Variant of the EOQ Model for Electronic Products with Short Life Cycle

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Abstract

The life cycle of electronic products is becoming shorter. Calculate the optimum quantity to buy and hold in inventory is a big challenge, must be incorporated the temporary variable into the mathematical model that describes the death of the product and therefore calculate the required inventory. Economic Order Quantity (EOQ) was used in this case study in coordination with the logistic function, originally applied to population growth, this is a convenient function for estimating economic batch versus time analogy. Thus, a nonlinear regression was used to estimate the parameters of a hybrid model, EOQ-curve S. Once applied this model, the total costs of inventories were reduced by about 67%, demonstrating the effectiveness achieved.

Keywords

Economic Order Quantity - EOQ, Product Life cycle, S-shaped curves, Logistics function, Inventory levels.

1. Introdution

The design of inventory models tailored to each company is important as they solve unique problems with the possibility of generalization. For the present case, we worked in a commercialization company of varied electronic products, which are sold by Internet. The main objective is to solve the problem of over inventory, and therefore, high costs of inventory maintenance, especially obsolescence products. The demand for short-lived products is described as an S-curve, allowing an optimal Q to be established over time, which minimizes costs substantially as observed in the results (Spedding & Chan, 2000). We used concepts of inventories, optimization, statistics and costs, a combination that together produced important results; Making the company more profitable. Then an empirical methodology was applied that allowed to formulate the model and to validate it by means of analysis of data, obtaining like result important theoretical savings.

The purpose of this article is to present an application of the Economic Order Quantity (EOQ) model considering the characteristics and variables of the product life cycle through the analysis of the S curves. One of the achievements was to design a dynamic model in the Time, sigmoid type adjusted to the life cycle of the product, which makes it closer to reality, unlike the original EOQ.

2. Literature review

Inventory management is a branch of operations management that arises from the need to optimize the economic resources allocated to the purchase or manufacture of products ensuring that these are available for sale when requested by the client. The questions that are answered with the methodologies of inventory analysis are related to the quantities to be ordered, the security inventory to be maintained, the quantity of orders to be made in the period, the frequency with which the orders are placed, and total logistic costs Generated by each model, (Krajewski, Ritzman & Malhotra, 2010).

The first model of inventory management was designed in 1913 by Ford Whitman Harris, this model is known as Economic Order Quantity (EOQ), today has innumerable variants and applications, see in detail the literature review of (Bushuev et al., 2015). The EOQ basically illustrates the behavior of inventories and allows to balance the costs of ordering and holding inventories, (Heizer & Render, 2007). Figure 1 graphically depicts the EOQ considering the behavior of inventory maintenance costs (CM), costs of ordering (CO), and the sum of both costs or total costs of the model (CT); The point Q is the quantity to be ordered that minimizes total costs (TC), (Ballou, 2004). The EOQ formula is derived from the equalization of the functions of the cost of ordering and the cost of maintaining. The cost of maintaining is proportional to the ordered quantity, while the cost of ordering is inversely proportional to the ordered quantity.

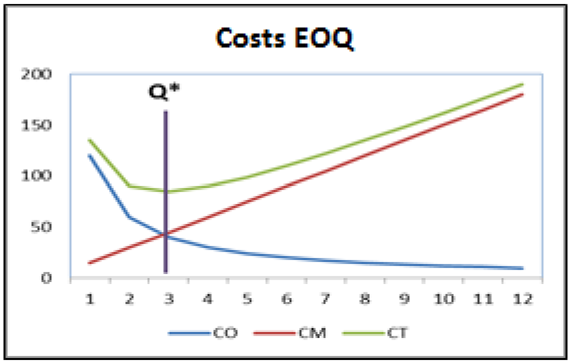

Figure 1. EOQ cost graph (Ballou, 2004)

Equation (1) expresses the equality:

$$CM = CO \qquad (1)$$

When performing the substitution of values, the equality is expressed (2):

$$\left(\frac{Q}{2}\right)h = \left(\frac{d}{Q}\right)k \qquad (2)$$

Finally, the formula (3) that determines the amount to be ordered considering both costs is:

$$Q^{*} = \sqrt{\frac{2kd}{h}} \qquad (3)$$

The notation of the presented model is:

k = Cost of ordering ($ / order)
h = Cost of maintenance (i * c)
c = Unit cost ($ / unit)
i = Inventory holding rate ($ / $ / time)
d = Demand rate (units / time)
Q = Economic order quantity (units)

On the other hand, a classification is necessary to delimit the study area. The best known tool is the Pareto or ABC classification to the analysis performed on a family of data whose purpose is to group them according to the percentage they represent of the basic criterion to be measured. The classification methodology holds that the highest income in the economy tends to be distributed according to the 80/20 rule, according to which 20 percent of the population concentrates 80 percent of the wealth of a nation. Pareto's assertion is that in all countries, a small fraction of society controls a high share of wealth (Aljure et al., 2010).

The application of the Pareto model in the management of operations commonly takes as valuation criteria the value of sales, the profit margin, the value of the inventory, among other aspects (Bowebox, 2007). The weights assigned to perform the classification in this work are listed below in Table 1.

Table 1. ABC classification elements

Classification	Accumulated percent contribution range	Percentage of sample size	Description
A	0%- 79%	0%- 9%	Data that require a greater degree of control, since they present the highest value of contribution to the basic criterion of measurement.
B	80%- 94%	10%-29%	Data that require a medium degree of control, since they present moderate value of contribution to the basic criterion of measurement
C	95%- 100%	30%- 100%	Data that require a low degree of control, since they present low value of contribution to the basic criterion of measurement

On the other hand, analyzing the product life cycle is an analogy of life, originated by neo-Darwinian biology (born, grow, reproduce and die) applied to goods or service. The dynamics of the market evolves significantly, the consumerism, the perception of the client and the constant creation of innovations, causes that the life cycle of the product today is very short. For electronic and digital products like telephones, computers, televisions and printers, the life cycle is for months, Chung (2012).

Table 2. Functional versus innovative products: differences in demand (Fisher, 1992)

Aspects of Demand	**Functional (Predictable Demand)**	**Innovative (Unpredictable Demand)**
Product Life Cycle	More than 2 years	3 months to 1 year
Contribution Margin*	5% to 20%	20% to 60%
Product Variety	Low (10-20 variants per category)	High (often millions of variants per category)
Average margin of error in the forecast at the time production is committed	10%	40% to 100%
Average stockout rate	1% to 2%	10% to 40%

Average forced end-of-season markdown as percentage of full price	0%	10% to 25%
Lead time required for made-to-order products	6 months to 1 year	1 day to 2 weeks
*The contribution margin equals price minus variable cost divided by price and is expressed as a percentage.		

The product life cycle (CVP) is defined as the period of time elapsed since a product goes through the design, growth, maturity, decline to disposal phases (Krishnamoorthi, 2012).

The product lifecycle drastically influences the organization's strategy, the supply chain and especially in inventory management, Fisher (1992), in Table 1, defines the short life cycle between 3 months and One year, Table 2.

Figure 2 shows the product life cycle stages and the sales levels (Q) for each period (t). During the introduction phase the product is placed on the market and acceptance is low; In growth the product begins to have better sales due to the efforts put in during the launch; In maturity, sales grow continuously and consolidate reaching the highest point of the cycle; And during the decline and discard sales begin to decline quite as the product is replaced by new products.

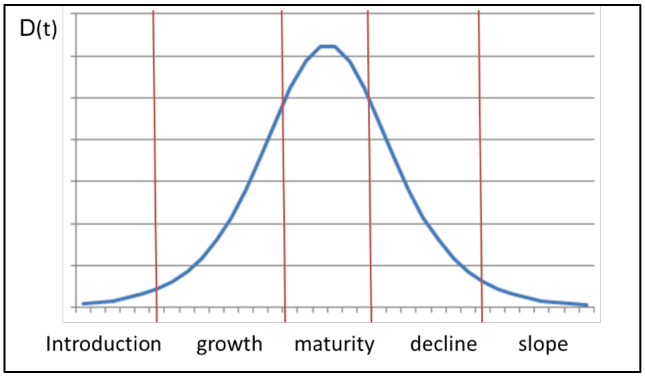

Figure 2. Product Life Cycle (Kucharavy et al., 2007).

The product life cycle can be represented by a S curve, provided that the performance function (sales or demand) is graphically accumulated historical data. Figure 3 presents the number of accumulated units that are placed on the market during the product life cycle (ΣQ (t), accumulated sales (Aguilar, Avalos, Giraldo, Quintero, Zartha & Cortes, 2012).

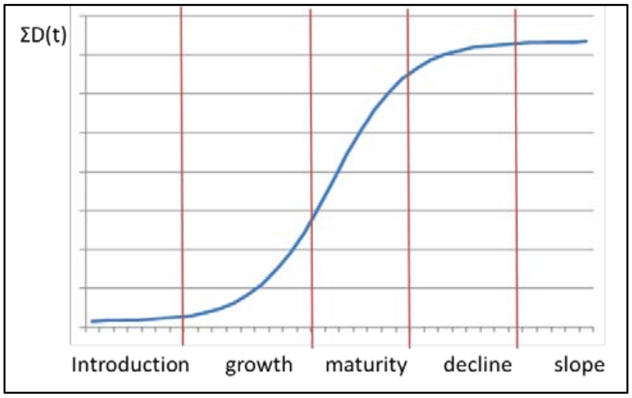

Figure 3. Curve S of the product life cycle (Buccini et al., 2011)

An S-shaped curve is usually adopted to define the product life cycle (ΣQ (t), accumulated sales), indicating the number of units that are placed on the market and therefore allows estimating the rate of return on the Based on the demand rate of a particular phase, as well as the calculation of the quantity to be kept in inventory (Buccini et al., 2011).

Logistic function as a model of population growth was first introduced by the Belgian mathematician Pierre-François Verhulst in 1838. The logistic equation was introduced to describe the self-limiting growth of a population. The equation is sometimes called the Verhulst-Pearl Equation. Today this model is often used to describe the dynamics of systems.

$$N(t) = \frac{m}{1+e^{-(ct+d)}} \qquad (4)$$

m = function asymptote
c = Growth parameter
d = Point of inflection

A product lifecycle can be divided into several stages characterized by revenue generated.

3. Description of the problem

This article aims to find a model for products with defined short life cycles between 12 and 52 weeks (Fisher, 1997), which minimizes the total inventory costs related to storage, ordering, missing and excess inventory.

The company is characterized by:

- Offer more than 250,000 products.
- To be the largest investment in online marketing in Latin America.
- Be addressed to all sectors and ages.
- Have a team of more than 1000 people.

It is a growing company that seeks to strengthen its operation in a technical way, with concrete results. Inventory problems focus on excess inventory, technological obsolescence, and high maintenance costs, together with the inability to predict shortages and product deaths. The methodology that will be presented in the development of this article was applied to data provided by the commercial company in Colombia, this company has the need to improve the model of replenishment considering, therefore, to increase the level of customer service and reduce costs. The initial situation of the trading company is described below:

- The company did not have effective inventory policies, so it had excess inventory of low demand codes and missing high demand codes.
- Control of inventories was done by means of a kardex systematized based on the Regional Repurchasing Model (own application). While the forecast of demand, quantities and frequency of purchase were calculated with a model based on the average weekly sales.
- According to a survey conducted by the company, 17% of customers were completely dissatisfied with the service provided by the company. The findings of the study indicated that the root cause of dissatisfaction was related to shortcomings and deficiencies in the inventory.

Leftovers and missing are basically caused by the current EOQ calculation, which they perform in a traditional way. For example, for A1 the forecasted demand is 13,040 units, the company calculates it based on average sales up to the current period 250 units (22 weeks), average multiplied by 52 weeks (this forecast raises orders substantially and by Both inventories), linearity is assumed for the projection. The costs of ordering and maintaining inventory are already calculated, for unit and per month, Table 3.

The stock or missing stock is calculated by subtracting the stock from stock. The estimated EOQ, for A1 is 53 units, the costs of ordering and storing are calculated, they add up to \$ 4,194,447, finally the costs for missing or surplus are added, corresponding to 50% of unit costs for the number of Units, for A1, \$ 26,561,503, for a total of \$ 800,161,720, only for 10 items and for 22 weeks of operation. Total inventory costs correspond to the sum of the costs of ordering more storage costs and the costs of singers or over-stock.

All calculations are for week 23 and are projected through week 52.

Table 3. Total inventory costs. Current cost situation for 10 items with own model

Name SKU	Average sales	Unit cost	Stock in Warehouse	Forcast	Cost Order	Cost Storage	EOQ (Q)	Over stock or missing	Costs Over stock and missing	Inventory costs	Total Costs of inventory
A1	250	1.002.321	154	13.040	16.207	41.622	101	53	26.561.503	4.194.447	30.755.950
B1	165	161.705	639	8.578	4.625	3.119	160	479	38.728.304	497.543	39.225.848
C1	151	706.987	0	7.859	28.617	11.226	201	- 201	71.052.217	2.247.166	73.299.382
D1	137	179.894	29	7.155	19.729	2.765	320	- 291	26.174.610	883.520	27.058.131
E1	116	24.226	3	6.021	119.395	2.669	734	- 731	8.854.669	1.959.064	10.813.733
F1	115	515.633	0	5.978	108.991	2.406	737	- 737	190.010.672	1.770.537	191.781.209
G1	112	22.267	24	5.846	126.255	2.110	837	- 813	9.051.458	1.764.763	10.816.221
H1	109	1.142.484	2	5.692	66.404	5.136	384	- 382	218.214.482	1.970.574	220.185.057
I1	108	276.415	0	5.628	84.027	2.615	602	- 602	83.200.885	1.572.781	84.773.666
J1	106	736.148	52	5.518	67.345	6.130	349	- 297	109.317.932	2.134.593	111.452.525
								Totales	**781.166.732**	**18.994.988**	**800.161.720**

4. Methodology

The EOQ assumes a constant demand and an improvement is to recognize that the demand varies with the time and in particular it diminishes, which as a direct consequence brings that the EOQ varied with the time. The 22-week sales database was taken for time series analysis; and for the life cycle time was defined as 52 weeks maximum. Finally, using a software, the parameters of the logistic function were optimized and by means of the minimization of the mean square error the adjustment curve was found. The product life cycle was divided into several stages characterized by revenue generated. The proposed model proposes to estimate the optimum production lot size (EOQ) and the total cost during the Product Life Cycle through the different phases.

For each t, a Q* is calculated, since the demand has a sigmoid behavior:

$$y_t = a + \frac{b}{1 + e^{-c(t-p)}} \tag{5}$$

Notation:

y_t = Value of the function in period t
$a + b$ = function asymptote
c = Growth parameter
p = Point of inflection

Where y_t is the cumulative sales function, which describes the product life cycle.

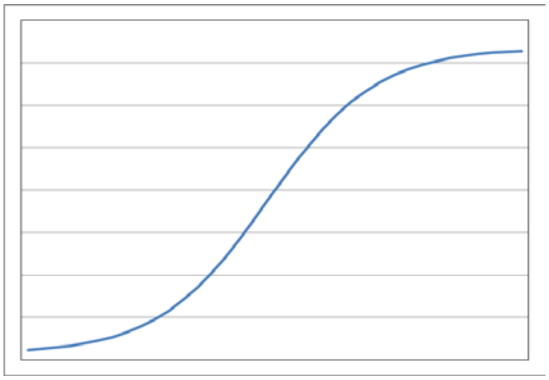

Figure 4. Example, Curve S, cumulative sales

The a and b is the upper bound or maximum estimated sale, c growth parameter, t the time or analysis index and p the point in time where sales start to decrease.

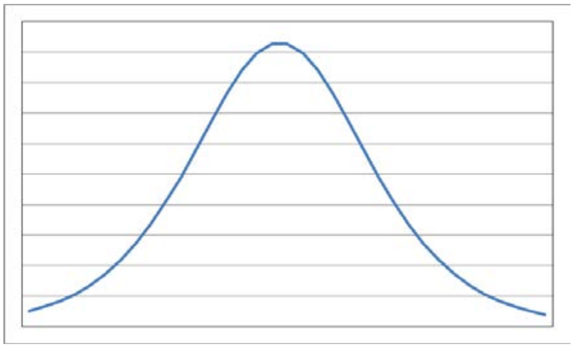

Figure 5. Example, S curve sales behavior

Therefore the demand d_t is equal to, the expected maximum of sales and less accumulated sales until the projected period t:

$$d_t = y_m - y_t \qquad (6)$$

Where:

$$y_m = \sigma/r + y_r \qquad (7)$$

The y_r are actual cumulative sales in period, r = 22, and t is the period with a planning horizon of 52 weeks.

$$d_t = y_m - \frac{b}{1 + e^{-c(t-p)}} \qquad (8)$$

Then the Q* is equal to:

$$Q_t^* = \sqrt{\frac{2k\left(y_m - \left(\frac{b}{1 + e^{-c(t-p)}}\right)\right)}{h}} \qquad (9)$$

So:

$$Q_t^* = \sqrt{\frac{2kd_t}{h}} \qquad (10)$$

5. Analysis of results

To estimate the parameters of the following logistic function:

$$y_t = \frac{b}{1 + e^{-c(t-p)}} \qquad (11)$$

The mean square error (MSE) was calculated, then by Excel solver, it is minimized by varying the parameters, until the optimum values are obtained.

$$Min\ MSE = \frac{1}{n}\sum_{i=1}^{n}(y_r - y_t)^2 \qquad (12)$$

Table 4. Curve-s estimation

Semanas	Yr	Yt	MSE	dt	Qt
1	1323	1539	46671	3732	617
2	2337	2404	4547	2866	536
3	3640	3323	100411	1948	437
4	4227	4092	18289	1179	337
5	4553	4617	4068	654	248
6	4716	4928	44789	343	178
7	4798	5096	89007	175	126
8	4948	5184	55504	87	88
9	4989	5228	56928	43	62
10	5030	5250	48182	21	43
11	5096	5260	27009	11	30
12	5129	5266	18684	5	21
13	5170	5268	9667	3	15
14	5183	5270	7502	1	10
15	5307	5270	1350	1	7
16	5407	5271	18614	0	5
17	5438	5271	27983	0	4
18	5483	5271	45031	0	3
19	5504	5271	54367	0	2
20	5504	5271	54358	0	2
21	5505	5271	54821	0	1
22	5509	5271	56708	0	1
Totales	**5509**	**5271**	**38386**	**11070**	**2773**

The values of the parameters obtained in the minimization were:

b = 5270.86
c = 0.71
p = 2.25

When replacing them, we have the curve y_t, in addition, *b* is the upper or the maximum estimated sale, *c* growth parameter, *t* the time or analysis index and *p* the point in time where sales begin to decrease, inflection point, figure 6.

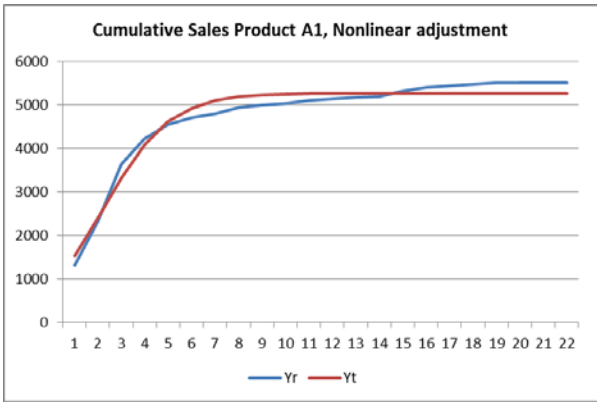

Figure 6. S curve, cumulative and adjusted sales.

In figure 7, it is evident that the Q_t decreases as the life cycle of the product A1 is exhausted.

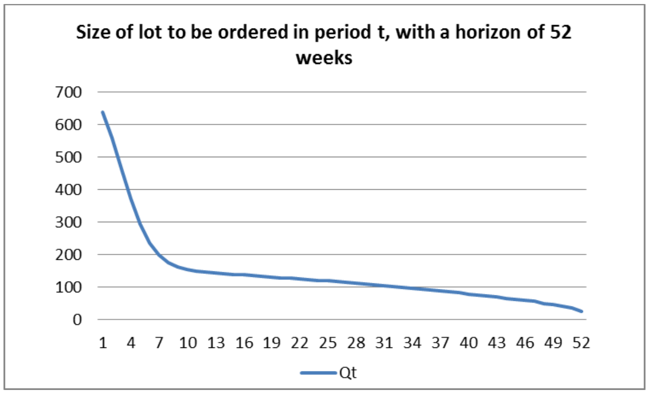

Figure 7. Curve Q* as a function of t.

Applying the dynamic EOQ model with short life cycle, sigmoid, logistic curve or S-curve, the order sizes decrease, that is, optimum Q as a function of time t, increasing the costs of ordering, but decreasing the costs of obsolescence of the inventory.

Table 5. Calculation of costs with the dynamic EOQ model. Cost situation for 10 items with the new model

Name SKU	Average sales	Maximum expected sales	Unit cost	Stock in Warehouse	Cost Order	Cost Storage	Q(t)	Over stock or missing	Costs Over stock and missing	Inventory costs	Total Costs of inventory
A1	250	5.530	1.002.321	154	16.207	41.622	4	150	75.174.064	22.487.930	97.661.994
B1	165	3.639	161.705	639	4.625	3.119	7	632	51.098.723	2.415.354	53.514.077
C1	151	3.337	706.987	0	28.617	11.226	10	- 10	3.534.936	9.606.659	13.141.595
D1	137	3.034	179.894	29	19.729	2.765	13	16	1.439.154	4.622.220	6.061.374
E1	116	2.551	24.226	3	119.395	2.669	28	- 25	302.827	10.916.117	11.218.944
F1	115	2.535	515.633	0	108.991	2.406	31	- 31	7.992.308	8.951.549	16.943.857
G1	112	2.481	22.267	24	126.255	2.110	39	- 15	167.001	8.073.560	8.240.561
H1	109	2.414	1.142.484	2	66.404	5.136	16	- 14	7.997.389	10.061.624	18.059.013
I1	108	2.396	276.415	0	84.027	2.615	34	- 34	4.699.053	5.965.235	10.664.289
J1	106	2.340	736.148	52	67.345	6.130	14	38	13.986.806	11.297.911	25.284.717
								Totales	**166.392.262**	**94.398.160**	**260.790.422**

The minimization was performed in the Solver optimizer to make the required nonlinear adjustment in the logistic function, finding the mean square error and estimating the optimal parameters. The results are very favorable for the proposed model, as shown in Table 5, for the first 10 products classified as A, in general achieves a cost reduction of 67.40%, in relation to the original model of the company.

Table 5. Summary of costs with the dynamic EOQ model

Model	Cost of Over Stock and Missing	Costs of Inventory	Total Cost the inventory
EOQ	781.166.732	18.994.988	800.161.720
Q(t)	166.392.262	94.398.160	260.790.422
Saving	**614.774.470**	**(75.403.172)**	**539.371.299**

The EOQ model is the one used by the company. Costs were calculated for the first 10 items (SKUs). When applying the dynamic model Q (t) based on the logistic function, the net saving is around $ 540 million. Tables 3 and 5.

In addition to the obvious achievements in cost reduction for inventory management, warehouse space was freed up and the efficiency of hand-to-hand operation over customer service increased.

Future research work should focus on a forecasting methodology and operations management for short-lived products.

CONCLUSIONS

It was possible to design a model for the control of inventories, for producer of short life cycle. This allowed us to reduce the costs of maintaining inventory and, in particular, to minimize obsolete inventories, also increasing turnover.

Today products have a very short lifecycle, which is not covered by most inventory models, but more and more often find sales structures based on innovation, reducing the life cycle of products and demanding a low level of inventory.

The techniques proposed here help to reduce inventory costs by approximately 67.40%, compared to the methodology used by the company to estimate the economic lot.

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