Stochastic inventory control and distribution of blood products

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Abstract

Inventory control in perishable products supply chain is one of the biggest challenges today, especially for medicines and blood products supply chain. Shortage can increase the mortality risk at hospitals, on the contrary, high levels of inventory could generate wastage of these resources. This paper studies the problem of inventory control and distribution of blood products. This study determines the number of blood units to be processed by the blood center and the number of units of blood products to be ordered by hospitals to minimize the total cost and the shortage and wastage levels in blood supply chain. Two optimization models are formulated: A Mixed Integer Linear Programming (MILP) Model for known demands and a Stochastic Programming (SP) Model for the case where demands are uncertain, considering multiple periods, types of blood and life time of products. Datasets are generated to evaluate the efficiency of proposed models for a multi-hospitals single-blood center system. An algorithm is developed to simulate the supply chain and evaluate the mathematical models. The results show that the SP Model obtains lower expected rates of shortage and wastage compared to the deterministic model. In this last, demands are approximated by their mean values.

Keywords
Optimization, Perishable products, Stochastic programming, Blood supply chain.

1. Introduction

Perishable products differ from nonperishable products in that the former can lose their usefulness if they are not properly stored and transported or if they are not used within their shelf life. Inventory control in perishable products supply chain is one of the biggest challenges today; especially for products requiring a very high service level such as medicines and blood products. Shortage of blood can increase the mortality risks at hospitals, on the contrary, high levels of inventory could generate wastage of this scares resource. Patients may receive Whole Blood (WB) or components of blood. Up to four components can be derived from donated blood. The main transfusable blood components include: Red Blood Cells (RBC), Platelets (PLT), plasma and cryoprecipitated AHF condition [1]. Components can be mechanically separated from a unit of WB, called whole blood-derived (WBD), or can be obtained using apheresis, an automated procedure that filter the desired components from the blood while the remaining components flow back into the donor [1]. Table 1 presents principal uses of main transfusable blood components and their shelf life. PLTs are the blood component with the shortest shelf life. Shortage and outdated units of blood are undesirables through supply chain. Transfusion of WB, RBC and Plasma units decrease between 2008 and 2011. On the contrary, the transfusion of PLT units to US patients in 2011 increased by 7.3% from 2008, especially the apheresis PLT transfusion with an increase of 11.9% [2]. The National Blood Collection and Usage Report presents data on outdated components as a percentage of the total number of units of each type processed for transfusion in 2011. PLT were the blood component with the greatest percentage of outdated units with 17.1% for WBD PLT and 12.8% for Apheresis PLT compared to WB and RBC (2.4%), plasma (2.2%) and Cryoprecipitate (3.3%) [2].

Table 1. Principal characteristics of blood type [1]

Blood component	**Uses**	**Shelf life**
Whole blood	Trauma and surgery	21/35 days[1]
RBCs	Trauma, surgery, anemia, any blood loss and blood disorders	Up to 42 days[1]
PLTs	Cancer treatments, organ transplants and surgery	5 days
Plasma	Burn patients, shock and bleeding disorders	1 year
Cryoprecipitated AHF	Hemophilia and hereditary coagulation abnormalities	1 year
[1]Shelf life of whole blood and red cells varies based on the anticoagulant type used		

Because of highly perishability and medical importance of PLT, we address a PLT inventory control and distribution problem and present mathematical programming models to minimize the total cost while minimizing outdate and shortage. The study can be extended to others transfusable blood products.
The rest of the document is structured as follows. Section 2 presents a review of the relevant literature on supply chain of perishable products with a focus on blood products. In section 3, the principal concepts of blood supply chain are explained and the problem studied is defined. The model formulation is shown in section 4. Numerical results of the models are given in section 5. Concluding remarks and opportunities for future work are presented in section 6.

2. Literature review

2.1 Works related to perishable products supply chains

Supply chain optimization of perishable goods has attracted much attention in the literature due to its relevance in real life applications. Perishable products are goods which obtain lower utility when they approach to the end of their maximum lifetime. Demirag et al. 2016 [3] study inventory ordering policies where the demand for the product is positively related with the freshness level. They build a classical Economic Order Quantity (EOQ) model to determine the optimal order quantity Q to maximize the profit. They solve the problem considering two different cases: the first case deal with linear demand function and the second with a demand function that is concave decreasing with the age of product. Other authors introduce stochastic parameters: Kouki et al. 2015 [4] consider an inventory system employing a continuous review (r, Q) policy and present an algorithm to compute the best r and Q parameters that minimize the total cost. They assume that demand follows a Gamma distribution. The Inventory Routing Problem (IRP) address the coordination of two supply chain components: the inventory management and the vehicle routing. Shaabani et al. 2016 [5] presents a multi-period multi-product IRP. A mathematical model is formulated considering deterministic demand. The IRP is divided into two subproblems: the vehicle routing subproblem is solved using a heuristic and its solution is used as input in the inventory subproblem. A Population-Based Simulated Annealing algorithm is proposed to moderate computational time. Soysal et al. 2015 [6] present a multi-period IRP model that includes an evaluation of CO_2 emission and fuel consumption. They formulate the IRP as a chance-constrained programming model to capture the risk associated with uncertain demand and present a deterministic approximation. After, a simulation model is proposed to evaluate the solutions of this deterministic approximation. Hybrid approaches proposed in the literature offer the advantages of simulation based methodologies together with the optimization capabilities of mathematical programming models, especially when models consider uncertain parameters. Keizer et al. 2015 [7] present a MILP model for multiple customers and multiple products in a cut flower supply chain, considering product quality decay. Results of the MILP are introduced in a discrete event simulation (DES) model which simulates fluctuations in supply quality, quantity, temperature, transportation duration and processing that follows a normal distribution function. A service level is obtained from the DES model which is used to iteratively update the products quality constraints in the MILP. Ahumada et al. 2012 [8] present a stochastic model for the production and distribution of fresh agricultural products, approached by a two-stage stochastic model, to minimize the expected revenue. The solution depends on the first stage decisions (planting), the random realizations of stochastic parameters, including the price and yield of products, and the second stage decisions (harvesting and distribution). The size of the model grows with the number of scenarios computed. For this reason, they consider the stochastic version of Bender's decomposition (L-shaped method) and multi-cut for risk stochastic programs.

2.2 Works related to blood products supply chains

Problems related to inventory control and distribution of blood products have been analyzed and solved using wide range of methods. Some studies use simulation methodologies and introduce hybrid metaheuristic. Duan et al. 2013 [9] propose a replenishment policy based on old inventory ratio which use only partial age information to measure the freshness of the blood inventory. They develop both centralized and decentralized models and propose a simulation optimization approach based on a hybrid metaheuristic algorithm to find near-optimal policies. In decentralized

control, each entity in the supply chain is treated as an individual company aiming to minimize its own outdate rate. However, the centralized control directly minimizes the system-wide outdate rate. Duan et al. 2014 [10] propose a new simulation optimization framework. Instead of treating RBC units as one unified product, the study considers all eight ABO/Rh(D) types and their compatibility for substitution. They develop a simulation optimization approach based on a new hybrid metaheuristic algorithm constituted of two cooperative metaheuristics algorithms, Threshold Accepting and Tabu Search to find near-optimal replenishment policies in acceptable computational time. Markov Processes are also used to handle stochastic parameters in some studies. For instance, Civelek et al. 2015 [11] study a discrete-time inventory where demand exists for product of different ages. Their model includes substitution costs incurred when a demand for a certain-aged item is satisfied by a different-aged item. They model the problem as a Markov Decision Process and propose a simple inventory replenishment and allocation heuristic to minimize the expected total cost.
The following authors suggest mathematical programming to model the problem which is then solved by mean of solvers. Some of them combine this tool with simulation or develop metaheuristics to obtain solutions in reasonable computational times: Hemmelmayr et al. 2010 [12] establish a two-stage stochastic program. First, they introduce an integer programming approach to handle stochastic demand considering demand realizations. Second, due to the size of the integer program which increase quickly with the number of realizations, the authors adapt a metaheuristic, a Variable Neighborhood Search approach, to solve the problem in shortest computational times. The study of Gunpinar et al. 2015 [13] presents integer programming models. The first model is a stochastic integer programming model that handles demand uncertainty. The second one is a stochastic integer programming model with two patient types: type 1, requires fresh/young blood and type 2 which could use blood of any age. The last model is a deterministic integer programming model with Crossmatch-to-Transfusion ratio (C/T ratio) (description in section 3).
One of the challenges of the blood supply chain that some authors consider is the issue of disasters. Zhou et al. 2011 [14] analyzes a periodic review inventory system under two replenishment modes: regular orders, placed at the beginning of a cycle, and expedited orders within the cycle. They first formulate the problem as a one-period model and then as a multicycle model and derive the necessary and sufficient conditions for the optimal policy. Second, an algorithm is designed based on the optimality conditions. Roni et al. 2015 [15] and Roni et al. 2016 [16] propose a stochastic inventory system facing regular demand and surge demand that has a lower arrival rate but higher demand volume per arrival. A MILP model is developed to obtain an optimal policy. Roni et al. 2016 [16] allow split deliveries which consider the scenario where there is a single supplier who replenishes all orders using multiple shipments and they develop a Tabu Search-Based algorithm to find high quality solutions in a reasonable amount of time. Hosseinifard et al. 2016 [17] propose the structure where some of the hospitals near of each other maintain centralized inventories to serve their demands and demands of other neighbor hospitals. They model the blood supply chain, where the items arrive to the blood center stochastically and stochastic demand is realized at hospitals.

3. Blood supply chain and description of the model

The blood supply chain, studied here, consists of two echelons. The first echelon corresponds to the blood center and the second to hospitals. The process starts with collections, which are done by blood centers or hospitals. Blood units collected by the blood center are obtained from donors either directly at a blood center or through mobile units. The supply of donor blood is irregular and there are several factors that affect the blood products availability such as number of the volunteering donors in a region and donation campaigns. Collected whole blood units or blood products through apheresis are tested to make sure that diseases cannot be transmitted through blood transfusion. After testing process, the whole blood units are sent to an initial storage in blood center and then, they are either stored for transfusion directly or mechanically separated into components. Tested components obtained through apheresis, WBD products and whole blood for transfusion remain in storage in the blood center until a hospital order arrives and then they are sent to supply the requirements of hospitals. Units in the inventory of a blood center can supply other blood centers. When requested blood products arrive at the hospital, they enter in an unassigned inventory. The hospital blood bank receives a doctor's order for a patient and then the required blood products enter in a process known as cross matching, in which the blood compatibility between the donor and the patient is determined. The cross-matched blood is moved from unassigned inventory to assigned inventory. Any untransfused cross-matched blood product return to the hospital blood bank. The relation between the total number of cross-matched units and transfused units is called crossmatch-to-transfusion ratio (C/T ratio). A patient should be transfused with his/her same blood group. If patient's blood group is unavailable, a compatible group must be provided. For clinical reasons, some compatible blood types are preferable to others. First and second echelon in blood supply chain can present outdate and shortage units. These situations are more undesirables in the second echelon. Outdated units in hospitals are more expensive due to transportation costs and shortage at hospitals which put the live of patients directly at risk.

We consider a two-echelon supply chain consisting of one blood center and several hospitals. The study considers all eight ABO/Rh(D) type of blood. We assume that the blood center is the only one in charge of making the collections and the supply of blood units is sufficient to meet the production orders of the blood center. At the beginning of each period, the blood center observes the total number of on-hand inventory of blood products and their age distribution and decides the number of units to produce for each blood type before the hospital orders arrive. The process of testing and producing blood units will take 1 day to complete so they will not be available for distribution until the next day. At the end of each day, hospitals observe their on-hand inventory and decide the number of units for each type of blood to order for satisfying the demand of the following day. The blood center receives orders from hospitals and satisfies each hospital order from its available stock following a FIFO policy at the beginning of the next day considering zero lead time for supply. If orders cannot be fulfilled from the available stock, a shortage for the blood center is registered. Hospitals face the demand in each period. First, we assume the demand as a deterministic parameter. Second, the demand is assumed as a stochastic parameter that follows a known distribution. To satisfy the demand, hospitals use their on-hand inventory following a FIFO policy. If demand is not satisfied, a shortage cost is incurred. Both in the blood center and in hospitals, if a unit of blood expires, a wastage cost is generated.
The objective is to minimize the total cost while minimizing the shortage and outdated units in blood supply chain. We consider a cost of replenishment per unit, a holding cost for every item carried from one period to the next, an expiration cost for units that outdate and a shortage cost for any demand that is no satisfied in any period.

4. Model formulation

Table 8, Table 9 and Table 10 summarize the indices, parameters and decision variables used in the models.

Table 2. Indices for models

a	Age of blood, $a = 1,2, \dots, A$ (days)	j	Hospitals, $j = 1,2, \dots, J$
g	Blood type, $g = 1,2, \dots, G$	k	Scenarios, $k = 1,2, \dots, K$
t	Period in the planning horizon, $t = 1,2, \dots, T$		

Table 3. Parameters for models

A	Lifetime of blood product	J	Number of hospitals
G	Blood type	T	Length of planning horizon
K	Number of scenarios	pc	Production cost per unit of product
c	Purchase cost per unit of product	wc	Wastage cost per unit of product
sc	Shortage cost per unit of product	M	Big number
h	Holding cost per unit of product held in stock per unit of time		
inv_{agj}	Units of a days old in initial inventory for blood type g in hospital j		
$invbc_{ag}$	Units of a days old in initial inventory for blood type g in the blood center		
$d_{gjt}^{(k)}$	Demanded units of blood type g by the hospital j in period t (for scenario k)		
p_k	Probability associated to scenario k		

Table 4. Decision variables for deterministic and stochastic models[1]

YFS_{agj}	Units of a days old of blood type g ordered by the hospital j at the end of the first period
$Y_{agjt}^{(k)}$	Units of a days old of blood type g ordered by the hospital j at the end of period t, ($t = 2, \dots, T$, for scenario k)
$F_{agjt}^{(k)}$	Units of a days old of blood type g used to satisfy the demand of hospital j in period t, $t = 2, \dots, T$, for scenario k
IFS_{agj}	Inventory level of a days old of blood type g in hospital j at the end of fist period
$I_{agjt}^{(k)}$	Inventory level of a days old of blood type g in hospital j at the end of period t, $t = 2, \dots, T$, for scenario k
OFS_{gj}	Outdated units of blood type g in hospital j at the end of fist period
$O_{gjt}^{(k)}$	Outdated units of blood type g in hospital j at the end of period t, $t = 2, \dots, T$, for scenario k
$S_{agjt}^{(k)}$	Shortage number of a days old of blood type g in hospital j in period t, ($t = 2, \dots, T$, for scenario k)
$V_{agjt}^{(k)}$	1 if a days old blood of type g is used to satisfy the demand of hospital j in period t, ($t = 2, \dots, T$, for scenario k), 0 otherwise

$W_{agjt}^{(k)}$	1 if all a days old blood of type g is used to satisfy the demand of hospital j in period t, ($t = 2, \dots, T$, for scenario k), 0 otherwise
XFS_g	Units of blood type g to produce by the blood center in first period
$X_{gt}^{(k)}$	Units of blood type g to produce by the blood center in period t, ($t = 2, \dots, T$, for scenario k)
$IBFS_{ag}$	Inventory level of a days old of blood type g in blood center at the end of first period
$IB_{agt}^{(k)}$	Inventory level of a days old of blood type g in blood center at the end of period t, ($t = 2, \dots, T$, for scenario k)
$OBFS_g$	Outdated units of blood type g in blood center at the end of first period
$OB_{gt}^{(k)}$	Outdated units of blood type g in blood center at the end of period t, ($t = 2, \dots, T$, for scenario k)
$SB_{agjt}^{(k)}$	Shortage number of a days old of blood type g for the hospital j in period t in blood center, ($t = 2, \dots, T$, for scenario k)
$VB_{agt}^{(k)}$	1 if the blood center uses a days old blood of type g to satisfy the demand in period t, ($t = 2, \dots, T$, for scenario k), 0 otherwise
$WB_{agt}^{(k)}$	1 if the blood center uses all a days old blood of type g to satisfy the demand in period t, ($t = 2, \dots, T$, for scenario k), 0 otherwise
[1]Notations in parenthesis are added to the notation for the stochastic model	

4.1 Deterministic programming model

In this section, the MILP formulation is presented for the inventory control and distribution problem described in section 3. The deterministic programming model is stated as follows:

$$\text{Min} \quad pc\left(\sum_{g=1}^{G}\sum_{t=1}^{T} X_{gt}\right) + c\left(\sum_{a=1}^{A}\sum_{g=1}^{G}\sum_{j=1}^{J}\sum_{t=1}^{T} Y_{agjt} - SB_{agjt}\right) + h\left(\sum_{a=1}^{A}\sum_{g=1}^{G}\sum_{j=1}^{J}\sum_{t=1}^{T} I_{agjt} + \sum_{a=1}^{A}\sum_{g=1}^{G}\sum_{t=1}^{T} IB_{agt}\right) + wc\left(\sum_{g=1}^{G}\sum_{j=1}^{J}\sum_{t=1}^{T} O_{gjt} + \sum_{g=1}^{G}\sum_{t=1}^{T} OB_{gt}\right) + sc\left(\sum_{a=1}^{A}\sum_{g=1}^{G}\sum_{j=1}^{J}\sum_{t=1}^{T} S_{agjt} + SB_{agjt}\right) \qquad (1)$$

Subject to,

$$I_{agjt} = inv_{agt}, \quad a = 1,2,\dots,A-1, t = 1, \forall g, j \qquad (2)$$

$$I_{agjt} = I_{(a-1)gj(t-1)} + Y_{agj(t-1)} - SB_{agjt} - F_{agjt}, \quad a = 1,2,\dots,A-1, t = 2,3,\dots,T, \forall g, j \qquad (3)$$

$$O_{gjt} = inv_{agj}, \quad a = A, t = 1, \forall g, j \qquad (4)$$

$$O_{gjt} = I_{(a-1)gj(t-1)} + Y_{agj(t-1)} - SB_{agjt} - F_{agjt}, \quad a = A, t = 2,3,\dots,T, \forall g, j \qquad (5)$$

$$d_{gjt} = \sum_{a=1}^{A} F_{agjt} + S_{agjt}, \quad t = 2,3,\dots,T, \forall g, j \qquad (6)$$

$$F_{agjt} \le M \cdot V_{agjt}, \quad t = 2,3,\dots,T, \forall a, g, j \qquad (7)$$

$$F_{agjt} \ge V_{agjt}, \quad t = 2,3,\dots,T, \forall a, g, j \qquad (8)$$

$$V_{(a-1)gjt} \le W_{agjt}, \quad a = 2,3,\dots,A, t = 2,3,\dots,T, \forall g, j \qquad (9)$$

$$Y_{agj(t-1)} - SB_{agjt} + I_{(a-1)gj(t-1)} - F_{agjt} \le \left(1 - W_{agjt}\right) \cdot M, \quad t = 2,3,\dots,T, \forall a, g, j \qquad (10)$$

$$Y_{agj(t-1)} - SB_{agjt} + I_{(a-1)gj(t-1)} - F_{agjt} \ge 1 - W_{agjt}, \quad t = 2,3,\dots,T, \forall a, g, j \qquad (11)$$

$$IB_{agt} = X_{gt}, \quad a = 1, \forall g, t \qquad (12)$$

$$IB_{agt} = invbc_{ag}, \quad a = 2,3,\dots,A-1, t = 1, \forall g \qquad (13)$$

$$IB_{agt} = IB_{(a-1)g(t-1)} - \sum_{j=1}^{J} Y_{agj(t-1)} - SB_{agjt}, \quad a = 2,3,\dots,A-1, t = 2,3,\dots,T, \forall g \qquad (14)$$

$$OB_{gt} = invbc_{ag}, \quad a = A, t = 1, \forall\, g \qquad (15)$$

$$OB_{gt} = IB_{(a-1)g(t-1)} - \sum_{j=1}^{J} Y_{agj(t-1)} - SB_{agjt}, \quad a = A, t = 2,3,\dots,T, \forall g \qquad (16)$$

$$\sum_{j=1}^{J} Y_{agj(t-1)} - SB_{agjt} \le M \cdot VB_{agt}, \quad t = 2,3,\dots,T, \forall a, g \qquad (17)$$

$$\sum_{j=1}^{J} Y_{agj(t-1)} - SB_{agjt} \ge VB_{agt}, \quad t = 2,3,\dots,T, \forall a, g \qquad (18)$$

$$VB_{(a-1)gt} \le WB_{agt}, \quad a = 2,3,\dots,A, t = 2,3,\dots,T, \forall g \qquad (19)$$

$$IB_{(a-1)g(t-1)} - \sum_{j=1}^{J} Y_{agj(t-1)} - SB_{agjt} \le \left(1 - WB_{agt}\right) \cdot M, \quad t = 2,3,\dots,T, \forall a, g \qquad (20)$$

$$IB_{(a-1)g(t-1)} - \sum_{j=1}^{J} Y_{agj(t-1)} - SB_{agjt} \geq 1 - WB_{agt}, \quad t = 2,3,\ldots,T, \forall a,g \tag{21}$$

$$Y_{agjt}, F_{agjt}, I_{agjt}, S_{agjt}, SB_{agjt} \in \mathbb{R}^+ \; \forall a,g,j,t \tag{22}$$

$$O_{gjt} \in \mathbb{R}^+ \; \forall g,j,t \tag{23}$$

$$I_{agt} \in \mathbb{R}^+ \; \forall a,g,t \tag{24}$$

$$X_{gt}, OB_{gt} \in \mathbb{R}^+ \; \forall g,t \tag{25}$$

$$V_{agjt}, W_{agjt} \in \{0,1\} \; \forall a,g,j,t \tag{26}$$

$$VB_{agt}, W_{agt} \in \{0,1\} \; \forall a,g,t \tag{27}$$

The objective function (1) seeks to minimize the production, purchase, holding, wastage and shortage costs. Constraint (2) states the inventory level of each hospital in the first period. Constraint (3) updates the inventory level in each hospital at the end of each period including the inventory level of the previous period, the units received from the blood center and units used to satisfy the demand. Constraints (4) and (5) identify the wastage levels of hospitals at the end of each period. Constraint (6) states the demand satisfaction introducing a shortage level. Constraints (7) - (11) guarantee the FIFO policy at hospitals. The production quantity in blood center at the beginning of each period is defined by constraint (12). The inventory level in blood center in the first period is ensured by constraint (13) and in the remaining periods is stablished by constraint (14) as a balance between the inventory level of the previous period and units sent to hospitals to satisfy the demand. Constraints (15) and (16) identify the wastage levels in the blood center at the end of each period. Constraints (17) - (21) guarantee the FIFO policy in the blood center. Constraints (22) - (25) show non-negative variables. Constraints (26) and (27) states binary variables.

4.2 Stochastic programming model

We follow conventions proposed in Shapiro et al. 2014 [18]. In the multistage setting, we have the stage index t, $t = 1,\ldots,T$. The uncertain data ξ_t is reveled gradually over time and decisions should be adapted to this random process. $\xi_t = (\xi_1,\ldots,\xi_T)$, represent demands at time t, with a specified probability distribution. $\xi_{[t]} \coloneqq (\xi_1,\ldots,\xi_t)$ denotes the history of the random process available at stage t. x_t represents the decision variables summarized in Table 10, f_t are the cost functions and depend on x_t and ξ_t. $\mathfrak{X}_t$, $t = 2,\ldots,T$ are measurable closed valued multifunctions state as constraints (2) - (27) and x_t must be valued in $\mathfrak{X}_t(x_{t-1},\xi_t)$. It is assumed that the first-stage data, i.e., ξ_1, f_1 and the set $\mathfrak{X}_1$ are deterministic. The values of the decision variables x_t, chosen at stage t, may depend on the information $\xi_{[t]}$ available at the time of decision and do not depend on future observations. This is the requirement of *nonanticipativity*. A T-stage stochastic programming problem can be written in the general nested formulation as follows,

$$\underset{x_1 \in \mathfrak{X}_1}{\text{Min}} \; f_1(x_1) + \mathbb{E}_{|\xi_1}\left[\inf_{x_2 \in \mathfrak{X}_2(x_1,\xi_2)} f_2(x_2,\xi_2) + \mathbb{E}_{|\xi_{[2]}}\left[\cdots + \mathbb{E}_{|\xi_{[T-1]}}\left[\inf_{x_T \in \mathfrak{X}_T(x_{T-1},\xi_T)} f_T(x_t,\xi_T)\right]\right]\right] \tag{28}$$

Nested formulation (28) allows to write the following *dynamic programming* equations. At the stage $t = T$,

$$Q_T(x_{T-1},\xi_{[T]}) = \inf_{x_T \in \mathfrak{X}_T(x_{T-1},\xi_T)} f_T(x_T,\xi_T) \tag{29}$$

At stages $t = 2,\ldots,T-1$,

$$Q_t(x_{t-1},\xi_{[t]}) = \inf_{x_t \in \mathfrak{X}_t(x_{t-1},\xi_t)} f_t(x_t,\xi_t) + \mathcal{Q}_{t+1}(x_t,\xi_{[t]}), \text{ with } \mathcal{Q}_{t+1}(x_t,\xi_{[t]}) \coloneqq \mathbb{E}_{|\xi_{[t]}}[Q_{t+1}(x_t,\xi_{[t+1]})] \tag{30}$$

The optimal values are denoted by $Q_t(x_{t-1},\xi_{[t]})$ and are called the *cost-to-go functions*. $\mathcal{Q}_{t+1}(x_t,\xi_{[t]})$ denote the *recourse functions*. The idea is to calculate the cost-to-go functions recursively, going back in time. At stage $t = 1$,

$$\inf_{x_1 \in \mathfrak{X}_1} f_1(x_1) + \mathbb{E}[Q_2(x_1,\xi_2)] \tag{31}$$

The optimal value of (31) gives the optimal value of the problem (28). We assume that the process $\xi_1,\ldots,\xi_T$ is *stagewise independent.* Then ξ_T is independent of $\xi_{[T-1]}$. The dynamic programming equation (30) can be written as

$$Q_t(x_{t-1},\xi_t) = \inf_{x_t \in \mathfrak{X}_t(x_{t-1},\xi_t)} f_t(x_t,\xi_t) + \mathcal{Q}_{t+1}(x_t), \quad \text{with } \mathcal{Q}_{t+1}(x_t) = \mathbb{E}[Q_{t+1}(x_t,\xi_{t+1})] \tag{32}$$

$$\begin{gathered} \text{Where, } f_t(x_t,\xi_t) \coloneqq c_t^{\mathrm{T}} x_t, \quad \mathfrak{X}_1 \coloneqq \{x_1 : A_1 x_1 = b_1, x_1 \geq 0\} \\ \mathfrak{X}_t(x_{t-1},\xi_t) \coloneqq \{x_t : B_t x_{t-1} + A_t x_t = b_t, x_t \geq 0\}, \quad t = 2,\ldots,T \end{gathered} \tag{33}$$

Below we present the dynamic formulation in the blood supply chain model. At stage T,

$$\begin{aligned} Q_T(x_{T-1},\xi_T) = \text{Min} \quad & pc\left(\sum_{g=1}^{G} X_{gT}\right) + c\left(\sum_{a=1}^{A}\sum_{g=1}^{G}\sum_{j=1}^{J} Y_{agjT} - SB_{agjT}\right) + h\left(\sum_{a=1}^{A}\sum_{g=1}^{G}\sum_{j=1}^{J} I_{agjT} + \right. \\ & \left. \sum_{a=1}^{A}\sum_{g=1}^{G} IB_{agT}\right) + wc\left(\sum_{g=1}^{G}\sum_{j=1}^{J} O_{gjT} + \sum_{g=1}^{G} OB_{gT}\right) + sc\left(\sum_{a=1}^{A}\sum_{g=1}^{G}\sum_{j=1}^{J} S_{agjT} + SB_{agjT}\right) \end{aligned} \tag{34}$$

Subject to constraints (3), (5) – (12), (14), (16) – (27) evaluated in $t = T$. The optimal value of $Q_T(x_{T-1}, \xi_T)$ depends on the decision variables $I_{(a-1)gj(T-1)}$, $Y_{agj(T-1)}$, $IB_{(a-1)g(T-1)}$ and data ξ_T. At stage $t = 2, \dots, T-1$,

$$Q_t(x_{t-1}, \xi_{[t]}) = \text{Min}\ pc\left(\sum_{g=1}^{G}\sum_{t=2}^{T-1} X_{gt}\right) + c\left(\sum_{a=1}^{A}\sum_{g=1}^{G}\sum_{j=1}^{J}\sum_{t=2}^{T-1} Y_{agjt} - SB_{agjt}\right) + h\left(\sum_{a=1}^{A}\sum_{g=1}^{G}\sum_{j=1}^{J}\sum_{t=2}^{T-1} I_{agjt}\right.$$
$$\left. + \sum_{a=1}^{A}\sum_{g=1}^{G}\sum_{t=2}^{T-1} IB_{agt}\right) + wc\left(\sum_{g=1}^{G}\sum_{j=1}^{J}\sum_{t=2}^{T-1} O_{gjt} + \sum_{g=1}^{G}\sum_{t=2}^{T-1} OB_{gt}\right) + sc\left(\sum_{a=1}^{A}\sum_{g=1}^{G}\sum_{j=1}^{J}\sum_{t=2}^{T-1} S_{agjt} + SB_{agjt}\right) \quad (35)$$
$$+ \mathbb{E}\left[Q_{t+1}(x_t, \xi_{[t+1]})\right]$$

Subject to constraints (3), (5) – (12), (14), (16) – (27). All of them evaluated in $t = 2, \dots, T-1$. At first stage we have,

$$\text{Min}\ pc\left(\sum_{g=1}^{G} X_{g1}\right) + c\left(\sum_{a=1}^{A}\sum_{g=1}^{G}\sum_{j=1}^{J} Y_{agj1}\right) + h\left(\sum_{a=1}^{A}\sum_{g=1}^{G}\sum_{j=1}^{J} I_{agj1} + \sum_{a=1}^{A}\sum_{g=1}^{G} IB_{ag1}\right)$$
$$+ wc\left(\sum_{g=1}^{G}\sum_{j=1}^{J} O_{gj1} + \sum_{g=1}^{G} OB_{g1}\right) + \mathbb{E}\left[Q_2(x_1, \xi_{[2]})\right] \quad (36)$$

Subject to constraints (2), (4), (12), (13), (15), and (22) – (27) evaluated in $t = 1$.

We consider that the random process $\xi_1, \dots, \xi_T$ has a finite number of realizations. A scenario tree is used to create a representation of all the possible K sequences of data. The scenario tree has nodes organized in levels which correspond to stages $1, \dots, T$. At stage 1 we have one *root node* associated with the value of ξ_1. At stage $t = 2$, we have as many nodes as realizations of ξ_2. For each node ι of stage $t = 2$ we create at least as many nodes at stage 3 as different values of ξ_3 and so on. The set of all nodes at stage t is denoted by Ω_t. The *ancestor* of a node $\iota \in \Omega_t$ is $a(\iota) \in \Omega_{t-1}$, $t = 2, \dots, T$. The set of *children nodes* of a node $\iota \in \Omega_t$ is denote by $C_\iota \in \Omega_{t+1}$, $t = 1, \dots, T$. Scenario is a path between the root node and a node in the last stage T and can be defined by its nodes $\iota_1, \dots, \iota_T$. We denote the probability of moving from $\iota \in \Omega_t$ to a node $\eta \in C_1$ by $\rho_{\iota\eta}$, where $\rho_{\iota\eta} \geq 0$ and $\sum_{\eta \in C_\iota} \rho_{\iota\eta} = 1$. The probability of a scenario is given by the product $\rho_{\iota 1 \iota 2}\rho_{\iota 2 \iota 3} \dots \rho_{\iota T-1 \iota T}$. We suppose a finitely number of scenarios K. Each scenario k has a probability p_k and a sequence of decisions. We present the stochastic programming model stated as a MILP model, using the indices, parameters and decision variables in Tables 8-10,

$$\text{Min}\quad pc\left(\sum_{g=1}^{G} XFS_g\right) + c\left(\sum_{a=1}^{A}\sum_{g=1}^{G}\sum_{j=1}^{J} YFS_{agj}\right) + h\left(\sum_{a=1}^{A}\sum_{g=1}^{G}\sum_{j=1}^{J} IFS_{agj} + \sum_{a=1}^{A}\sum_{g=1}^{G} IBFS_{ag}\right) +$$
$$wc\left(\sum_{g=1}^{G}\sum_{j=1}^{J} OFS_{gj} + \sum_{g=1}^{G} OBFS_g\right) + \sum_{k=1}^{K}\sum_{g=1}^{G} p_{gk}\left[pc\left(\sum_{t=2}^{T} X_{gt}^k\right) + c\left(\sum_{a=1}^{A}\sum_{j=1}^{J}\sum_{t=2}^{T} Y_{agjt}^k - SB_{agjt}^k\right) + \right. \quad (37)$$
$$\left. h\left(\sum_{a=1}^{A}\sum_{j=1}^{J}\sum_{t=2}^{T} I_{agjt}^k + \sum_{a=1}^{A}\sum_{t=2}^{T} IB_{agt}^k\right) + wc\left(\sum_{j=1}^{J}\sum_{t=2}^{T} O_{gjt}^k + \sum_{t=2}^{T} OB_{gt}^k\right) + sc\left(\sum_{a=1}^{A}\sum_{j=1}^{J}\sum_{t=1}^{T-1} S_{agjt}^k + SB_{agjt}^k\right)\right]$$

Subject to,

$$IFS_{agj} = inv_{agj}, \quad a = 1,2,\dots,A-1, \forall g,j \quad (38)$$
$$I_{agjt}^k = IFS_{a-1}gj + YFS_{agj} - SB_{agjt}^k - F_{agjt}^k, \quad a = 1,2,\dots,A-1, t = 2, \forall g,j,k \quad (39)$$
$$I_{agjt}^k = I_{(a-1)gj(t-1)}^k + Y_{agj(t-1)}^k - SB_{agjt}^k - F_{agjt}^k, \quad a = 1,2,\dots,A-1, t = 3,\dots,T, \forall g,j,k \quad (40)$$
$$OFS_{gj} = inv_{agj}, \quad a = A, \forall g,j \quad (41)$$
$$O_{gjt}^k = IFS_{(a-1)gj} + YFS_{agj} - SB_{agjt}^k - F_{agjt}^k, \quad a = A, t = 2, \forall g,j,k \quad (42)$$
$$O_{gjt}^k = I_{(a-1)gj(t-1)}^k + Y_{agj(t-1)}^k - SB_{agjt}^k - F_{agjt}^k, \quad a = A, t = 3,\dots,T, \forall g,j,k \quad (43)$$
$$d_{gjt}^k = \sum_{a=1}^{A} F_{agjt}^k + S_{agjt}^k, \quad t = 2,3,\dots,T, \forall g,j,k \quad (44)$$
$$F_{agjt}^k \leq M \cdot V_{agjt}^k, \quad t = 2,3,\dots,T, \forall a,g,j,k \quad (45) \qquad F_{agjt}^k \geq V_{agjt}^k, \quad t = 2,3,\dots,T, \forall a,g,j,k \quad (46)$$
$$V_{(a-1)gjt}^k \leq W_{agjt}^k, \quad a = 2,3,\dots,A, t = 2,3,\dots,T, \forall g,j, \quad (47)$$
$$YFS_{agj} - SB_{agjt}^k + IFS_{(a-1)gj} - F_{agjt}^k \leq (1 - W_{agjt}^k) \cdot M, \quad t = 2, \forall a,g,j,k \quad (48)$$
$$Y_{agj(t-1)}^k - SB_{agjt}^k + I_{(a-1)gj(t-1)}^k - F_{agjt}^k \leq (1 - W_{agjt}^k) \cdot M, \quad t = 3,\dots,T, \forall a,g,j,k \quad (49)$$
$$YFS_{agj} - SB_{agjt}^k + IFS_{(a-1)gj} - F_{agjt}^k \geq 1 - W_{agjt}^k, \quad t = 2, \forall a,g,j,k \quad (50)$$
$$Y_{agj(t-1)}^k - SB_{agjt}^k + I_{(a-1)gj(t-1)}^k - F_{agjt}^k \geq 1 - W_{agjt}^k, \quad t = 3,\dots,T, \forall a,g,j,k \quad (51)$$

$$IBFS_{ag} = XFS_g, \quad a = 1, \forall g \tag{52}$$

$$IB_{agt}^k = X_{gt}^k, \quad a = 1, t = 2,3,\ldots,T-1, \forall g,k \tag{53}$$

$$IBFS_{ag} = invbc_{ag}, \quad a = 2,3,\ldots,A-1, \forall g \tag{54}$$

$$IB_{agt}^k = IBFS_{(a-1)g} - \sum_{j=1}^{J} YFS_{agj} - SB_{agjt}^k, \quad a = 2,3,\ldots,A-1, t = 2, \forall g,k \tag{55}$$

$$IB_{agt}^k = IB_{(a-1)g(t-1)}^k - \sum_{j=1}^{J} Y_{agj(t-1)}^k - SB_{agjt}^k, \quad a = 2,3,\ldots,A-1, t = 3,\ldots,T, \forall g,k \tag{56}$$

$$OBFS_g = invbc_{ag}, \quad a = A, \forall g \tag{57}$$

$$OB_{gt}^k = IBFS_{(a-1)g} - \sum_{j=1}^{J} YFS_{agj} - SB_{agjt}^k, \quad a = A, t = 2, \forall g,k \tag{58}$$

$$OB_{gt}^k = IB_{(a-1)g(t-1)}^k - \sum_{j=1}^{J} Y_{agj(t-1)}^k - SB_{agjt}^k, \quad a = A, t = 3,\ldots,T, \forall g,k \tag{59}$$

$$\sum_{j=1}^{J} YFS_{agj} - SB_{agjt}^k \leq M \cdot VB_{agt}^k, \quad t = 2, \forall a,g,k \tag{60}$$

$$\sum_{j=1}^{J} Y_{agj(t-1)}^k - SB_{agjt}^k \leq M \cdot VB_{agt}^k, \quad t = 3,\ldots,T, \forall a,g,k \tag{61}$$

$$\sum_{j=1}^{J} YFS_{agj} - SB_{agjt}^k \geq VB_{agt}^k, \quad t = 2, \forall a,g,k \tag{62}$$

$$\sum_{j=1}^{J} Y_{agj(t-1)}^k - SB_{agjt}^k \geq VB_{agt}^k, \quad t = 3,\ldots,T, \forall a,g,k \tag{63}$$

$$VB_{(a-1)gt}^k \leq WB_{agt}^k, a = 2,3,\ldots,A, t = 2,3,\ldots,T, \forall g,k \tag{64}$$

$$IBFS_{(a-1)g} - \sum_{j=1}^{J} YFS_{agj} - SB_{agjt}^k \leq \left(1 - WB_{agt}^k\right) \cdot M, \quad t = 2, \forall a,g,k \tag{65}$$

$$IB_{(a-1)g(t-1)}^k - \sum_{j=1}^{J} Y_{agj(t-1)}^k - SB_{agjt}^k \leq \left(1 - WB_{agt}^k\right) \cdot M, \quad t = 3,\ldots,T, \forall a,g \tag{66}$$

$$IBFS_{(a-1)g} - \sum_{j=1}^{J} YFS\_agj - SB_{agjt}^k \geq 1 - WB_{agt}^k, \quad t = 2, \forall a,g,k \tag{67}$$

$$IB_{(a-1)g(t-1)k} - \sum_{j=1}^{J} Y_{agj(t-1)}^k - SB_{agjt}^k \geq 1 - WB_{agt}^k, \quad t = 3,\ldots,T, \forall\, a,g \tag{68}$$

$$YFS_{agj}, IFS_{agj} \in \mathbb{R}^+ \quad \forall a,g,j \tag{69}$$

$$Y_{agjt}, F_{agjt}, I_{agjt}, S_{agjt}, SB_{agjt} \in \mathbb{R}^+ \quad t = 2,3,\ldots,T, \forall a,g,j \tag{70}$$

$$OFS_{gj} \in \mathbb{R}^+ \quad \forall g,j \tag{71}$$

$$O_{gjt} \in \mathbb{R}^+ \quad t = 2,3,\ldots,T, \forall g,j \tag{72}$$

$$IFS_{ag} \in \mathbb{R}^+ \quad \forall a,g \tag{73}$$

$$I_{agt} \in \mathbb{R}^+ \quad t = 2,3,\ldots,T, \forall a,g \tag{74}$$

$$XFS_g, OBFS_g \in \mathbb{R}^+ \quad \forall g \tag{75}$$

$$X_{gt}, OB_{gt} \in \mathbb{R}^+ \quad t = 2,3,\ldots,T, \forall g \tag{76}$$

$$V_{agjt}, W_{agjt} \in \{0,1\} \quad t = 2,3,\ldots,T, \forall a,g,j \tag{77}$$

$$VB_{agt}, WB_{agt} \in \{0,1\} \quad t = 2,3,\ldots,T, \forall a,g \tag{78}$$

The scenarios that have the same history $\xi_{[t]}$ cannot be distinguished, so we enforce the *nonanticipativity constraints*:

$$Y_{agjt}^k = Y_{agjt}^{k'}, F_{agjt}^k = F_{agjt}^{k'}, I_{agjt}^k = I_{agjt}^{k'}, S_{agjt}^k = S_{agjt}^{k'}, V_{agjt}^k = V_{agjt}^{k'}, W_{agjt}^k = W_{agjt}^{k'}, SB_{agjt}^k = SB_{agjt}^{k'}, \quad t = 2,\ldots,T-1, \forall a,g,j, \quad \forall k,k' \text{ for which } \xi_{[t]}^k = \xi_{[t]}^{k'} \tag{79}$$

$$O_{gjt}^k = O_{gjt}^{k'}, \quad t = 2,\ldots,T-1, \forall g,j, \quad \forall k,k' \text{ for which } \xi_{[t]}^k = \xi_{[t]}^{k'} \tag{80}$$

$$IB_{agt}^k = IB_{agt}^{k'}, VB_{agt}^k = VB_{agt}^{k'}, WB_{agt}^k = WB_{agt}^{k'}, \quad t = 2,\ldots,T-1, \forall a,g, \quad \forall k,k' \text{ for which } \xi_{[t]}^k = \xi_{[t]}^{k'} \tag{81}$$

$$X_{gt}^k = X_{gt}^{k'}, OB_{gt}^k = OB_{gt}^{k'}, \quad t = 2,\ldots,T-1, \forall g, \quad \forall\, k,k' \text{ for which } \xi_{[t]}^k = \xi_{[t]}^{k'} \tag{82}$$

Equations (37) – (82) form the stochastic programming model for blood supply chain.

5. Computational study

PLTs have been the product chosen due to their high perishability with a shelf life of 5 days and we include the 8 ABO/Rh(D) types of blood. The planning horizon defined is 8 periods to face the demand of a week because in the first period, the model decides how much to produce in the blood center and the order quantity of each hospital to supply the demand of the next day. The demands for a week of each blood type are assumed to follow a Poisson distribution with varying daily means λ and independent of each other. We use the weekly demand data used by Haijema et al. 2009 [19] as the demand of one hospital. We consider that the second hospital has a 20% higher demand than the first one. The demand distribution by blood type is proportional to the percentage of each blood type in the population [10]. The computational experiments are based on the daily Poisson mean demand data exposed in Table 12. Table 13 summarize the values of cost parameters for PLTs that are used in the models. For this study, we do not consider the initial inventory neither for the blood center nor for hospitals.

Table 5. Demand data

Blood type	**US (%)** [10]	**Mean Demand Data Hospital 1**[1]							**Mean Demand Data Hospital 2**[1]						
		Mo	**Tu**	**We**	**Th**	**Fr**	**Sa**	**Su**	**Mo**	**Tu**	**We**	**Th**	**Fr**	**Sa**	**Su**
O+	37,40%	49	38	60	38	49	14	25	59	45	72	45	59	16	30
O-	6,60%	9	7	11	7	9	3	5	11	8	13	8	11	3	6
A+	35,70%	47	36	58	36	47	13	24	56	43	69	43	56	15	28
A-	6,30%	9	7	11	7	9	3	5	10	8	13	8	10	3	5
B+	8,50%	12	9	14	9	12	3	6	14	11	17	11	14	4	7
B-	1,50%	2	2	3	2	2	1	1	3	2	3	2	3	1	2
AB+	3,40%	5	4	6	4	5	2	3	6	5	7	5	6	2	3
AB-	0,60%	1	1	1	1	1	1	1	1	1	2	1	1	1	1
[1]Mo=Monday, Tu=Tuesday, We=Wednesday, Th=Thursday, Fr=Friday, Sa=Saturday and Su=Sunday															

Table 6. Cost parameters

Parameter	**Value**	**Parameter**	**Value**	**Parameter**	**Value**
Production cost [2]	538 $/unit	Wastage cost [19]	150 $/unit	Holding cost [13]	1.25 $/unit·day
Purchase cost [13]	100 $/unit	Shortage cost [14]	1500 $/unit		

MATLAB R2010b was used to generate a total of 54 datasets to test the SP model. 18 datasets considering two realizations of the demand, 18 considering three realizations and the remaining 18 considering five realizations. These are generated from a Poisson distribution using the mean values in Table 12. In each group of the 18 datasets, we generate 3 datasets for 6 different planning horizons: 3, 4, 5, 6, 7 and 8 days to evaluate the performance of the mathematical model in terms of computational time.
The deterministic programming model described by equations (1) - (27) and the stochastic programming model described by (37) - (82) are solved using IBM ILOG CPLEX with GAMS version 23.5.2 on a computer with a processor Intel(R) Core(TM) i5, 2.67 GHz and 4 GB memory. Table 15 contains the computational times required to solve the mathematical models. For the stochastic case, only 15 instances are solved due to the problem complexity. Indeed, this last grows with the time horizon length and the number of scenarios. Table 16 shows the results obtained from the objective function in each of the mathematical models. This total cost increases in the stochastic model with respect to the deterministic model, since the second one considers the costs that can be incurred when the demand is not met.

Table 7. Computational times of solutions

Periods	**MILP Model (s)**	**Realizations of demand**[1]			**Periods**	**MILP Model (s)**	**Realizations of demand**[1]		
		2	3	5			2	3	5
3	0.103	0.5495	45.343	2002.331	6	0.149	OOM	OOM	OOM
4	0.038	39.564	OOM	OOM	7	0.073	OOM	OOM	OOM
5	0.043	5132.913	OOM	OOM	8	0.158	OOM	OOM	OOM
[1]Average time of solutions (s), OOM=Out of memory									

Table 8. Total cost

Periods	**MILP Model**	**Realizations of demand**[1]		
		2	3	5
3	$333,049	$351,032	$352,914	$359,840
4	$587,821	$588,736	-	-
5	$708,289	$725,830	-	-
[1]Average of the expected total cost				

For the planning horizon with 3 periods of time we obtained solution. Figure 3 shows a behavioral pattern of associated costs. We can observe that when the expected cost of production increases, i.e., the number of blood units available in the supply chain is higher, the expected cost of shortage decreases.

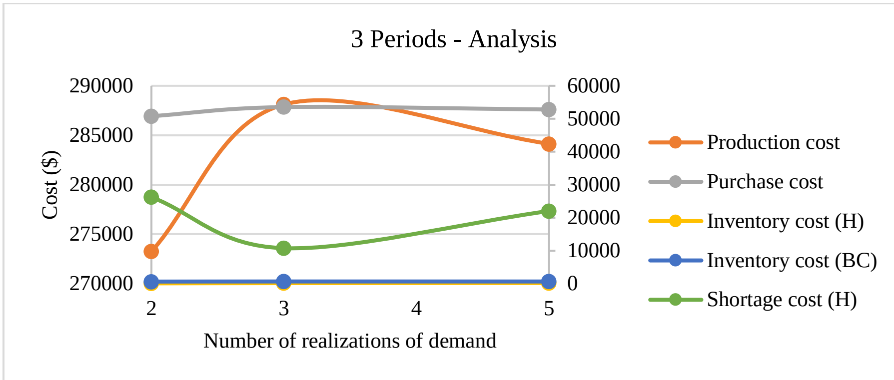

Figure 1. Costs for the 3 periods case

5.1 Simulation and results

In order to evaluate the production quantities of blood center and order quantities of hospitals in different periods of the planning horizon obtained as results of mathematical models, we develop the Algorithm 1 using the indices, parameters and variables from Table 8 to Table 10.

Algorithm 1. Simulation of blood supply chain

Input: Number of simulations N, parameters A, G, J, T, cost parameters pc, c, h, wc, sc, initial inventory $inv_{agj}, invbc_{ag}$, demand d_{gjt} and solutions from mathematical models X_{gt}, Y_{agjt}. **Output:** Average costs and average shortage rates in hospitals.

for $t = 1$
  Calculate IB_{agt} considering X_{gt} and $invbc_{ag}$
  Calculate OB_{gt}
  Calculate I_{agjt} considering inv_{agj}
  Calculate O_{gjt}
end
for $n = 1{:}N$
  for $t = 2{:}T$
    Define the available units in blood center at the beginning of period t, CB_{agt}, to satisfy the hospital orders
    if $\sum_{j=1}^{J} Y_{agjt} \leq CB_{agt}$
      Calculate units, FB_{agt}, to satisfy the order quantities from hospitals considering $\sum_{j=1}^{J} Y_{agjt}$
    else
      Calculate FB_{agt} considering CB_{agt}
      Define SB_{agjt}
    end
    Define the available units in hospitals at the beginning of period t, C_{agjt}, to satisfy the demand
    Generate a random number $drandom_{gjt} \sim P(d_{gjt})$
    if $drandom_{gjt} \leq \sum_{a=1}^{A} C_{agjt}$
      Calculate $\sum_{a=1}^{A} F_{agjt}$ considering $drandom_{gjt}$
    else
      Calculate F_{agjt} considering C_{agjt}
      Define S_{agjt}
    end
    At the end of period t:
    Calculate IB_{agt} considering X_{gt} and $CB_{agt} - FB_{agt}$
    Calculate OB_{gt}
    Calculate I_{agjt} considering $C_{agjt} - F_{agjt}$
    Calculate O_{gjt}
  end
  Calculate the production, purchase, inventory, wastage, shortage and total costs for each simulation n
  Calculate the shortage rate in hospitals as $sr = \frac{\sum_{g=1}^{G}\sum_{j=1}^{J}\sum_{t=1}^{T} drandom_{gjt}}{\sum_{a=1}^{A}\sum_{g=1}^{G}\sum_{j=1}^{J}\sum_{t=1}^{T} S_{agjt}}$ for each simulation n
end
Calculate the average costs and the average shortage rates in hospitals

Table 23 shows the outputs: expected costs and the new evaluation parameter defined in the Algorithm 1, the shortage rate in hospitals. The number of simulations defined is $N = 100$. The simulation of results of the deterministic model registers levels of inventory and shortage of blood units in hospitals. If the blood supply chain does not consider demand as a stochastic parameter and the blood center produces the units established by the deterministic model just as the hospitals order the units established for each period, it is expected that the supply chain present the higher shortage rate compared with the expected shortage rate considering the results of the stochastic programming models. The expected total costs increase if the expected cost of shortage and the shortage rate decreases.

Table 9. Solutions by mathematical programming models and simulation

Results	**Expected cost[1]**						
	Production	Purchase	Inventory in hospitals	Inventory BC	Shortage hospitals	Total costs	**Expected shortage rate**
Deterministic							
Model	$280,298	$52,100	$0	$651	$0	$333,049	0%
Simulation	$280,298	$52,100	$77	$651	$55,980	$389,106	7.08%
Stochastic[2]							
2 RM	$273,253	$50,790	$96	$635	$26,257	$351,032	-
2 RS	$305,763	$56,833	$176	$710	$24,025	$387,508	3.03%
3 RM	$288,114	$53,553	$164	$670	$10,762	$353,262	-
3 RS	$327,821	$60,700	$263	$762	$17,895	$407,441	2.26%
5 RM	$284,134	$52,813	$188	$661	$22.046	$359.842	-
5 RS	$348,624	$64,767	$327	$810	$8,580	$423,108	1.08%
[1]Considering the defined number of simulations N, [2]Average of expected cost, RM=Realizations model, RS=Realizations simulation.							

6. Conclusions

This document presented a study to optimize the blood supply chain for a single-blood center multiple-hospitals system considering products of 8 different blood types. The challenge of inventory control and distribution of perishable products, especially blood products, inspired us to start this research. A deterministic and stochastic programming models are developed to minimize the total cost and the shortage and wastage levels in blood supply chain within a planning horizon. Datasets are generated to evaluate the efficiency of the proposed models and an algorithm is used to simulate and evaluate the results from mathematical models. One of the challenges of stochastic programming is the size of the problems and the resolution of these in reasonable computational times. For future research related to the blood supply chain modeled as a multi-stage problem, it is proposed to evaluate scenarios reduction methods. In addition to this, approximation methods can be used such as the L-Shape Method or Cutting Plane Approximation in the case of multi-stage problems. Heuristics, Markov processes or other solution methods can be developed to optimize the blood supply chain proposed. The mathematical model developed has possibilities of expansion: to include ABO compatibility or include other stochastic parameters such as blood supply or C/T ratio.

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Biography

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