

Value at Risk when covariance is misspecified

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Abstract

We evaluate the impact of covariance matrix misspecification on a portfolio's value at risk, by means of Monte Carlo simulation. The portfolio's composition is found using four risk-based allocation methods, the minimum-variance portfolio, the inverse-volatility weighted portfolio, the equal-risk contribution portfolio, and the maximum-diversification portfolio. The covariance matrix is estimated historically and also by EWMA and multivariate GARCH methods. Covariance misspecification is considered in three possible ways. We consider the value at risk sensitivity due to misspecification of the variance and covariance terms. We also consider misspecification of both simultaneously. Our results show that multivariate GARCH methods are more accurate to estimate the covariance components and are less sensitive to covariance misspecification.

Keywords

Value at Risk, Covariance Misspecification, Risk-based Portfolios, Monte-Carlo Simulation

1. Introduction

Investors and traders are increasingly using minimum variance and maximum diversification strategies to select the best portfolio composition as defined by the portfolio weights. To select the best set of weights a risk-based asset allocation method is commonly used. In this paper four methods are considered. The minimum-variance weights, the inverse-volatility weighted weights (Leote De Carvalho et al., 2012), the equal-risk contribution weights (Maillard et al., 2010), and the maximum-diversification weights (Choueifaty and Coignard, 2008).

Ardia et al. (2017) analyze the impact of covariance misspecification on the optimal weights from the aforementioned methods. To extend their results, in this paper we assess the effect on the portfolio's Value at Risk (VaR) when the covariance matrix is misspecified.

The risk-based asset allocation methods require knowledge of the covariance matrix of the portfolio's returns. Two common methodologies to estimate it are the sample covariance matrix and the Ledoit-Wolf (2003) weighted average of the sample covariance matrix. Since these methods do not consider the changes over time of the conditional covariance matrix, we also consider the exponentially weighted moving average (EWMA) covariance matrix estimate, and the multivariate GARCH estimate as suggested by Engle (2002).

Our objective is to assess the impact of the covariance matrix estimate on the portfolio's performance as given by the portfolio's value at risk (VaR).

We use Monte Carlo simulation to compare the portfolio's performance with weights found by the four risk-based allocation methods. We restrict our comparisons to portfolios with non-negative weights (when short selling is not possible). All the portfolios are a combination of the stocks in the Dow Jones Industrial Average during 2017. Simulation is performed using the *R* language with library *rmgarch*.

Misspecification of the portfolio's covariance is analyzed in three different ways. We consider the VaR sensitivity due to misspecification of the variance terms and due to correlation components, simultaneously and separately. Therefore we analyze three scenarios, misspecification of the covariance matrix, of only the variance terms, and of only correlations.

We find that the DCC estimation provides the closest estimates to the true covariance matrix components. It also provides value at risk values that are closest to the corresponding value at risk values. This conclusion applies to all risk-based allocation methods and for all forecasting horizons considered.

2. Risk-based Portfolios and Covariance Matrix Estimation

In this section we describe the methods to find the best set of weights. We describe the optimal weights found by the minimum-variance, the inverse-volatility, the equal-risk contribution, and the maximum-diversification criteria. We also describe the methods used to estimate the covariance matrix.

2.1 Risk-based Allocation Methods

Let us consider a portfolio that is composed of n risky assets with weights denoted by $w = (w_1, w_2, \dots, w_n)$. We also denote the portfolio's return by $r = (r_1, r_2, \dots, r_n)$ and the covariance matrix by

$$C = \begin{pmatrix} \sigma_{11} & \cdots & \sigma_{1n} \\ \vdots & \ddots & \vdots \\ \sigma_{n1} & \cdots & \sigma_{nn} \end{pmatrix}$$

The vector of weights of the min-variance portfolio is given by

$$w = \arg \max_{w \in A} w^T C w \quad (1)$$

where $A = \{w: w \in R_+^N, \sum_i w_i = 1\}$.

The weights of the inverse volatility portfolio are given by

$$w = \left(\frac{1/\sigma_1}{\sum \sigma_i}, \dots, \frac{1/\sigma_N}{\sum \sigma_i} \right) \quad (2)$$

The weights of the equal-risk-contribution portfolio are given by

$$w = \arg \min_{w \in A} \sum_i \left(\frac{w_i(Cw)_i}{w^T C w} - \frac{1}{N} \right) \quad (3)$$

The goal of this method is to find the optimal weight that makes every asset contribute equally to the overall portfolio's volatility.

Finally, the weights of the maximum diversification portfolio are given by

$$w = \arg \max_{w \in A} \frac{w^T \sigma}{\sqrt{w^T C w}} \quad (4)$$

2.2 Covariance Matrix Estimation

We consider static and dynamic estimation of the covariance matrix. For static estimation we use the sample-based estimator and the method suggested by Ledoit-Wolf (LW). For dynamic estimation we consider the Exponential Weighted Moving Average (EWMA) and the Dynamic Conditional Correlation (DCC) introduced by Engle (2002). In these two last methods the variances and covariances are modeled by a GARCH(1,1) model estimated by maximum likelihood techniques.

3. Monte-Carlo Simulation

We based our results using the Dow Jones index as the investment portfolio. We consider daily prices from the period 2012 to 2016. Value at risk values are estimated for a one dollar investment. 1-day value at risk values are compared for all cases. For all assets in the Dow Jones index weights using each of the risk-based allocation methods. For each asset the returns are found using the adjusted closing prices. We use close price to reflect the most cutting-edging valuation of stocks until commence on the next trading date. This price reflects the effects of both short side and long side, which stands for the market's opinion about certain stocks. What is more, most investors not only individual but institution prefer to make their decision based on this price, which forms overwhelming power of the close price. Nevertheless, close prices will perform abnormally and deliver misleading information when companies distributing cash dividends to their shareholders. Other company behaviors such as split and new offering can lead to similar results as well. All this information should not be the source of stocks price variation since none of them alter the real value of the stocks and only nominal price change. Considering what the investors really care about ought to be the real change of the stock price, we need to make adjustment to smooth those variances. To fix this issue, the final data we are going to consider is the adjusted close price, the price amended to include corporate actions prior to next trading day.

3.1 Simulation Process

For a time-horizon of T trading days, we fit a dynamic conditional correlation GARCH (DCC-GARCH) model to the covariance matrix of returns of the prices of the thirty-asset Dow Jones portfolio. Then we use the fitted model to generate h future observations. To get the true covariance matrix for time $T+h$, we proceed as follows.

Consider predicting the one period ahead returns up to time $T+h$. For example, to predict one period ahead. At time $T+h$, the average of the sum of the h -period simulated covariance matrix is defined as the *true covariance*. This is used to benchmark the various estimators.

Misspecification of the portfolio's covariance is analyzed in three different ways. We consider the VaR sensitivity due to misspecification of the variance and covariance terms. We also consider misspecification of both. This last case is to be called overall misspecification. To estimate the effect of variance misspecification, we replace the true covariances with the estimated ones obtained by each construction method. To estimate the effect of covariance misspecification, we replace the true variances with the variances estimated by each method. Overall misspecification is the result of using the covariance matrix that results from the construction method, ignoring the elements from the true covariance matrix.

4. Simulation Results

Figures 1 to 3 show the result for a time horizon $h=20$. The plots show the VaR using different covariance estimation methods, and different portfolio construction methods. Figure 1 present overall misspecification situation and figure 2 and 3 is the variance misspecification and covariance misspecification respectively.

When the whole overall covariance matrix is misspecified (overall misspecification), except for the minimum variance method, all other 3 is almost insensitive to the misspecification. In the minimum variance portfolio construction method, DCC estimation share the same excellent performance with the true covariance matrix in the sense of risk level. Whilst EWMA method in this setting has the highest risk.

When the variance is misspecified, inverse volatility and equal risk contribution methods is not affected by the misspecification by large extent and true covariance matrix VaR nearly overlap with other estimation methods. Maximum diversification method indicates that EWMA has the worst performance from risk perspective. It is surprising to see that it is sample based estimation that has the lowest risk. Minimum variance portfolio method maintains the same feature with previous overall misspecification except for LW estimation has the highest level of risk.

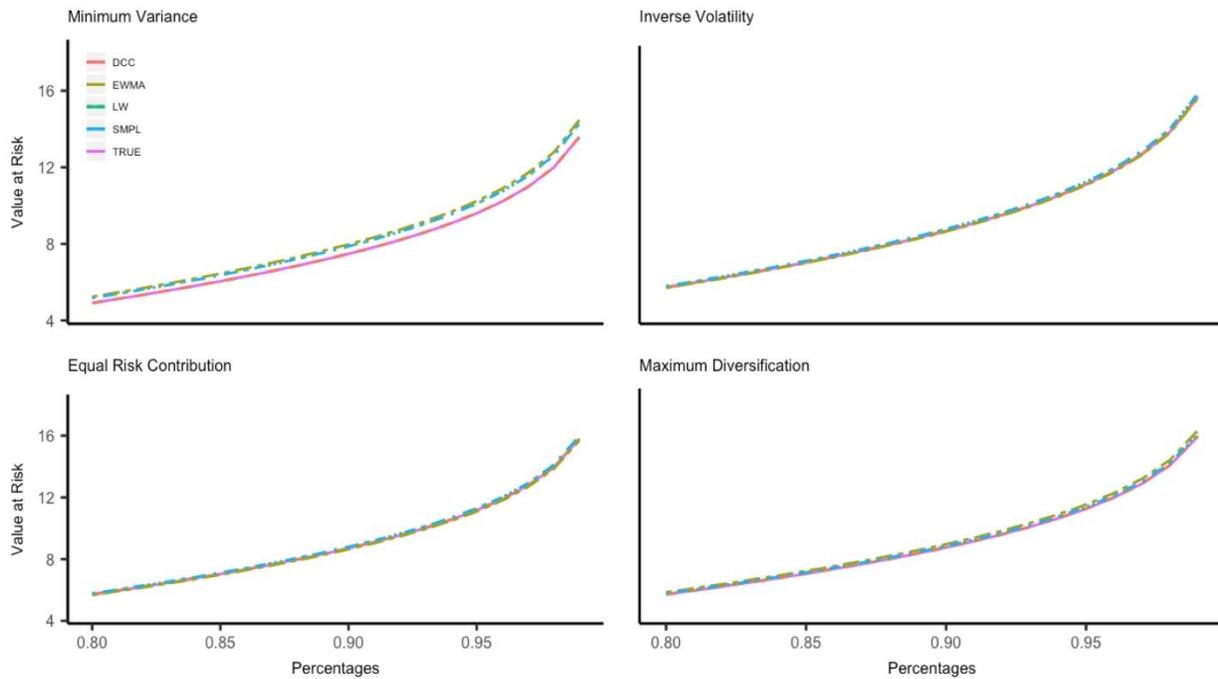


Figure 1. VaR values for covariance and variance misspecification

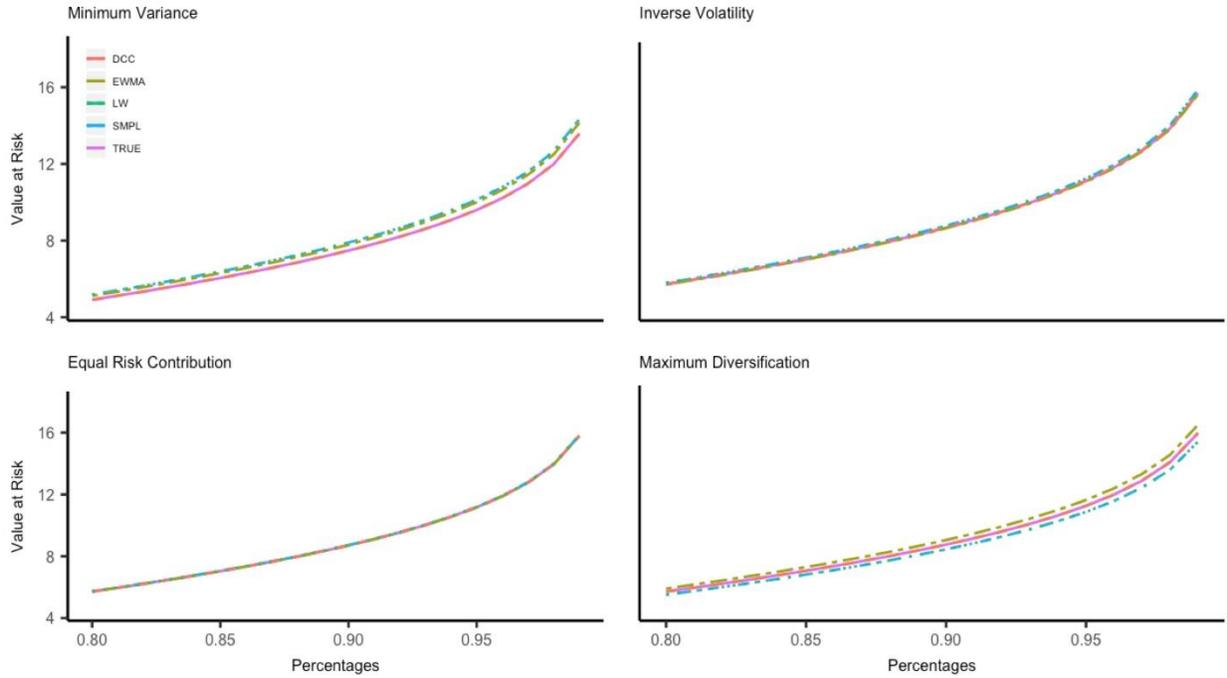


Figure 2. VaR values for variance misspecification

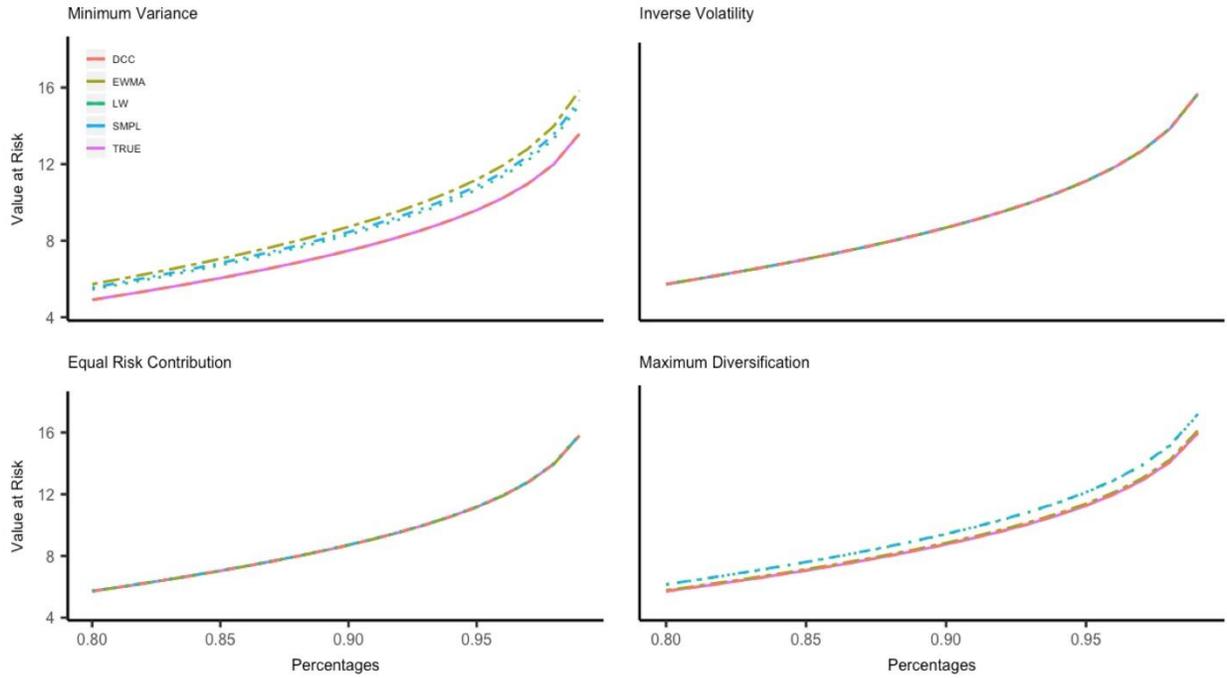


Figure 3. VaR values for covariance misspecification

Table 1. Value at Risk estimates

Time Horizon	Risk-Based Allocation	True	Overall Misspecification				Variance Misspecification				Correlation Misspecification			
			DCC	LW	SMPL	EWMA	DCC	LW	SMPL	EWMA	DCC	LW	SMPL	EWMA
h=1	minimum variance	0.00939	0.00953	0.01014	0.01015	0.01023	0.00971	0.01007	0.01007	0.01004	0.00945	0.01112	0.01139	0.01201
	inverse volatility	0.0113	0.01134	0.01152	0.01152	0.01128	0.01134	0.01152	0.01152	0.01128	0.0113	0.0113	0.0113	0.0113
	equal risk contribution	0.01119	0.01123	0.01141	0.01141	0.01112	0.0112	0.01122	0.01122	0.01119	0.01122	0.01136	0.01136	0.01113
	maximum diversification	0.01108	0.0111	0.01124	0.01127	0.01168	0.01106	0.01048	0.01048	0.01137	0.01137	0.0124	0.01242	0.01159
h=5	minimum variance	0.00942	0.00943	0.01006	0.01007	0.01018	0.00942	0.01003	0.01003	0.00993	0.00942	0.01097	0.01115	0.01158
	inverse volatility	0.01123	0.01123	0.01139	0.01139	0.01119	0.01123	0.01139	0.01139	0.01119	0.01123	0.01123	0.01123	0.01123
	equal risk contribution	0.01112	0.01112	0.01127	0.01128	0.01104	0.01112	0.01114	0.01114	0.01111	0.01112	0.01124	0.01125	0.01105
	maximum diversification	0.01102	0.01102	0.01114	0.01117	0.01153	0.01103	0.0105	0.0105	0.01137	0.01103	0.01204	0.01206	0.01133
h=20	minimum variance	0.0096	0.0096	0.0101	0.0101	0.01023	0.0096	0.01013	0.01013	0.01	0.0096	0.01067	0.01084	0.01119
	inverse volatility	0.01128	0.01128	0.0114	0.0114	0.01123	0.01128	0.0114	0.0114	0.01123	0.01128	0.01128	0.01128	0.01128
	equal risk contribution	0.01117	0.01117	0.01129	0.01129	0.01109	0.01117	0.01119	0.01119	0.01116	0.01117	0.01126	0.01127	0.0111
	maximum diversification	0.01107	0.01108	0.01116	0.01119	0.01134	0.01108	0.0107	0.0107	0.01145	0.01108	0.0119	0.01193	0.01119

Overall, minimum variance and maximum diversification portfolio tend to be affected by the misspecification of any parameters while inverse volatility and equal risk contribution portfolio is relative stable. In almost all the cases, DCC estimation and true covariance has the lowest risk level in terms of VaR. What is more, our simulation results show that when the time horizon for prediction h is increased, all estimation methods in each portfolio construction tilt toward to converge.

Table 1 shows the 95% VaR values found with four risk-based allocation methods estimated by the four construction methods (DCC, LW, SMPL, EWMA) for three different time horizons ($h=1, 5$, and 20). From the table, it can be seen that, about the allocation methods, the minimum variance method is always producing the minimum VaR and the inverse volatility is always giving the maximum variance except the covariance misspecification due to the weights are exactly same if the variance part is the same.

When the covariance is misspecified, the main result is almost the same as the situation when variance is misspecified. Inverse volatility and equal risk contribution portfolio is insensitive to the misspecification while maximum diversification and minimum variance portfolio is liable to the misspecification. Maximum diversification portfolio shows that DCC and true covariance has the same best performance but sample based estimation is the worst. Minimum variance portfolio is highly affected by the misspecification. Similar as before, DCC estimation and true covariance matrix has the lowest risk level in terms of the VaR but EWMA is the riskiest estimation method.

5. Conclusions

Value at risk values are estimated using Monte Carlo simulation when the portfolio's covariance matrix of returns is misspecified. It is found that the DCC estimation provides the best estimates to the true covariance matrix components. It also provides value at risk values that are closest to the corresponding value at risk values obtained from the true covariance matrix. This conclusion applies to all risk-based allocation methods and for all forecasting horizons considered.

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Biographies

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