A Multi-fractal Spectrum Analysis for online Structural Health Monitoring

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Abstract

The nonlinear and nonstationary nature of structural damage brings a great challenge to structural health monitoring (SHM). Chaos theory and nonlinear time-series analysis domain suggests many effective candidates to capture system dynamic and measure the complexity of dynamical system. From different candidates, this paper focuses on multi-fractal spectrum analysis for online structural health monitoring. Results show that the quasi recessive correlation dimension (QRCD) is not only the best fractal dimension for detecting different defect levels, but also it has less computational complexity than the singularity spectrum used for extracting multi-fractal spectrum. On the other hand, the multi-fractal spectrum analysis is an effective damage quantifier for analyzing data which exhibit multi-fractal behavior and it has a better diagnosis capability for monitoring non-stationary process and those attractors which exhibit phase-transition.

Key words: Structural health monitoring; Multi-fractal Analysis; Singularity spectrum; Ergodic theorem.

1. Introduction and motivation

Chaos theory and nonlinear time-series analysis domain suggests many effective candidates to capture system dynamic and measure the complexity of dynamical system (Moon et al., 1992). From different candidates, this paper focuses on the multi-fractal spectrum analysis for the purpose of structural health monitoring. The importance of the multi-fractal analysis in connection to dynamical system comes from the fact that it is an efficient technique to determine the existence of strange attractors, thus it allows a statistical description of the underlying dynamics system's attractor (Moon et al., 1992). A more convenient way to describe the global scaling properties of the attractor is by using a spectrum of singularities, which will be discussed in this paper. Scaling in Chaos analysis almost always reduce the object or pattern into smaller replicas of the original according to a fixed ratio (Garnett and Williams, 1997). The point is to find out how certain results change as the length of the measuring device gets smaller and smaller, which probes the strange attractor to a much finer scale.

The importance of the multi-fractal analysis in connection to dynamical system comes from the fact that it is an efficient technique to determine the existence of strange attractors, thus it allows a statistical description of the underlying dynamics system's attractor (Grassberger, 1990). A more convenient way to describe the global scaling properties of the attractor is by using a spectrum of singularities, which also will be discussed in this paper.

The multi-fractal analysis has the promising properties to characterize a system dynamics. To choose the best potential methods for structural health monitoring application in the chaotic nonlinear research domain, this paper investigates exponent dimensions as well as the multi-fractal spectrum methods for online damage detection of structures.

2. Review of the related research

Multifractals could be seen as an extension of fractals and have common characteristics such as they have a statistical geometric regularity and a noninteger number that quantifies the scaling of their complexity over a range of scales (Garnett and Williams, 1997). The recurrence of the same pattern over a range of scales which is called self – similarity. So, any part of the line, surface, or pattern looks alike over a wide range of scales (Garnett and Williams, 1997).

There is a wide variety of system that exhibit multi-fractal properties. Examples include stock market data, branching of the lungs, web data, physiological data, frequency-intensity distribution of earthquakes (Schuster and Just, 2006), oil and gas field distribution (Fraedrich and Wang, 1993) and earth gravity field (Thorarinsson and Magnusson, 1990).

There are many types of variations of dimension as a scaling exponent. In no particular order of importance, some that are mentioned quite commonly in the literature are: similarity dimension, capacity dimension, Hausdorff dimension and correlation dimension. The different types of exponent dimensions are all related (Abraham et al., 2013). Thus, some of them have the same numerical value for certain conditions.

Most fractal dimensions consist of two groups (Garnett and Williams, 1997); the first group includes similarity dimension capacity dimension and Hausdorff dimension. This category aims at measuring only the attractor's geometry, in the sense that, it takes no account of how often the trajectory visits different points in the state space (Abraham et al., 2013). On the other hand, there is another group such as the correlation dimension and the information dimension. This group not only consider the attractor's geometry, but also, the probabilistic aspects of the trajectories in terms of visiting some state space neighborhoods more often than others (Garnett and Williams, 1997).

Many damages of structures occur in localized areas and exhibit a nonlinear and nonstationary dynamic behavior (Wang et al., 2001). As a damage quantifier, the best candidate from the fractal dimensions should consider not only the geometry of the attractor (its apparent size and shape), but also accounts for the frequency at which a trajectory visits different regions on the attractor. Thus, the exponent dimensions in the first group are not good candidates to describe fully the attractor. In addition, the correlation dimension can examine thoroughly the scaling of the attracting set to a very small length scales r based on the pairwise distances (Garnett and Williams, 1997). On the other hand, if the same small length r is used in a capacity dimension algorithm, then many of the small boxes would be empty.

According to (Garnett and Williams, 1997), the Haudorff dimension is one of the least useful in analyzing real-world data. The reason is that it applies only at the theoretical limit of $(r \to 0)$. Also, in calculating the box-counting dimension D_0, the covering open set is restricted to balls of uniform radius, then D_0 is not a good option for those frequencies which are not uniform or even having singularities somewhere in the phase space (Zaslavsky, 1985). In addition, it has been reported that it is so hard and impractical for high dimensional systems (Garnett and Williams, 1997).

The correlation dimension has many advantages over many types of exponent dimension: the most common estimation of attractor dimension that is practical as its calculation is relatively simple and fast; it has a consistent estimation (Theiler, 1990); it is less sensitive to low noise (Trendafilova, 2002); it has a high sensitivity to the dynamical change; and, for a given dataset, it explores the attractor too much finer scale than other exponent dimension (Garnett and Williams, 1997). The true value of the correlation dimension has an important practical implication: the next highest integer value above the correlation dimension describes the level of complexity of the underlying dynamic system, by representing the minimum number of active degree of freedom needed to model the system (Garnett and Williams, 1997). Moreover, it has an operational and more rigorous mathematical definition which provides different estimation algorithms to deal with not only experimental time series data, but also simulation data (Grassberger and Procaccia, 1983).

In application of correlation dimension, Logan and Mathew (Grassberger, 1990) applied the correlation dimension in the fault diagnosis domain. Their studies demonstrated that the correlation dimension is an effective approach in classifying three major rolling element bearing faults. The applicability of correlation dimension using Grassberger-Procaccia (GP) algorithm for diagnosis of large rotating machinery is reported in (Grassberger and Procaccia, 1983). This investigation indicates that the correlation dimension is useful in reflecting the different kinematic mechanisms.

Brief introduction of multi-fractals spectrum analysis

In this section, the generalized fractal dimension, D_q is introduced first (Sec. 3.1), and then the singularity spectrum, $f(\alpha)$ using the Legendre transform is presented (Sec. 3.2).

3.1 Generalized fractal dimension

A spectrum of dimensions was introduced in (Grassberger and Procaccia, 1983) as

$$D_q = \frac{1}{q-1} \lim_{r\to 0} \frac{\log(\sum_i \mu^q(C_i))}{\log r} \quad , -\infty < q < \infty \tag{1}$$

or

$$D_q = \lim_{r\to 0} \frac{1}{\log r} \log C_q(r) \ , -\infty < q < \infty$$

where

$$C_q(r) = \lim_{N_m\to\infty} \left\{ \frac{1}{N_m} \sum_{i=1}^{N_m} \left\{ \frac{1}{N_m - 1} \sum_{i,j=1,j>i}^{N_m} I(r - \|x(i,m) - x(j,m)\|) \right\}^{q-1} \right\}^{\frac{1}{(q-1)}}$$

$C_q(r)$ is the generalized correlation integral, $\mu(A)$ is the probability measure for elements of A in **A** ($\mu(A) \geq 0$ for any A and $\mu(A) = 1$) when **A** is the entire phase space and q is a continuous index, I is the Heaviside step function, such that $I(x) = 0 \ if \ x \leq 0 \ and \ I(x) = 1 \ for \ x > 0$, $\|...\|$ indicates the Euclidean-norm.

From the previous comprehensive definition, D_q is nonincreasing as a function of q: $D_i \geq D_j$ if $i \leq j$ (Hentschel and Procaccia, 1990). There is infinitely many number of dimensions. For q=0, equation 1 gives (if the limit exists)

$$D_0(A) = \lim_{r\to 0} \frac{\log N(r,A)}{\log\left(\frac{1}{r}\right)},$$

If not, then $D_0(A) = \limsup_{r\to 0} \frac{\log N(r,A)}{\log\left(\frac{1}{r}\right)}$, where A is a compact metric space and $N(r,A)$is the minimum number of open balls of uniform radius r needed to cover A.

$D_0(A)$ is called the capacity dimension or the Box-counting dimension (Zaslavsky, 1985), which is independent of the frequency with which a trajectory visits the different parts of the attractor.

Let q=1 in equation 1 and applying L'Hopital's rule (Zaslavsky, 1985), then we get

$$D_1 = \lim_{r\to 0} \frac{\sum_i \mu(C_i) \log \mu(C_i)}{\log \ r},$$

which is called the information dimension. It is more accessible experimentally and theoretically than the capacity dimension (McEliece, 2002), because instead of counting each cube which contains part of attractor, the information dimension counts how much of the attractor is contained within each cube. Both are equal for the case with evenly distributed points.

In the case of q=2,

$$D_2 = \lim_{r \to 0} \frac{\log \sum_i \mu^2(C_i)}{\log\ r},$$

This is the correlation dimension from Grassberger-Procaccia algorithm, by which we can examine thoroughly the scaling of the attracting set to a very small length scales r based on the pairwise distances. If the squared Euclidean distance between all pairs of data points in (m+1) embedding space obtained from the squared Euclidean distance in an (m) embedding space, then the resulting factual dimension is called quasi-recursive correlation dimension (QRCD) (Mistarihi et al., 2012).

3.2 Singularity spectrum

Another way to characterize the complexity of chaotic attractors is using singularity spectrum (Moon et al., 1992). Consider a covering of the attractor with m-box size r. Different regions of the attractor leads to different singularities in the measure. For some I, the measure associated with the box

$$p_i(r) \propto r^{\alpha_i} \quad , \tag{2}$$

where α_i is the exponent given the singularity. In other words, α_i is the pointwise dimension at the point on the attractor on which the boxes are centered. The number of boxes with the measure r^{α_i} varies according to

$$N(\alpha_i) \propto r^{-f(\alpha_i)} \tag{3}$$

where $N(\alpha_i)$ represents the number of volume elements with scaling exponent.

Typical plot for the $f(\alpha)$ singularity spectrum is shown in Figure 1. The $f(\alpha)$ curve is always convex with a single maximum, which corresponds to the capacity dimension (D_0). Also, when $|q|$ approaches infinity, it picks out the regions where the measure is densest ($q > 0$) or least concentrated ($q < 0$) (Schuster and Just, 2006). The intersection of the $f(\alpha)$ curve with the x-axis gives $D_{\pm\infty}$.

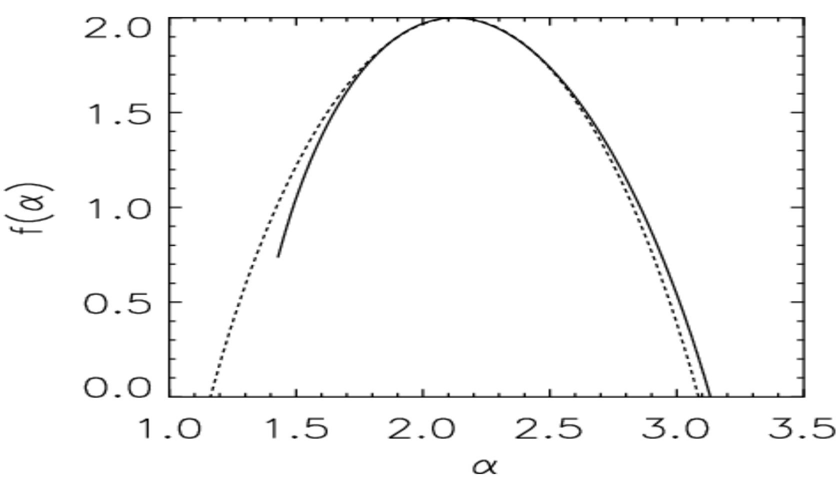

Figure 1. Typical plot for the $f(\alpha)$ singularity spectrum

3. The proposed technique for capturing nonlinear system dynamic

The proposed technique can be done with the help of Singularity spectrum analysis and Ergodic theorem. Covering the attractor of a multi-fractal chaotic system with volume elements of diameter r, can be used to evaluate the generalized fractal dimension D_q in equation 1. With the assistant of Legendre transformation, the D_q and the $f(\alpha)$ are related by the following equation

$$D_q = \frac{1}{q-1}\left[q\alpha(q) - f\big(\alpha(q)\big)\right] \tag{4}$$

The previous relationship can be inverted as

$$\alpha = \frac{d}{dq}\left[(q-1)D_q\right]$$

and then

$$f(\alpha) = -(q-1)D_q + q\alpha$$

Some statistical quantities of the attracter of a dynamical system are robust and not affected by noisy data (Todd et al., 2001). According to (Trendafilova, 2003), the variance and the skewness demonstrated sensitivity and regular dependence on damage. Ergodic theorem is used to investigate the best sensitive feature for SHM between multi-fractal dimensions based on the linkage between the identified dimension as nonlinear invariant in characterizing a dynamic system in its state space with the statistical analysis of

the attractor distribution. Since the QRCD is an invariant measure such that QRCD $\left(f^{-t}(A)\right) = \text{QRCD}\ (A), t > 0$. Here A is a subset of R^n , f is evolution function and $f^{-t}(A)$ is the set obtained by evolving each point in A backwards for a time t. Then, the QRCD will be further compared with different multi-fractals spectrum analysis for SHM. In practice this means that the correlation dimension is invariant for smooth changes of the coordinate system.

Let $\mu(X)$ be an ergodic natural measure, we would like to show that the time averages are identical to space averages. That is to show that the phase space average over the measure $\mu(X)$ can be approximated by the time averages over a typical trajectory. In symbols, we need to show that

$$\langle \mu(x) \rangle \cong \frac{1}{N}\sum_{j=1}^{N} \mu(x_j), \quad (5)$$

where, $\langle \mu(x) \rangle$ is the phase space average over x with respect to the natural measure μ, x_j is a point on the time series and N is a large number to represent a long term behavior of the underlying dynamic system. Since $\mu(X)$ is an ergodic natural measure, then

$$\mu(x_j) = \frac{1}{N}\sum_{i \neq j}^{N} I\left(r - \left|x_i - x_j\right|\right),$$

Hence

$$\frac{1}{N}\sum_{j}^{N} \mu(x_j) = \frac{1}{N^2}\sum_{j=1}^{N}\sum_{i \neq j}^{N} I\left(r - \left|x_i - x_j\right|\right),$$

As N approaches infinity, the previous formula is a proximately identical to the definition of correlation integral.

4. Case Study: Two – Scale Cantor Set

The comparison between multi-fractal spectrum analysis and QRCD analysis for structure damage detection includes the computational complexity, accuracy and suitability based on the characteristics of data will be demonstrated by the following case study and a discussion.

To evaluate the effectiveness of the QRCD algorithm and the generalized dimension D_q computational time and accuracy, it has been used well-known deterministic fractal, namely two-scale Cantor set. The values of fractal dimension and the singularity spectrum can be theoretically determined for the two-scale Cantor set. It has been found that in cases which the correlation integral must be calculated over all the distinct pairs of points, the quasi-recursive correlation dimension algorithm has achieved a high reduction in CPU time over the generalized multi-fractal spectrum as shown in Figure 2. On the other hand, a comparison between the analytical and obtained fractal dimension using QRCD algorithm and the generalized dimension D_q for the two-scale Cantor set is summarized in Table 1.

The percentage decrease in computational time depends on data length and maximum embedding dimension. The larger the embedding dimension along with a longer data size gives more advantages for the QRCD over the $f(\alpha)$ algorithm. Per previous calculations, using the QRCD algorithm significantly reduces the computational complexity in terms of computation time by 40% over the multi-fractal spectrum technique. That is due to the fact that the QRCD uses consecutive overlapping segmentation technique, while the $f(\alpha)$ requires heavy mathematical calculations via

Legendre transform and digital signal processing. Moreover, the sorting procedure of the data takes some time while evaluating the multi-fractals spectrum.

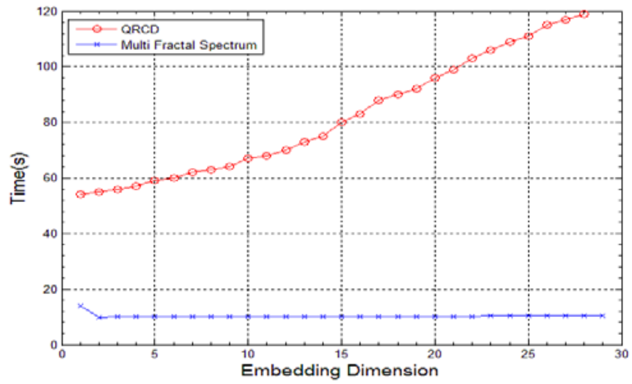

Figure 2. Comparison of CPU time between the QRCD and D_q for two-scale Cantor set

Table 1. Computational results of QRCD and the generalized dimension D_q algorithms for the two-scale Cantor set example

Signal model	D_q	$\widetilde{D_q}$ $f(\alpha)$	$\widetilde{D_q}$ QRCD	CPU time (s) $f(\alpha)$	CPU time (s) QRCD	Signal length
				m_{max}=30	m_{max}=30	
Two-scale Cantor set	1.2618	1.257	0.98	1620	114	2^{20}

The exact computational complexity for the quasi-recursive correlation dimension algorithm is of $O(N^2-(N-M)^2.r_i)$, which belongs to $O(MN-M^2)$, taking $(M \ll N)$ then the overall computational complexity belongs to (N). Here M is the overlapping segment such that $(M \ll N)$.

The overall running time of the algorithm can be expressed as a linear function $O(N)$, which describes an algorithm whose performance grows linearly and indirect proportion to the sample size of the input data set.
Under the best case scenario (Grassberger, 1983), the computation of the generalized dimension D_q is on the order of $O(N\ logN)$ for each $q \in (-\infty, \infty)$. But only integer values for $q \geq 0$ have physical meaning, that is Δq =integer. Suppose $q \in \{0,1,2\}$, then

$$D_0 \in O(N \log N),\ D_1 \in O(N \log N) \text{ and } D_2 \in O(N \log N)$$

This implies that

$$D_{q_{=0,1,2}} \in O[(N \log N)^3]$$

In summary, using big O notation and synthetical example, the QRCD has less computational complexity than the singularity spectrum used for extracting multi-fractal spectrum and the QRCD algorithm is much faster than the multi-fractal spectrum.

5. Discussion of the suitability of multi-fractals spectrum versus the QRCD as a damage quantifier

The characteristics of the data play a main role in choosing the best damage detection technique. The date, whether comes from physical measurements or numerical modeling, almost have some problems such as limited size of the total data, the data are non-stationary and/or the data represent nonlinear processes. The suitability of the multi-fractals spectrum as a damage quantifier is discussed in the following subsections.

Size of the dataset

The correlation dimension requires a large sample size. An appropriate sample size (in practice, often very large) is needed especially at small radii. Otherwise, very fewer points will be used in computation of the correlation integral. Thus, the data will not adequately represent the attractor when embedded in several dimensions. As a result of that, the correlation dimension will be less than the true correlation dimension. In fact, the required sample size is application dependent (Garnett and Williams, 1997). In general, applying the standard G-P algorithm to calculate the correlation dimension, the sample size should be at least $10^{\left(\frac{D_2}{2}\right)}$ (Jiang, 1995), while applying the QRCD algorithm using the consecutive overlapping segmentation technique will highly reduce the desirable sample size for the purpose of online monitoring to as minimum as the size of the sliding window.

Regarding the generalized fractal dimensions, in the linear scaling reign, the true fractal nature of the underlying attractor is revealed under the condition that the local slopes are constant for increasing embedding dimension. The scaling reign structure will be lost due to limited dataset. Moreover, the multi-fractal spectrum is very sensitive to lack of points, especially the right part of the singularity spectrum $f(\alpha)$curve as shown in Figure 3, where a theoretical $f(\alpha)$ curve (solid line) was plotted for comparison.

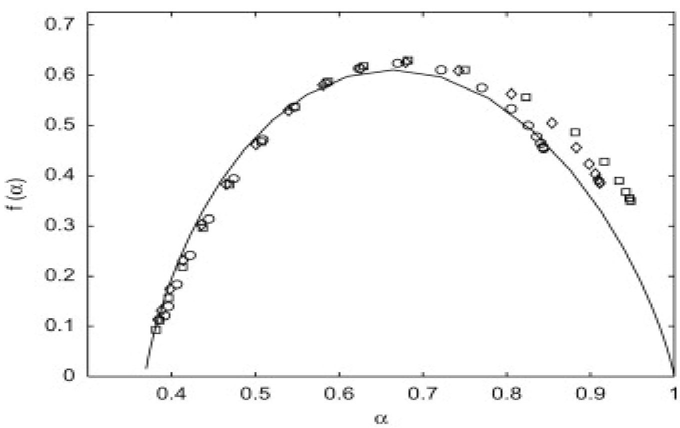

Figure 3. Multi-Multifractal spectrum for a two-scale Cantor set with 1014 points fitted using different ranges (De Souza and Rostirolla, 2011)

From Figure 3, we notice that, even $f(\alpha)$ spectrum is relatively robust to change in values of cutoff for positive moments ($\alpha < \alpha(max[f(\alpha)])$), curve is very sensitive for negative moments. For certain applications, with small datasets ($N < 5000$) (Fowler and Roach, 1993) , the monotonic rule of the generalized fractal dimensions D_q is violated ($D_i \geq D_j$ if $i \leq j$) (Hentschel and Procaccia, 1990) as shown in Figure 4.

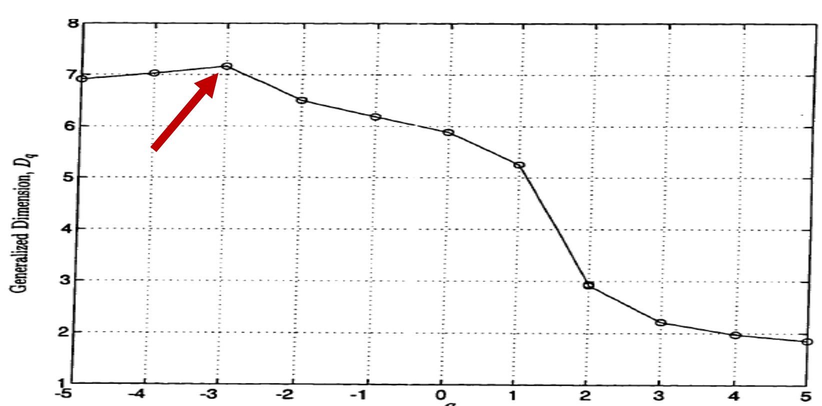

Figure 4. Invalid D_q curve where the monotonic decreasing has been violated around $q < -3$

Noisy dataset

The standard G-P correlation dimension algorithm does not perform well with noisy signals (Yu et al., 2000). That is because the linear scaling reign does not fully hold. In other words, the existent of noise will disturb in the slope of the empirical best–fit regression line of $\log \mathrm{C(r)}$ and $\ell\mathrm{og(r)}$. The disturbance occurs significantly when the value of radius equals the magnitude of the noise (r_{noise}). In other cases, the linear scaling reign exists with too much systematic error at the upper end and too much statistical error at the lower end (Theiler, 1987). Moreover, choosing the radii (r_i) to be greater than the magnitude of the noise will be meaningless while embedding in higher dimensions with a high level of noise.

If the noise level is high, then there is no guarantee that the choice of (r_i) will satisfies $(r_i) > (r_{noise})$. Thus, then all points will be covered from the first choice of the radius in embedding dimension 2. That makes the correlation integral be constant with different radii. Consequently, the correlation dimension will be meaningless under the very high level of noise.The existence of the noise may hide the presence of the low dimension attractors, so it is so crucial to choose the radii (r_i) to be greater than the magnitude of the noise.

Regarding the influence of noise on the theoretical fractals and experimentally gained fractal, the absolute value of the fractal dimension would be altered as illustrated in Figure 5, where "N" depicts the horizontal region caused by noise, "T" the transition range between the horizontal noise influenced region and the unaltered range for the larger ε, and "L" the range limited by the side length of the image.

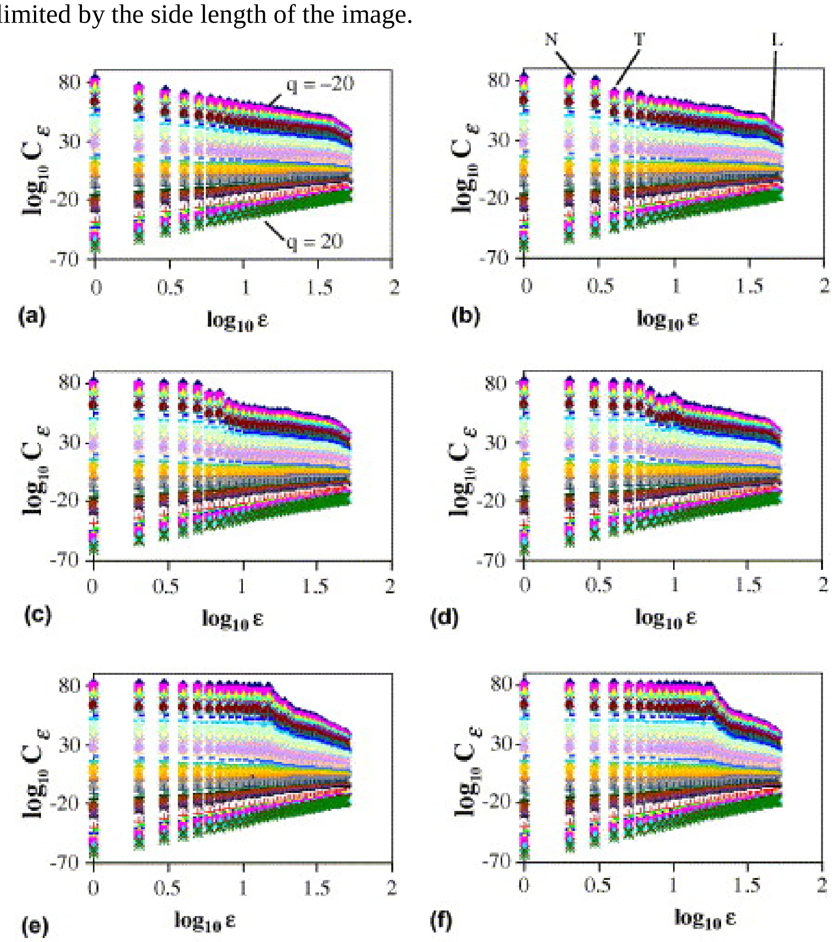

Figure 5. Plots of the general correlation integral $C_\varepsilon(q)$ versus the distance r for $-20 \leq q \leq 20$.(a) Image without noise; (b) the same image with noise of variance $\sigma^2 = 1$; (c) $\sigma^2 = 5$; (d) $\sigma^2 = 10$; (e) $\sigma^2 = 50$; (f) $\sigma^2 = 100$ (Ahammer and DeVaney, 2005)

Non-stationary dataset

The basic G-P correlation dimension algorithm measures static or stationary states (i.e. in transition from one quasi-steady state to another) of the system of interest. As the system migrates or transition to another steady state with a new geometric representation, the correlation dimension may or may not change (Zaslavsky, 1985).The quasi-recursive correlation dimension partially overcomes the non-stationary problem using consecutive overlapping segmentation technique, especially, when the data is not highly non-stationary.

The multi-fractal spectrum analysis works well with even highly non-stationary data (Adeyemi, 1997). It works efficiently for attractors that exhibit multi-fractal behavior as well as phase transitions if the generalized fractal dimension D_q satisfies the monotonic rule of the generalized fractal dimensions D_q is violated ($D_i \geq D_j$ if $i \leq j$).
The Logistic Map, a notable chaotic attractor which exhibits phase-transitions was taken to test the suitability of multi-fractal spectrum for analyzing non-stationary data. The generalized fractal dimensions D_q curve as a function of q for the logistic Map is shown in Figure 6. The discontinuity in the derivative of the D_q curve occurs at q=2.

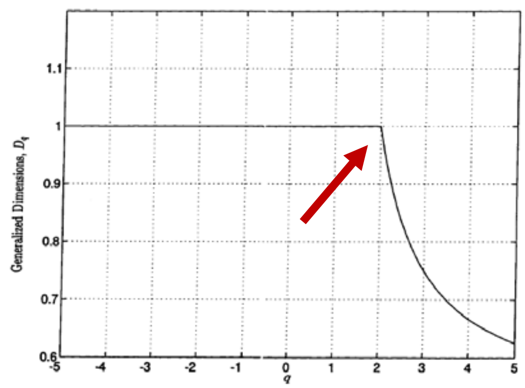

Figure 6. Schematic plot of the D_q vs q showing for the Logistic Map

6. Conclusion

In this paper, a multi-fractal spectrum analysis is used for online SHM. Results indicate that the generalized fractal dimension has fairly high computational complexity of $O(N \, logN)$ for each $q \in (-\infty, \infty)$ compared with the QRCD, whose computational complexity of O(N). On the other hand, the multi-fractal spectrum has a better diagnosis capability for monitoring non-stationary process and those attractors which exhibit phase-transition.
It should also be pointed out that, for a noisy data, the capabilities of both the QRCD and the multi-fractal spectrum are affected as damage detection features. Based on the characteristics of date, each of the correlation dimension and the multi-fractal spectrum has its own application and practical considerations. In future work, making no assumptions about the data and looking at the dynamical changes within an adaptive sliding window will be further investigated via Recurrence Quantification analysis or/and Hilbert Huang Transform.

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Biography

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