cost function of this system, we partitioned the system into four blocks. The cost function of each block is formulated by using conditional probability and queuing network analyses. Finally, the cost function of the system is derived by adding up the cost function of four blocks.

Furthermore, an SA algorithm is developed to generate the near optimum values of inventory control policy. Through the comparison between our proposed SA and an existing real GA developed by Roy et al. (2008), we proved that the answers provided by SA are reliable. Future studies may focus on more advanced solution procedures such as Neural network (Avsar and Aliabadi, 2015), agent-based algorithms (Aliabadi *et al.*, 2017). Besides, in section 6 some numerical investigations are presented to provide some managerial insight to the problem. The results indicate that when shortage cost or the demand rate rises, increase of the capacity of product storage is recommended to keep total operation cost as low as possible. Also, to overcome the effect of increase of disposal cost, one needs to increase the capacities of all storages.

For future work in this area, researches can focus on deriving the cost function with considering setup costs for part and product manufacturing processes. Also, considering the non-zero lead time for material purchase process is another interesting issue. Furthermore, the current works usually focus only on inventory models. One interesting direction is integrating the inventory model with finding the optimal location for warehouses (for example by a similar model as in Shahraki *et al.*, 2015). Given that return of product induces more transportation, dose the manufacturing really reduce the emissions and lead to sustainability? To do this, one needs to model the supply chain transportation network and its emissions (see Shahraki and Turkay, 2014).

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Appendix

In this appendix, values of $\pi^p(\gamma)$, $\pi^{pa}(0)$, $\pi^{pa}(\beta)$, and $1 - \pi^p(0)$ that are used to formulate the closed-form total cost function of understudy remanufacturing system are calculated. Also, the approach used to derive these steady-state probabilities is explained.

The steady-state probabilities of $\pi^p(\gamma)$, $\pi^{pa}(0)$, $\pi^{pa}(\beta)$, and $1 - \pi^p(0)$ are calculated as follows:

$$\pi^p(\gamma) = \frac{A \times C}{1 - A \times B + A \times C} \quad (\text{A.1})$$

$$\pi^{pa}(0) = 1 - \frac{\pi^p(\gamma)}{A} \quad (A.2)$$

$$\pi^{pa}(\beta) = (1 - \pi^p(\gamma)) \times E \quad (A.3)$$

$$1 - \pi^p(0) = \pi^{pa}(0) \times F + (1 - \pi^{pa}(0)) \times G \quad (A.4)$$

$$\begin{aligned} \text{where } A = \pi^p\left(\gamma \middle| \lambda = \frac{1}{M_p}, \mu = \lambda_d\right) &= \frac{\left(1 - \frac{1}{\lambda_d M_p}\right) \left(\frac{1}{\lambda_d M_p}\right)^\gamma}{1 - \left(\frac{1}{\lambda_d M_p}\right)^{\gamma+2}}, B = 1 - \pi^{pa}\left(0 \middle| \lambda = \lambda_{pa} + \frac{1}{M_{pa}}, \mu = 0\right) = 1, \\ C = 1 - \pi^{pa}\left(0 \middle| \lambda = \lambda_{pa} + \frac{1}{M_{pa}}, \mu = \frac{1}{M_p}\right) &= 1 - \frac{\left(1 - \frac{\lambda_{pa} + \frac{1}{M_{pa}}}{\frac{1}{M_p}}\right)}{1 - \left(\frac{\lambda_{pa} + \frac{1}{M_{pa}}}{\frac{1}{M_p}}\right)^{\beta+2}}, D = \pi^{pa}\left(\beta \middle| \lambda = \lambda_{pa} + \frac{1}{M_{pa}}, \mu = 0\right) = 0, \\ E = \pi^{pa}\left(\beta \middle| \lambda = \lambda_{pa} + \frac{1}{M_{pa}}, \mu = \frac{1}{M_p}\right) &= \frac{\left(1 - \frac{\lambda_{pa} + \frac{1}{M_{pa}}}{\frac{1}{M_p}}\right) \left(\frac{\lambda_{pa} + \frac{1}{M_{pa}}}{\frac{1}{M_p}}\right)^\beta}{1 - \left(\frac{\lambda_{pa} + \frac{1}{M_{pa}}}{\frac{1}{M_p}}\right)^{\beta+2}}, F = 1 - \pi^p(0 \middle| \lambda = 0, \mu = \lambda_d) = 1, \\ G = 1 - \pi^p\left(0 \middle| \lambda = \frac{1}{M_p}, \mu = \lambda_d\right) &= 1 - \frac{\left(1 - \frac{1}{\lambda_d M_p}\right)}{1 - \left(\frac{1}{\lambda_d M_p}\right)^{\gamma+2}}. \end{aligned}$$

Proof: In the understudy remanufacturing system, as one can infer from Fig 1, input and output rates of each block depends on the fullness or emptiness of other blocks. Considering this fact, steady-state probabilities can be formulated by conditioning on the fullness or emptiness of the beforehand or afterward storage. In this appendix, by using conditional probability, we compute $\pi^p(\gamma)$, $\pi^{pa}(0)$, $\pi^{pa}(\beta)$, and $1 - \pi^p(0)$ as shown by Eqs. (A.5) to (A.8). Other steady-state probabilities can be obtained directly using classical queuing theory, e.g. $\pi^p(\gamma | \text{part storage is not empty})$ is calculated from the existing closed-form relations for steady-state probabilities of a classic $M/M/1/\gamma$ queuing system.

$$\pi^p(\gamma) = (1 - \pi^{pa}(0)) \times \pi^p\left(\gamma \middle| \begin{array}{l} \text{part storage} \\ \text{is not empty} \end{array}\right) + \pi^{pa}(0) \times \pi^p\left(\gamma \middle| \begin{array}{l} \text{part storage} \\ \text{is empty} \end{array}\right) \quad (A.5)$$

$$\begin{aligned} 1 - \pi^{pa}(0) &= \pi^p(\gamma) \times \pi^{pa}\left(\begin{array}{l} \text{part storage} \\ \text{is not empty} \end{array} \middle| \begin{array}{l} \text{product storage} \\ \text{is full} \end{array}\right) + \\ &\quad (1 - \pi^p(\gamma)) \times \pi^{pa}\left(\begin{array}{l} \text{part storage} \\ \text{is not empty} \end{array} \middle| \begin{array}{l} \text{product storage} \\ \text{is not full} \end{array}\right) \end{aligned} \quad (A.6)$$

$$\pi^{pa}(\beta) = \pi^p(\gamma) \times \pi^{pa}\left(\beta \middle| \begin{array}{l} \text{product storage} \\ \text{is full} \end{array}\right) + (1 - \pi^p(\gamma)) \times \pi^{pa}\left(\beta \middle| \begin{array}{l} \text{product storage} \\ \text{is not full} \end{array}\right) \quad (A.7)$$

$$\begin{aligned} 1 - \pi^p(0) &= \pi^{pa}(0) \times \pi^p\left(\begin{array}{l} \text{product storage} \\ \text{is not empty} \end{array} \middle| \begin{array}{l} \text{part storage} \\ \text{is empty} \end{array}\right) + \\ &\quad (1 - \pi^{pa}(0)) \times \pi^p\left(\begin{array}{l} \text{product storage} \\ \text{is not empty} \end{array} \middle| \begin{array}{l} \text{part storage} \\ \text{is not empty} \end{array}\right). \end{aligned} \quad (A.8)$$

where,

$$\pi^p\left(\gamma \middle| \begin{array}{l} \text{part storage} \\ \text{is not empty} \end{array}\right) \rightarrow \pi^p\left(\gamma \middle| \lambda = \frac{1}{M_p}, \mu = \lambda_d\right); \text{ Let's call this value } A.$$

$$\pi^p\left(\gamma \middle| \begin{array}{l} \text{part storage} \\ \text{is empty} \end{array}\right) \rightarrow \pi^p(\gamma | \lambda = 0, \mu = \lambda_d); \text{ This value is equal to } 1.$$

$$\pi^{pa} \left(\begin{array}{c|c} \text{part storage} & \text{product storage} \\ \text{is not empty} & \text{is full} \end{array} \right) \rightarrow 1 - \pi^{pa} \left(0 \middle| \lambda = \lambda_{pa} + \frac{1}{M_{pa}}, \mu = 0 \right); \text{ Let's call this value } \mathbf{B}.$$

$$\pi^{pa} \left(\begin{array}{c|c} \text{part storage} & \text{product storage} \\ \text{is not empty} & \text{is not full} \end{array} \right) \rightarrow 1 - \pi^{pa} \left(0 \middle| \lambda = \lambda_{pa} + \frac{1}{M_{pa}}, \mu = \frac{1}{M_p} \right); \text{ Let's call this value } \mathbf{C}.$$

$$\pi^{pa} \left(\beta \middle| \begin{array}{c} \text{product storage} \\ \text{is full} \end{array} \right) \rightarrow \pi^{pa} \left(\beta \middle| \lambda = \lambda_{pa} + \frac{1}{M_{pa}}, \mu = 0 \right); \text{ Let's call this value } \mathbf{D}.$$

$$\pi^{pa} \left(\beta \middle| \begin{array}{c} \text{product storage} \\ \text{is not full} \end{array} \right) \rightarrow \pi^{pa} \left(\beta \middle| \lambda = \lambda_{pa} + \frac{1}{M_{pa}}, \mu = \frac{1}{M_p} \right); \text{ Let's call this value } \mathbf{E}.$$

$$\pi^p \left(\begin{array}{c|c} \text{product storage} & \text{part storage} \\ \text{is not empty} & \text{is empty} \end{array} \right) \rightarrow 1 - \pi^p(0 | \lambda = 0, \mu = \lambda_d); \text{ Let's call this value } \mathbf{F}.$$

$$\pi^p \left(\begin{array}{c|c} \text{product storage} & \text{part storage} \\ \text{is not empty} & \text{is not empty} \end{array} \right) \rightarrow 1 - \pi^p \left(0 \middle| \lambda = \frac{1}{M_p}, \mu = \lambda_d \right); \text{ Let's call this value } \mathbf{G}.$$

By solving simultaneous equations (A.5) to (A.8), we get the steady-state probabilities as in equations (A.1) to (A.4).

Biography:

Narges Shahraki has completed her Ph.D. program in Industrial Engineering and Operations Management at Koc University in August 2015. Currently, she is a Master Scheduler at Michael foods Inc. Her current research focuses on production and inventory analysis.