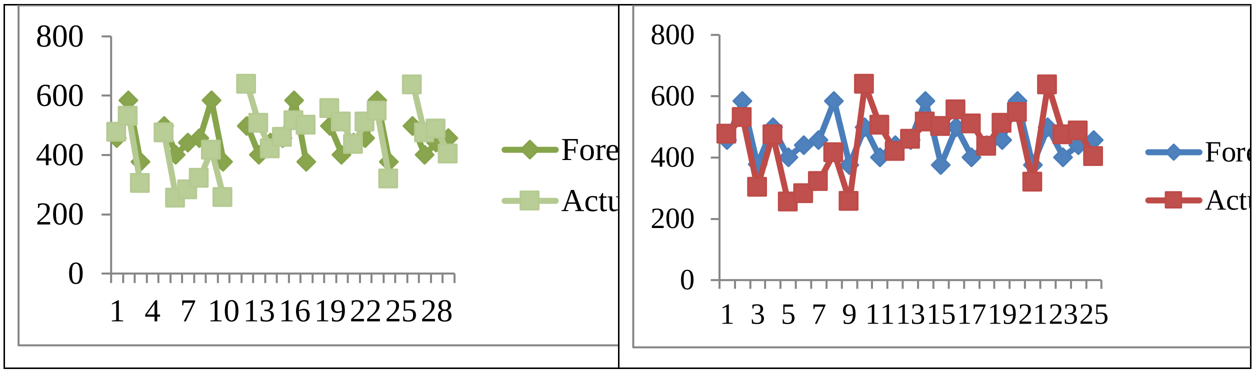

Figure 3: Comparison between Actual values and forecasted values including and excluding weekends.

In bank management system, we know that banking hour is 6. In this problem we have found that on an average 474 customer per day bank is open enter into the bank for service.

So, per hour average number of customer entering the bank is = $\frac{474}{6}$ = 79

The bank has 5 tellers and an average of 79 customers enter into the bank per hour in order to have service and they wait in a single line if there is no idle teller. To serve a customer it takes 3 minutes on an average. Assume that inter arrival times and service times are exponential. We determine (for January):

(a) The expected number of customers present in the bank

(b) The expected length of time a customer wait in the line and total time spend in the bank

(c) A particular teller's idle time (fractions).

Solution (a): We have an M/M/2/GD/∞/∞ system where $\lambda = 79$ customers per hour and μ = 20 customers per hour. Therefore $\rho = \frac{79}{5*20} = 0.79 < 1$, so the state indicates steady state. From Table 4, $P(j \geq 5)$ = 0.54. Then (v) yields

$$L_q = \frac{P(j \geq s)\rho}{1-\rho} = \frac{.79*.54}{1-.79} = 2.03 \approx 2 \; customers$$

And from (vii), $L = L_q + \frac{\lambda}{\mu}$ = 2.03 + $\frac{79}{20}$ = 5.98 ≈ 6 customers.

(b) Since, $W = \frac{L}{\lambda} = \frac{6}{79} = 0.07595 \; hour$ = 4.56 minutes or 4 minutes and 33 seconds.

(c) To determine the fraction of time that a particular server is idle , note that he or she is idle during the entire time that $j = 0$ and one fifth (by symmetry) the time that $j = 1$. The probability that a server is idle is given by $\pi_0 + 0.2\pi_1$. Using the fact that $P(j \geq 5) = .54$, we obtain from(iv):

$$\pi_0 = \frac{s!P(j \geq s)(1-\rho)}{(s\rho)^2} = \frac{5!(.54)(1-.79)}{3.95^2} = \frac{13.608}{15.6025} = 0.87$$

Now (ii)b yields

$\pi_1 = \frac{(s\rho)^j \pi_0}{j!} = \frac{(3.95)^1(.87)}{1!} = 3.44$

Thus the probability that a particular teller is idle is $\pi_0 + 0.2\pi_1$ = 0.87+0.2(3.44) = 1.56>1 indicates that a particular (any of the) teller were idle (free) for all time.

Now, if the manager of the bank want to determine how many tellers should work on February 12, Wednesday? For every minute a customer stand in line, the manager believes that a delay cost of Tk.5 is incurred. An average of 1.42 customers per minute arrive at the bank. On the average, it takes a teller 2.5 minutes to complete a customer's transaction. It cost the bank Tk.420 per hour to hire a teller. Inter arrival times and service times are exponential. To minimize the sum of service costs and delay costs, how many tellers should the bank have working on Wednesday where forecasted number of customer is 511?

Solution: Since λ = 1.42 customers per minute and μ = 0.4 customer per minute $\frac{\lambda}{s\mu} < 1$ requires that $\frac{1.42}{s(.4)} < 1$ or $s \geq 4$. Thus, there must be at least 4 tellers, or the number of customers present will "blow up". We now compute, for $s = 4,5,6,7, \ldots \ldots$

$$\frac{Expected\ service\ cost}{Minute} + \frac{Expected\ delay\ cost}{Minute}$$

Since each teller is paid $\frac{420}{60}$ = Tk.7 per minute,

$$\frac{Expected\ service\ cost}{Minute} = 7 * s$$

Now, we know

$$\frac{Expected\ delay\ cost}{Minute} = \left(\frac{Expected\ customers}{Minute}\right)\left(\frac{Expected\ delay\ cost}{Customer}\right)$$

But

$$\left(\frac{Expected\ delay\ cost}{Customer}\right) = 5 * W_q$$

Since an average of 1.42 customers arrive per minute,

$$\frac{Expected\ delay\ cost}{Minute} = 1.42 * 5 * W_q = 7.1 * W_q$$

For s = 4, $\rho = \frac{1.42}{.40*4}$ = 0.89 and $P(j \geq 4)$ = 0.79.

$$W_q = \frac{0.79}{4 * 0.4 - 1.42} = 4.39\ minutes$$

$$\frac{Expected\ delay\ cost}{Minute} = \frac{7.1 * 4.39}{4} = Tk. 7.80$$

$$\frac{Total\ Expected\ cost}{Minute} = 7 * 4 + 7.80 = Tk. 35.8$$

Since s = 5 has a service cost per minute of 5*7 = 35, so 4 tellers cannot have a lower total cost than 5 tellers. Hence, having 5 tellers service is optimal. Putting it another way, adding an additional tellers can save the bank at most Tk. 0.8 per minute in delay costs.

In addition to a customer's expected time in the system, the distribution of a customer's waiting time is of interest. For example, if all the customers who have to wait more than 5 minutes i.e.(2.5+4.39) = 6.89 minutes at the bank counter decide to switch to another bank, the probability that a given customer will switch to another bank equals $P(W > 6)$.

To determine this probability, we needed to know the distribution of a customer's waiting time. For an M/M/s/GD/∞/∞ queuing system, it can be shown that

$$P(W > t) = e^{-\mu t}\left\{1 + P(j \geq s) * \frac{1 - exp[-\mu t(s - 1 - s\rho)]}{s - 1 - \rho}\right\} \dots\dots\dots (ix)$$

$$P(W_q > t) = P(j \geq s) exp\,[-s\mu(1-\rho)t] \dots\dots\dots (x)$$

To illustrate the use of (ix) and (x), suppose that in this example (for s = 4), the bank manager wants to know the probability that a customer will have to wait in line for more than six minutes. For s = 4, $\rho = 0.89$, $P(j \geq 4) = 0.79$ and μ = 0.4 customer per minute. So (x) yields

$$P(W_q > 6) = 0.79 * exp\,[-4 * 0.4 * (1 - 0.89) * 6]$$
$$= 0.79 * e^{-1.06} = 0.2748$$

Thus, the bank manager can be sure that the chance of a customer's having to wait more than 6 minutes is small but not quite small.

4.4 An Application Using Primary Data

The Table 7 shows the number of customer each day on April 2017 enter into the Bangladeshi state owned renowned bank's (Janata Bank Ltd.) branch in Dhaka. We have forecasted for the 1st week of April where actual data is also given.

Day (Month)	Day (Week)	Customer	Day (Month)	Day (Week)	Customer
1	1		17	3-Monday	457
2	2-Sunday	H	18	4-Tuesday	401
3	3-Monday	502	19	5-Wednesday	391
4	4-Tuesday	362	20	6-Thursday	388
5	5-Wednesday	296	21	7-Friday	
6	6-Thursday	315	22	1-Saturday	
7	7-Friday		23	2-Sunday	584
8	1-Saturday		24	3-Monday	H

9	2-Sunday	452	25	4-Tuesday	515
10	3-Monday	306	26	5-Wednesday	325
11	4-Tuesday	350	27	6-Thursday	412
12	5-Wednesday	326	28	7-Friday	
13	6-Thursday	392	29	1-Saturday	
14	7-Friday	H	30	2-Sunday	466
15	1-Saturday				Average = 406
16	2-Sunday	474			

Table 7: Customers arrived each day in the Bank in April 17.

Let x_t= number of customers entered into the bank on day t. We postulate that x_t= B*DW_t*ε_t
We estimate B = average number of arrivals per day bank is open = 406,
DW_t for Sunday = $\frac{494}{406}$ = 1.216749,
DW_t for Monday = 1.038588,
DW_t for Tuesday = 1.002463,
DW_t for Wednesday = 0.823892,
DW_t for Thursday = 0.927956.

Day (April) M	Day W	Customer	Day (May) M	Day W	Forecast	Actual	ABS Error
1	1		1	3		H	
2	2	H	2	4	468.05	511	42.95
3	3	502	3	5	384.675	408	23.325
4	4	362	4	6	433.2625	448	14.7375
5	5	296	5	7			
6	6	315	6	1			
7	7		7	2	568.1	527	41.1
8	1		8	3	484.9167	446	38.91667
9	2	452	9	4	468.05	419	49.05
10	3	306	10	5	384.675	326	58.675
11	4	350	11	6	433.2625	454	20.7375
12	5	326	12	7		H	
13	6	392	13	1			
14	7	H	14	2	568.1	613	44.9
15	1		15	3	484.9167	458	26.91667
16	2	474					

17	3	457					
18	4	401					
19	5	391					
20	6	388					
21	7						
22	1						
23	2	584					
24	3	H					
25	4	515					
26	5	325					
27	6	412					
28	7						
29	1						
30	2	466					

Table 8: Forecast of the number of customers by the Ad Hoc Forecasting Technique.

Table 8, shows the solution of real life example using Ad Hoc Forecasting Technique with absolute error. Now, we draw the following figure using Ad Hoc Forecasting Technique comparing the actual value and forecasted value.

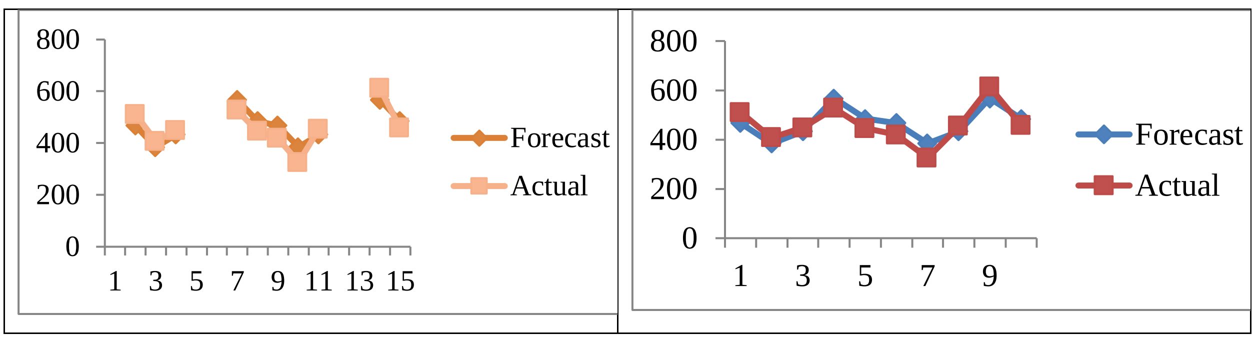

Figure 4: Comparison between Actual values and forecasted values including and excluding weekends.

In this section, we have discussed the advanced forecasting (Ad Hoc Forecasting) technique, some examples highly applied in banking sectors. We made comparison between the forecasted value with the actual value & also have shown graphical representation of actual value and calculated value.

5. Conclusion

The purpose of this thesis paper was to develop a sophisticated forecasting technique for the banking sector of Bangladesh. For this, we first analyzed existing methods such as Exponential Smoothing Method, Holt's Method & Winter Method etc. using different types of data sets. But problems raised to assign the value of random error terms, because forecast accuracy depends on them. In this thesis, we have developed an advance technique to predict required number of speaker or service men in order to serve the customer by forecasting the number of customer for each day so that the customer need not wait for getting service for long time as well as any banker will not pass time idly. We have also determined the length of time a customer is willing to wait to get service and minimized the total cost after calculating the optimum number of bank tellers. We have discussed real life problems along with comparison between actual value and forecasted value by showing graphical representation.

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