When the variance is misspecified, inverse volatility and equal risk contribution methods is not affected by the misspecification by large extent and true covariance matrix VaR nearly overlap with other estimation methods. Maximum diversification method indicates that EWMA has the worst performance from risk perspective. It is surprising to see that it is sample based estimation that has the lowest risk. Minimum variance portfolio method maintains the same feature with previous overall misspecification except for LW estimation has the highest level of risk.

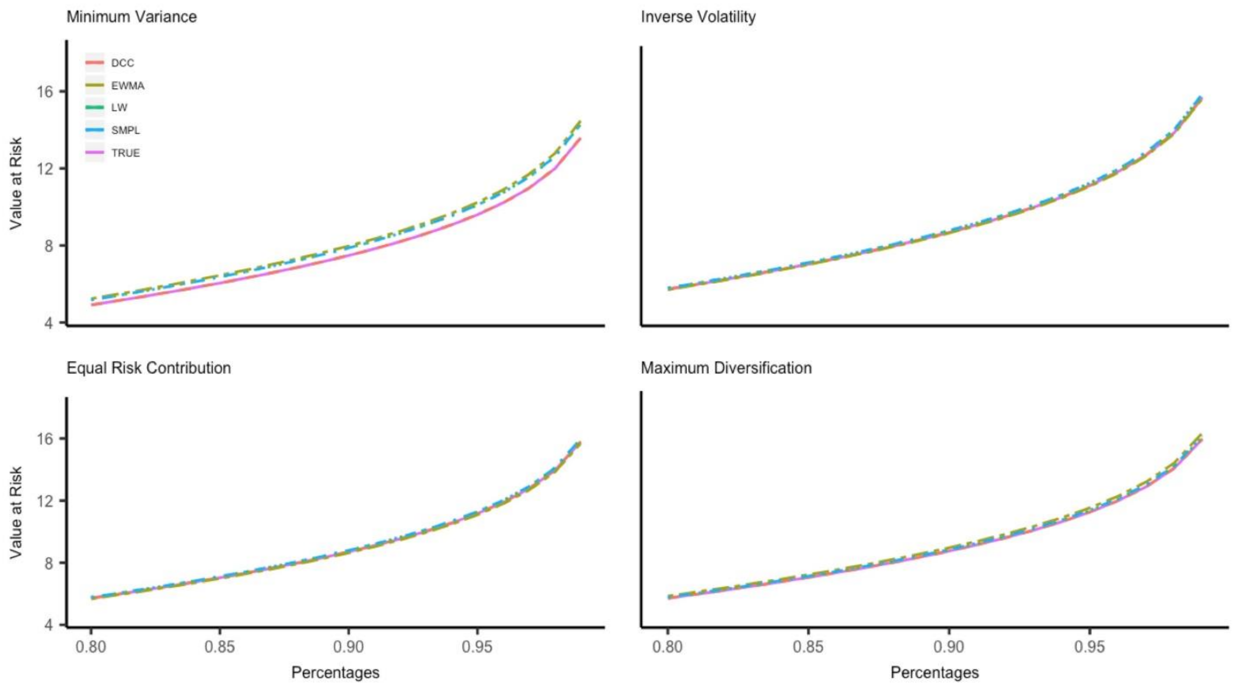

Figure 1. VaR values for covariance and variance misspecification

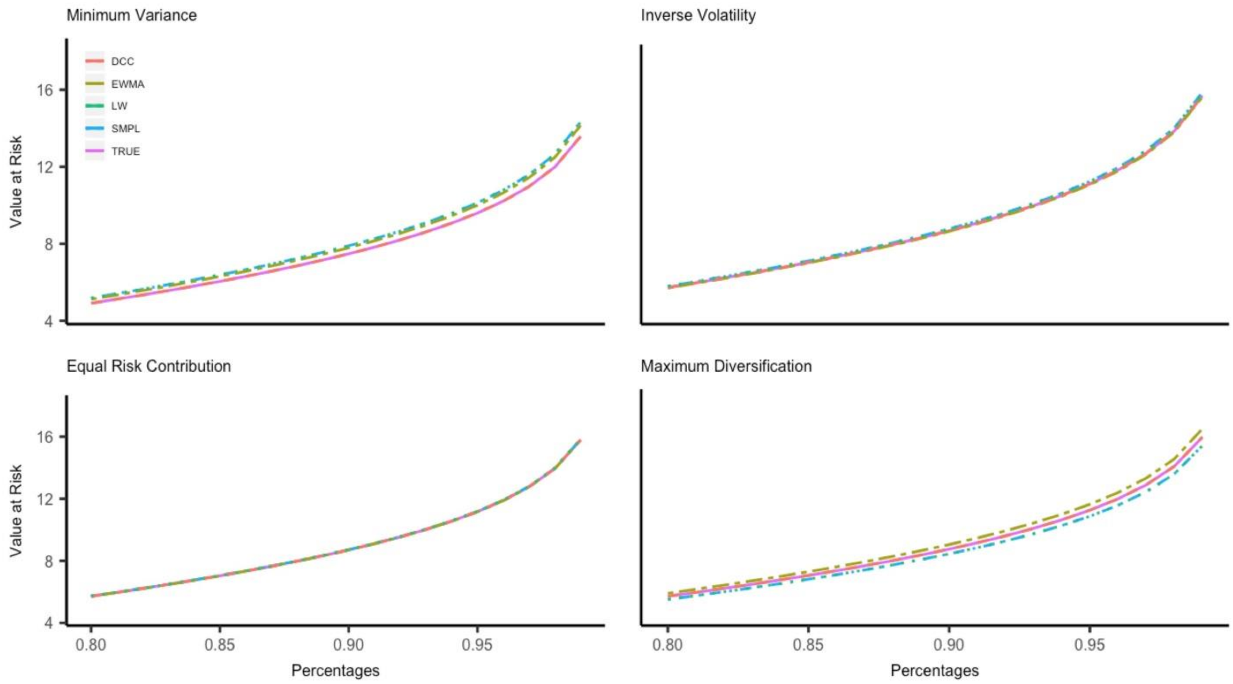

Figure 2. VaR values for variance misspecification

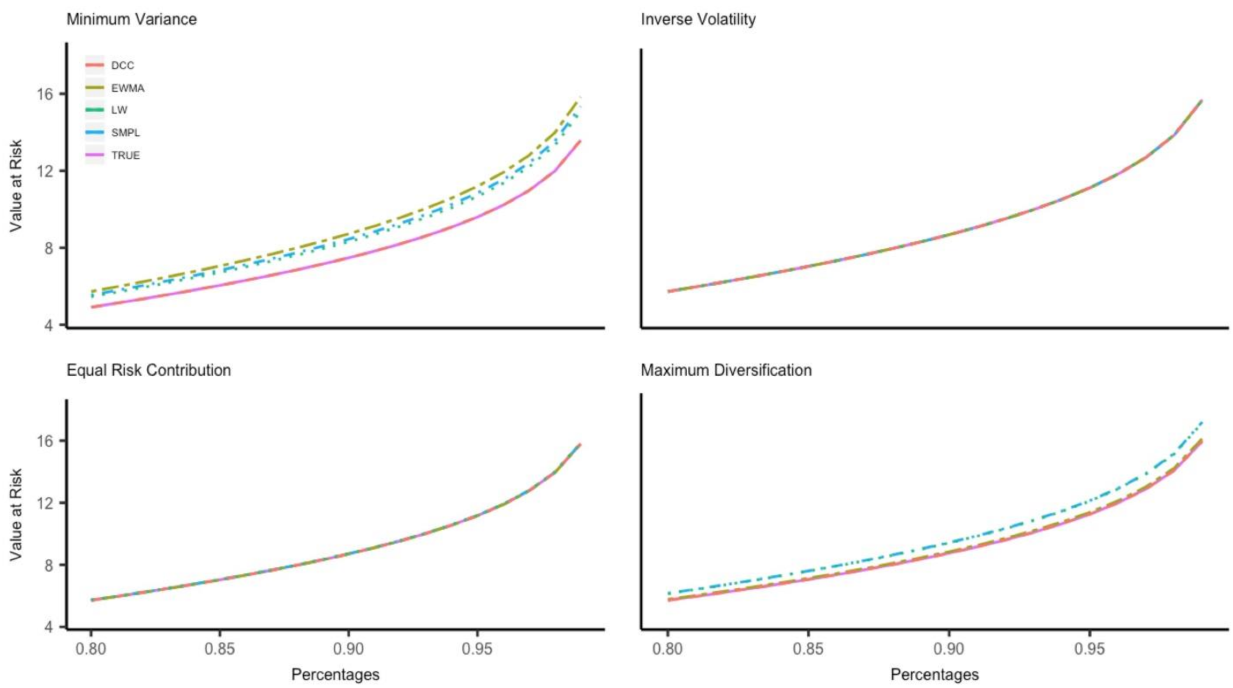

Figure 3. VaR values for covariance misspecification

Table 1. Value at Risk estimates

Time Horizon	Risk-Based Allocation	True	Overall Misspecification				Variance Misspecification				Correlation Misspecification			
			DCC	LW	SMPL	EWMA	DCC	LW	SMPL	EWMA	DCC	LW	SMPL	EWMA
h=1	minimum variance	0.00939	*0.00953*	0.01014	0.01015	**0.01023**	*0.00971*	**0.01007**	**0.01007**	0.01004	*0.00945*	0.01112	0.01139	**0.01201**
	inverse volatility	0.0113	*0.01134*	0.01152	**0.01152**	0.01128	0.01134	**0.01152**	**0.01152**	*0.01128*	*0.0113*	*0.0113*	*0.0113*	*0.0113*
	equal risk contribution	0.01119	*0.01123*	0.01141	0.01141	0.01112	0.0112	**0.01122**	**0.01122**	*0.01119*	*0.01122*	**0.01136**	**0.01136**	0.01113
	maximum diversification	0.01108	*0.0111*	0.01124	0.01127	**0.01168**	*0.01106*	0.01048	0.01048	**0.01137**	*0.01137*	0.0124	**0.01242**	0.01159
h=5	minimum variance	0.00942	*0.00943*	0.01006	0.01007	**0.01018**	*0.00942*	**0.01003**	**0.01003**	0.00993	*0.00942*	0.01097	0.01115	**0.01158**
	inverse volatility	0.01123	*0.01123*	**0.01139**	**0.01139**	0.01119	*0.01123*	**0.01139**	**0.01139**	0.01119	*0.01123*	*0.01123*	*0.01123*	*0.01123*
	equal risk contribution	0.01112	*0.01112*	0.01127	**0.01128**	0.01104	*0.01112*	**0.01114**	**0.01114**	0.01111	*0.01112*	0.01124	**0.01125**	0.01105
	maximum diversification	0.01102	*0.01102*	0.01114	0.01117	**0.01153**	*0.01103*	**0.0105**	**0.0105**	0.01137	*0.01103*	0.01204	**0.01206**	0.01133
h=20	minimum variance	0.0096	*0.0096*	0.0101	0.0101	**0.01023**	*0.0096*	**0.01013**	**0.01013**	0.01	*0.0096*	0.01067	0.01084	**0.01119**
	inverse volatility	0.01128	*0.01128*	**0.0114**	**0.0114**	0.01123	*0.01128*	**0.0114**	**0.0114**	0.01123	*0.01128*	*0.01128*	*0.01128*	*0.01128*
	equal risk contribution	0.01117	*0.01117*	**0.01129**	**0.01129**	0.01109	*0.01117*	**0.01119**	**0.01119**	0.01116	*0.01117*	0.01126	**0.01127**	0.0111
	maximum diversification	0.01107	*0.01108*	0.01116	0.01119	**0.01134**	*0.01108*	0.0107	0.0107	**0.01145**	*0.01108*	0.0119	**0.01193**	0.01119

Overall, minimum variance and maximum diversification portfolio tend to be affected by the misspecification of any parameters while inverse volatility and equal risk contribution portfolio is relative stable. In almost all the cases, DCC estimation and true covariance has the lowest risk level in terms of VaR. What is more, our simulation results show that when the time horizon for prediction h is increased, all estimation methods in each portfolio construction tilt toward to converge.

Table 1 shows the 95% VaR values found with four risk-based allocation methods estimated by the four construction methods (DCC, LW, SMPL, EWMA) for three different time horizons ($h=1,5,$ and 20). From the table, it can be seen that, about the allocation methods, the minimum variance method is always producing the minimum VaR and the inverse volatility is always giving the maximum variance except the covariance misspecification due to the weights are exactly same if the variance part is the same.

When the covariance is misspecified, the main result is almost the same as the situation when variance is misspecified. Inverse volatility and equal risk contribution portfolio is insensitive to the misspecification while maximum diversification and minimum variance portfolio is liable to the misspecification. Maximum diversification portfolio shows that DCC and true covariance has the same best performance but sample based estimation is the worst. Minimum variance portfolio is highly affected by the misspecification. Similar as before, DCC estimation and true covariance matrix has the lowest risk level in terms of the VaR but EWMA is the riskiest estimation method.

5. Conclusions

Value at risk values are estimated using Monte Carlo simulation when the portfolio's covariance matrix of returns is misspecified. It is found that the DCC estimation provides the best estimates to the true covariance matrix components. It also provides value at risk values that are closest to the corresponding value at risk values obtained from the true covariance matrix. This conclusion applies to all risk-based allocation methods and for all forecasting horizons considered.

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