A linear programming model for integrated baked production and distribution planning with Raw material and routing consideration.

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Abstract
A linear programming model for integrated production planning study for methods, procedures and practices of a South African bread producing firm. The study considers the entire planning activity in the firm as an integrated process involving several closely related functions. The mathematical modelling considers production costs, production throughput rates, customer specifications and demands, sales process and facility capacities and designs an optimal production planning. The firm produces two perishable products that should be stored for fixed periods. There is a series a of vehicles responsible for delivering the products from the firm to distribution centers in specific quantities. Computation of results and analysis reveals that the integrated methodology is feasible and practical.

Keywords
Production planning, distribution planning, supply chain management, Baking industry, integrated systems.

1. Introduction

In the past years, managers in the food industry have encountered difficulties and to instantaneously reform, adapt and up their game as they were faced with international changes within the food industry, because of safety regulations and legislations, advancement in technology, market globalization and emerging quality objectives. Organizations are now placed under pressure to improve certain aspects in their businesses such as manufacturing, quality control, marketing, etc. Managers need to understand that such changes, need to be accompanied by supply chain management, to ensure that the control process does not halt at the manufacture but continues to distribution and after sales firms. The goal of supply chain is to efficiently create a link between suppliers, producers, distributors, storage facilities, sellers and customers (Liang, 2008), Therefore supply chain integration should provide the end customer with maximum value of quality products, while coordinating a balance between requirements of customers with materials coming from suppliers, including supply chain issues of designing low cost products, and keeping inventory at a minimum.
This paper takes a supply chain management approach, the paper takes a discussion on the facilities and processes that are required in the production and delivery of products from supplier to customers at a bakery that process pita bread and wraps. The paper further illustrates Scheduling and management of raw materials, manufacturing, packing, storage and distribution services.

1.1 The case study.

The bakery is a supplier of pita bread and Wraps, it also specializes in producing Shawarma's and hamburgers. The bakery supplies major retail stores, including coffee shops and supermarkets.
When analyzing the bakery there mainly two areas that stand out, the production area and the distribution Scheduling area. Production planning that is carried out by the production manager, addressing and tackling decisions of how to

transform unprocessed materials into the required end products that are of high quality and specifications conforming to meet customer demands while adhering to production time with minimum cost. Production planning within the bakery is done by the production manager, the manager receives all orders of major retail stores in the morning of the scheduled delivery day, coffee shops supermarkets and other franchise orders come in during the day, till 23:30 at night. The manager then determines the total amount of mixes/products to be produced, he then calculates the quantity to be produced, keeping in mind the amount of damages, rework and other factors that hinder the perfect production of an in-specification product. This is very crucial as he needs to take into consideration the time required to complete these products, processing schedule to ensure minimum downtime, and delivery slot times that are set by customers and distribution centers. In line with Scheduling of achieving service levels, it is important to consider the number and sequence of production, amount of raw materials required, resources constraint's, use of shared machines, deterioration of items used, demand type during and at off peak sessions.
Distribution Scheduling is a factor goes hand in hand with production planning, planning for distribution therefore challenges decisions of delivering the final product from the bakery to the consumer respecting to fulfil service levels, meet customer demands on time with minimum cost and provide high quality products that are with required specifications. A major challenge in distribution Scheduling is Routing. Routing makes calculated decisions on critical paths to consumer premises or distribution centers need to be determined and distribution to be scheduled accordingly in order to reduce or minimize supply chain costs.

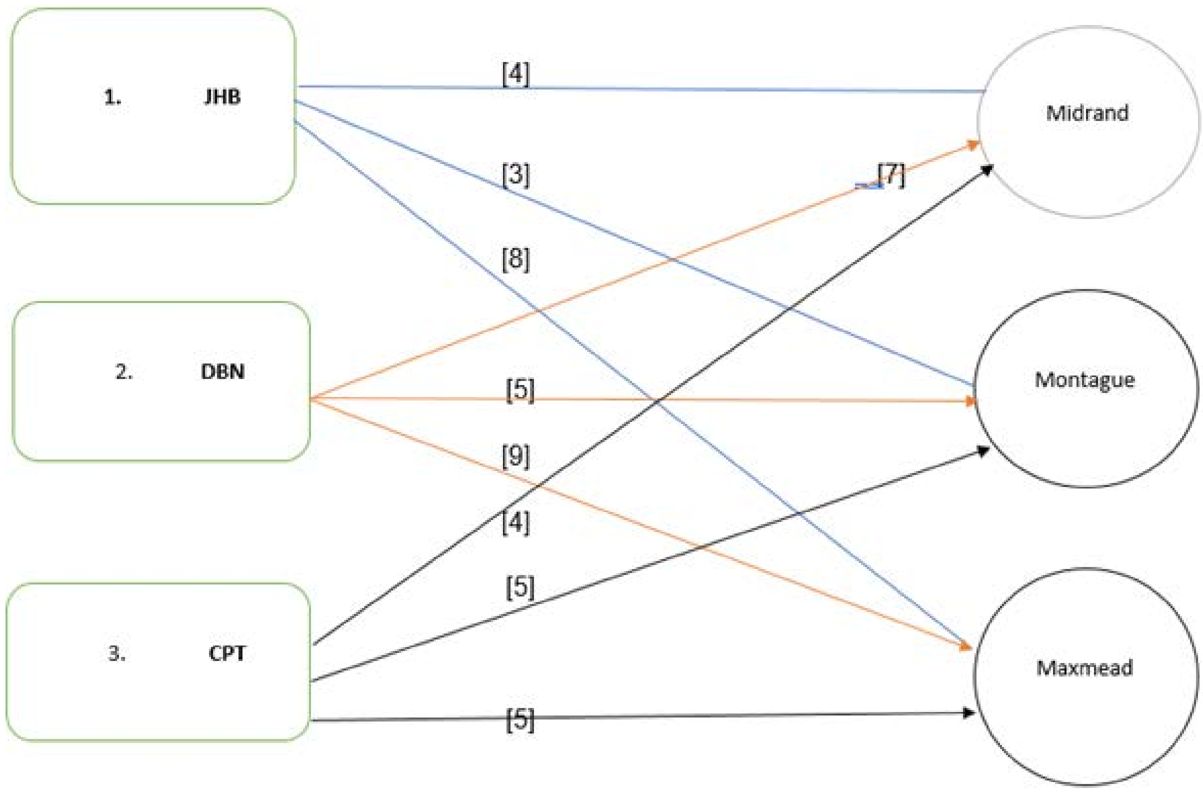
2. Literature review

In the past, researchers have provided a lot of attention to Integrated production and distribution Scheduling problems. Researchers such as (Sarmieto & Nagi, 1999) (Ivanov, et al., 2013)and (Chen, 2010) have provided reviews on the subject.
(Ivanov, et al., 2013) Stated that in supply chains planning for production and distribution has been referenced research problem, where production distribution planning decides on the total product flows within the supply chain, with the reason of creating a balanced supply and demand within the middle-range period to successfully meet consumer needs while at the same time improving efficiently improving operations and performance.
(Liang, 2008) argues that the HMMS (Holt, Modigliani, Muth and Simon) rule that was offered in 1995, is a linear decision rule which seeks to stipulate an ideal degree of production and levels of labor that reduce the total production costs of regular payoff, overtime, hiring layoffs, and inventory through a series of quadratic cost curves.
The existing production planning and distribution models and optimizations objectives vary regarding industries and organizations (Ivanov, et al., 2013), they further state that understanding the integration of production and distribution Scheduling can significantly enhance service level and reliable delivery, along with improving costs associated with transportation and inventory.
(Bard & Nananukul, 2009) expressed a united batch sizing and inventory transportation problem as a mixed integer program with a clear outline of maximizing the output. (Bard & Nananukul, 2009) further established a twostep process that estimates delivery quantities and then further resolves a transportation routing problem. (Boudia & Prins, 2009) studied an NP- hard multiperiod production- distribution problem to reduce sum of the three costs. (set up cost of production, inventory levels, and distribution. This was explained by a memetic algorithm with population management.
This research paper aims to tackle the integration problem of planning for production, including distribution Scheduling of baked products with levels of inventory and Transportation consideration.

3. Problem formulation

the bakery is a single production firm that manufactures baked bread and wrap products. The bread has a fixed shelf life that is calculated by a number of scheduling periods, the products can either be warehoused in the manufacturing facility or in distribution centers. The Scheduling is divided into equal separate time intervals that each is respectively a Scheduling period. The manufacturing size is subject to a limited quantity of raw material can be temporarily stored in the manufacturing firm with unit holding cost of. There are several established customer centers isolated from the manufacturing facilities that the products are to be delivered to. Each customer outlet and manufacturing firm has a nonnegative demand in the Scheduling period that must be encountered, that is there can be no shortages. Only a partial number of finished products can be kept in a manufacturing firm, but it contains a unit holding cost. A fleet of capacitated vehicles is responsible for delivering the products from facilities to customer outlets. The research therefore, aims to reduce the number of vehicle trips. In accumulation, applying two assumptions to the deliveries: 1:

each plant has its own transport capacity, and 2: each vehicle can visit a distribution center one time per Scheduling period.



(figure 2) current model of transportation from the bakery and other manufacturing plants to the customers outlets.

Figure 2 indicates the current situation of transportation happening from the bakery and other manufacturing plants to outlets, it further indicates the transportation costs associated of moving the products. Figure 2 is a two- stage supply network model consisting of manufacturing firms and customer outlets. The bakery response to the demand of customers using the volume of inventory it has available as well inventory levels of other branches that partners itself with.

3.1. Research Questions:

RQ_1 How could the bakery determine whether the production process, is capable that all products are produced in the provided time frame and yielding maximum profit?

RQ2 What should the bakery implement to ensure that raw materials inventory is readily available for production in order to fulfill service levels?

RQ3 How can the transportation model of the bakery be enhanced in order to ensure that transportation vehicles will reach the distribution centers on time, thereby minimizing fuel costs and vehicle depreciation?

Hypotheses are derived from the research question above and are as follows:

RQ_1- H_1: the correlation between the production process rate and the production time frame to yield maximum profit.

RQ2- H2: the correlation between the availability of inventory and fulfilment of service levels.

RQ3- H3: the correlation between the transportation model and fuel costs including vehicle depreciation.

4. Finding and discussions

The bakery is a single production facility that produces two products namely pita bread and Wraps. It takes four (4) hours to complete the manufacturing process on a single batch of pita bread and it takes two hours to complete manufacturing a batch of wraps, the company operates twenty hours per day.
It takes a single bag of flour to produce one batch of wraps and a batch of Pita bread, with six (6) bags of flour available in stock. A loaf of bread sells at R8.00 while a packet of bread sells at R6.00.

	Product 1 (X1= Pita bread)	**Product 2 (Y1= wraps)**	**Resources Available**
Production Time (hours) *(a)*	4 hours	2 hours	20 hours
Bags of flour required/ Batch *(b)*	1 bag	1 bag	6 bags
Profit	R 8.00	R6.00	

4.1. Hypotheses testing

4.1.1. Testing RQ_1- H_1

In order to regulate the correlation between the production process rate (independent variable) and the production time frame to yield maximum profit (dependent variable), a linear function model is projected as follows taking consideration of the maximum hours of production that are available:

$$y(PTFYMP)= 4(a_1) + 2(a_2) \quad (1)$$

where by:

$y(PTFYMP)$ = The dependent variable (production time frame to yield maximum profit)
a_1 = The time it takes to produce a single batch of pita bread
a_2 = The time it takes to produce a single batch of wraps

therefore, if $y(PTFYMP)$ is held constant, while a_2 increases by one-unit a_1 would decrease by 0.5`

4.1.2. Testing RQ_2- H_2

In order to regulate the correlation between availability of inventory (independent variable) and services levels (dependent variable), a linear function model is projected as follows:

$$y(SL)=(b_1) + (b_2)$$
(2)

where by:

$y(SL)$ = The dependent variable (service levels)
b_1 = The amount of inventory required to produce a batch of pita bread
b_2 = The amount of inventory required to produce a batch of wraps

therefore, if $y(SL)$ is held constant, while b_2 increases by one unit a_1 would decrease by a single unit.

4.1.3. Testing RQ_3- H_3

In order to regulate the correlation between the transportation Model (independent variable) and fuel costs and vehicle depreciation (dependent variable), a linear function model is projected as follows:

$$y(FcVd)=4X_{1A} + 3X_{1B} + 8X_{1C} +7X_{2A} + 5X_{2B} + 9X_{2C} + 4X_{3A} + 5X_{3B} + 5X_{3C}. \tag{3}$$

where by

y*(FcVd)*	=	The dependent variable (fuel costs and vehicle depreciation)		
Xi-Xj	=	cost of vehicle trips from warehouses to customer centers		
		i	=	1,2,3…
		j	=	*A,B,C...*

4.2. Mathematical framework and Solution leading to validation of hypotheses

The framework has been presented as a linear objective function. symbols used in formulating the problem are as follows:

X1	=	Product 1 (number Pita bread to be produced)
Y1	=	Product 2 (number of wraps to be produced)
Max Z	=	Profit to be achieved through production.

4.2.1 Objective function is to maximize profit

Therefore, $Max\ Z = R8x1 + R6y$ (4)

Subjected to: $4x1 + 2y1 \ \leq 20\ hours$ (5)

$1x1 + 1y1 \ \leq 6$ bags (6)

$x1; y1 \ \geq 0$ (7)

Coordinate calculations

$4x1 + 2y1 \ \leq 20\ hours$			$1x1 + 1y1 \ \leq 6$ Bags	
Let $x1 = 0$ Therefore, 4(0) + 2 $y1$= 20 $y1 = 10$ Coordinate: (0;10)	Let $y = 0$ Therefore, $4x1 + 2(0) = 20$ $x1 = 5$ Coordinate (5;0)		Let $x1 = 0$ Therefore, 1(0) +1 $y1 = 6$ $y1 = 6$ Coordinate (0;6)	Let $y1 = 0$ Therefore, 1 $x1 + 1(0) = 6$ $x1 = 6$ Coordinate (6;0)

Table1: calculation of y and x intercepts along a cartesian plane

Graph

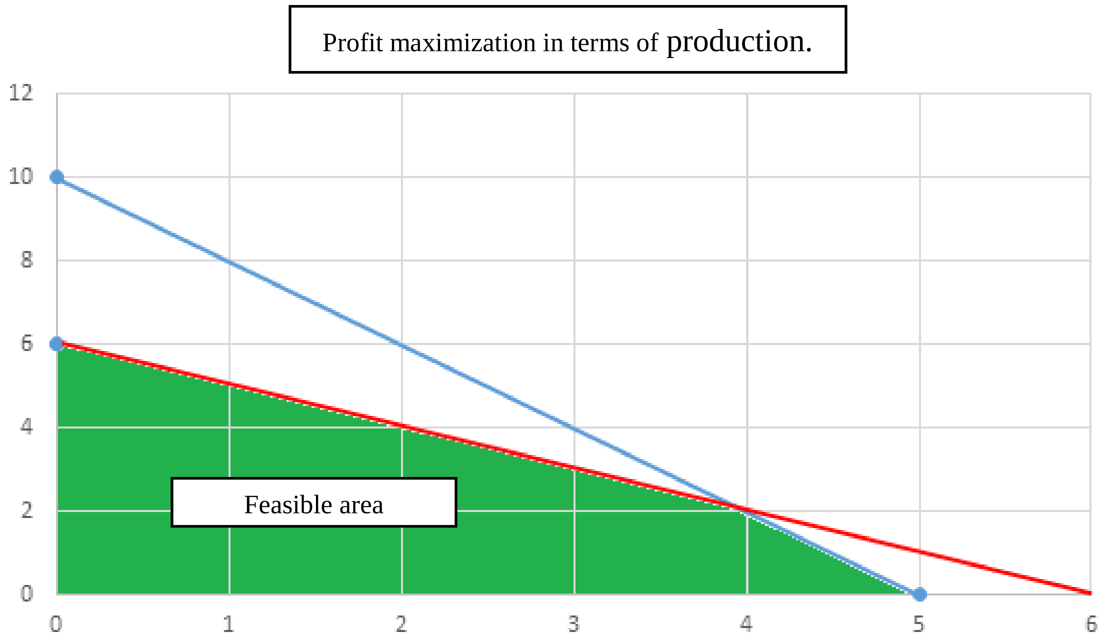


Figure 1. profit maximization in terms of production planning.

Figure1 indicates that firm can produce a certain number of X_1 *or* Y_1 products that lie within the feasible region but place consideration on that the maximum of twenty (20) hours to produce during a given day, further the company based on inventory levels the company can only use a maximum of six (6) bags of flour for the provided period

<u>Corner Points</u>

A= (0;6); B=(4;2); C=(5;0)

Calculation for point B

$$4x1 + 2y1 = 20 \quad (5)$$
$$1x1 + 1y1 = 6 \quad (6)$$

Therefore,
$$4(1x1 + 1y1) = 6 \quad (8)$$

Equation 2 – Equation 3

$2y1 = 4$
y1 = 2

Substitute (y1 = 2) into equation 2

$1x1 + 1(2) = 6$
x1= 4

4.2.2. Determining optimum solution using objective function.

Objective function: $Max\ Z = R8x1 + R6y1$.

A) $Max\ Z = R8x1 + R6y1$ $Max\ Z = R8(0) + R6(6)$ $Max\ Z = R36.00$	**B)** $Max\ Z = R8x1 + R6y1.$ $Max\ Z = R8(4) + R6(2)$ $\boldsymbol{Max\ Z = R44.00}$	**C)** $Max\ Z = R8x1 +$ $R6y1$ $Max\ Z = R8(5)$ $+ 6(0)$ $Max\ Z = R40.00$

Table 2: feasible solutions for production.

Table 2 indicates the different feasible solution that the frim can produce in order to yield the optimum solution. All these solutions lie within the indicated feasible area indicated in figure 1. According to the objective function, the firm yields a maximum profit of eight (8) Rands for every *X1* products it produces and a maximum profit of Six (6) Rands for every *Y1* products it produces. If the firm decides to produce zero *X1* products and six *Y1* products it will yield a maximum of thirty six Rand in profits, If the firm decides to produce four *X1* products and two *Y1* products it will yield a maximum of forty four Rands in profits and If the firm decides to produce five *X1* products and zero *Y1* products it will yield a maximum of forty Rands in profits, therefore producing four *X1* products and two *Y1* products is best decision the firm take as it yields a maximum profits according to the constraints the firm is challenged with.

4.2.3 validation of hypotheses

RQ_1- H_1

RQ_1- H_1 states that $y(PTFYMP)= 4(a_1) + 2(a_2)$ *whereby*, if $y(PTFYMP)$ is held constant, while a_2 increases by one-unit a_1 would decrease by 0.5.

The equation $4x1 + 2y1 \leq 20\ hours$ in section 4.2.1. provided the variable for a_1 as 5 and a_2 as 10. Therefore, if $y(PTFYMP)$ is held constant at 20, while a_2 increases by one-unit (i.e. from 5 to 6) the value of a_1 will result as 2, and this still lie within the feasible region presented in figure 1, Thus satisfying the hypothesis of correlation between the production process rate (independent variable) and the production time frame to yield maximum profit (dependent variable).

RQ2- H2

RQ2- H2 states that $y(SL)=(b_1) + (b_2)$, whereby if $y(SL)$ is held constant, while b_2 increases by one-unit b_1 would decrease by a single unit.

The equation $1x1 + 1y1 \leq 6$ bags in section 4.2.1. provided the variable for b_2 as 6 and b_1 as 0. Therefore, if b_2 increased by one unit (i.e. from 6 to 7) the value of b_1 will result as -1, and this will still lie within the feasible region presented in figure 1, thus satisfying the hypothesis of correlation between availability of inventory (independent variable) and services levels (dependent variable).

4.3. Transportation Model

Maximization of transportation costs of the final product to distributions centers, coffee shops and other franchises or end consumers.

The bakery has three Manufacturing plants in South Africa they are in Johannesburg, Durban and Cape town, the firms transport the baked products from the three plants to three Distribution centers located in Midrand, Maxmead, and Montague.

The production plants can be able to supply the following number of products on daily basis.

Manufacturing plant	Supply Capacity
1. JHB	300
2. DBN	300
3. CPT	100

Though the three distribution centers require the following in terms of capacity so that can be able to meet the demands of their customers.

Distribution centers	Demand required
Midrand	200
Montague	200
Maxmead	300

The cost of transportation for 1 kg of the baked products from each plant to distribution center is reflected in the table below:

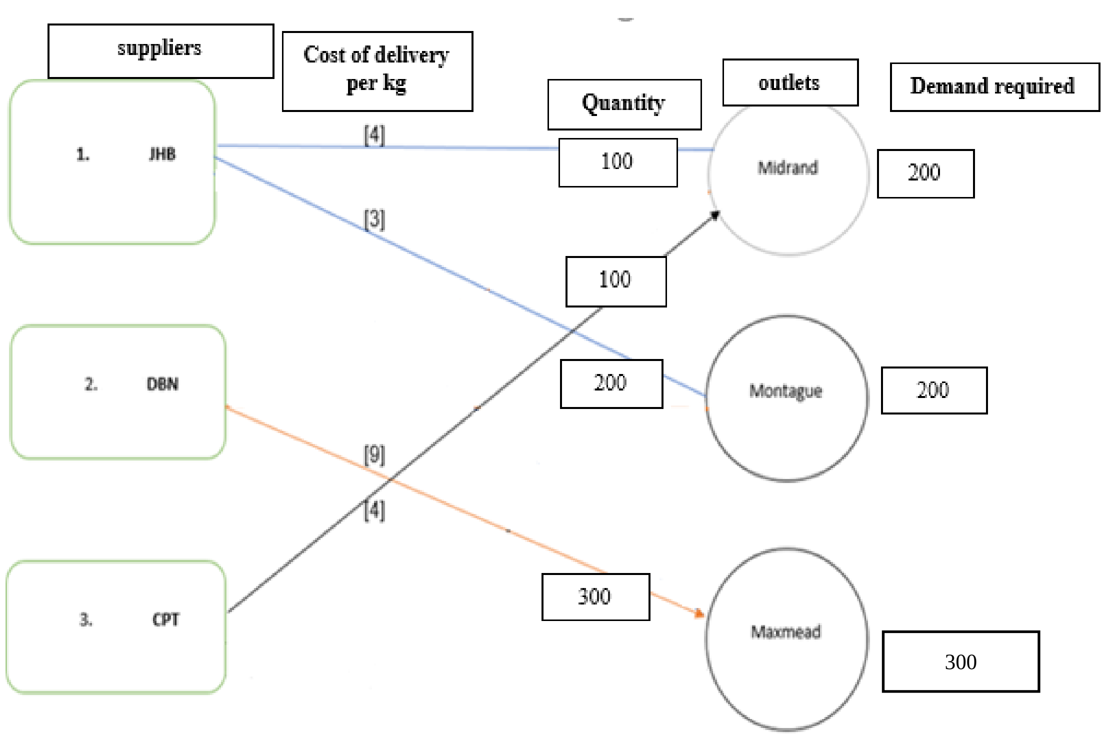
Table 1 (transportation cost table)

	Distribution centers					
Manufacturing plants		To	Midrand	Montague	Maxmead	Supply available (Kg)
	from					
	JHB		4	3	8	300
	DBN		7	5	9	300
	CPT		4	5	5	100
	Demand Required (kg)		200	200	300	

Below (figure 3) is an Illustration of the transportation model developed, as a recommendation to minimize transportation costs where as the main objective was to minimize fuel costs, vehicle depreciation and further satisfy the market requirements. The development of the model in figure 3 was aided by (Huseseini, et al., 2015) three stage supply network model where by three stage supply network model was developed with the intention of minimizing total costs of inventory acquired from partners, cost of distribution and costs associated with inventory shortage.

The objective function of figure 3 is represented as:

$$Z= 4X_{1A} + 3X_{1B} + 8X_{1C} +7X_{2A} + 5X_{2B} + 9X_{2C} + 4X_{3A} + 5X_{3B} + 5X_{3C}. \qquad (5)$$



(figure 3) Suggested Transportation model that contributed to lowest possible fuel costs.
In regard to figure 3, the Model yields a minimum cost of R 3 100 in relation to the objective function developed (5), thus also satisfying RQ_3- H_3.
It is noted that within this problem the total supply is 300 + 300 + 100 = 700 and the total demand is 200 +200 +300 = 700. Therefore, in this case total demand is equivalent to total supply making it a balanced transportation problem.

5. Conclusion

In this research, we were presented formulations for integrating production planning and distribution Scheduling of baked products within a supply chain. The problems are in relation to the supply chain network (a single manufacturing facility and multi delivery centers). There is also a fleet of internal vehicles that is in control for transporting the products from facility to distribution centers. In each period of Scheduling, a vehicle can at most make a single trip to a distribution center. The produced items are perishable, that is they can be stored for only a fixed period. The manufactured capacity is limited and there can be no shortage in distribution centers. The decisions to be made in each scheduling period are product quantities, distribution centers that should be visited, and the levels of inventory to be delivered. Based on the above a linear programming and a balanced transportation model where developed formulated. The proposed method is efficient considering solution for quality and time performance. In addition, it can provide robust results. In addition, it is proven that the complexity of the proposed method depends on linear multiplication of three parameters, number of distribution centers, vehicles and Scheduling period. The studied program also contains some limitations into decision making for other companies within this industry. Therefore, it can be concluded that the linear programming model for integrated Baking production and distribution Scheduling with raw materials considerations, is working well rather than independent production to distribution.

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Biography

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