A Capacitated Location-Inventory Model with Stochastic Demand and Coverage Radius Constraint

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Abstract

In this paper we introduce the capacitated warehouse location model with risk pooling and coverage radius (CLMRPCR). The CLMRPCR models a logistic system in which a single plant ships one type of product to a set of retailers, each with an uncertain demand, through the capacitated warehouses. The newly important issue is considered in this model, is considering coverage radius which is extremely important for some pragmatic cases. The CLMRPCR minimizes the sum of the fixed facility location costs, transportation costs and safety stock and working inventory costs. This model simultaneously determines warehouse location, assignment of retailers to warehouses, shipment sizes from the plant to warehouses, working inventory and safety stock levels at the warehouses considering the coverage radius each DC could service a retailer. We show that this problem can be formulated as a nonlinear integer program in which the objective function is neither concave nor convex. Resorting to a recent work, a Lagrangian relaxation solution algorithm is proposed. This algorithm provides near-optimal solutions with reasonable computational requirements for large problem instances.

Keywords Location-inventory, Capacitated EOQ, Lagrangian Relaxation, Integrated Supply Chain Design

1. Introduction

Considering a supply chain network as a whole by means of integrating the three levels of decision included strategic, tactical and operational could be very efficient and lead to high cost saving and large customer service. In reality these decision are dependent to each other for example strategic location decisions have a major effect on shipment and inventory costs (Aliabadi et al. 2013, Bagherpour et al. 2009, and Frenk et al. 2014). Each of these decisions separately has been considered in literature. For example, in seminal books Hopp and Spearman (2011), Daskin (2011), and Axater (2006) focus on inventory control and discuss the inventory policies for filling retailer orders. Alternatively, location models tend to focus on determining the number and the location of facilities, as well as retailer assignments to them. For a complete review on location modeling see Daskin and owen (2003).

Our paper relates to the extensive literature on deterministic inventory models as well. The Economic Order Quantity (EOQ) models are an important building block of research on supply chain for many years. The EOQ model is the building block of deterministic inventory models. For a complete review of EOQ models, we refer the readers to Choi (2014); For the treatment of EOQ models with general cost functions, Frenk et al. (2014) is a good source; for an application of EOQ model to a distribution network see Sajadifar and Pourghannad (2010) and Pourghannad et al. (2015). Also, although we focus on the deterministic approximation models, we acknowledge that there is a body of inventory literature focusing on stochastic inventory models; Axsater (1996) and Pourghannad (2013) apply the deterministic EOQ formula in stochastic inventory control. Bagherpour et al. (2009) propose a novel approach for an inventory model of a remanufacturing system with stochastic decomposition Process. Shahrazi et al. (2015) find optimal locations of electric public charging stations using stochastic real world vehicle travel patterns. Sajadifar and Pourghannad (2012) propose the cost function of an integrated dyadic supply chain with uncertainty in the supply.
One of the early works in incorporating location models and inventory costs is Baumol and Wolf (1958). They state that inventory costs should add a square root term to the objective function of the uncapacitated fixed charge location problem. Shen (2000), Shen and Qi (2007), and Daskin et al. (2002) present a joint location-inventory model in which location, shipment and nonlinear safety stock inventory costs are included in the same model. In fact, Shen et al. (2007) and Daskin et al. (2002) present the location model with risk pooling (LMRP). Shen and Qi (2007) develop a model in supply chain system with uncertainty in demands. They should determine the number and location of the DCs and also the assignment of retailers’ demands to the DCs. They apply routing costs instead of direct shipments which is much more realistic and use Lagrangian relaxation as solution algorithm. Sourirajan et al. (2007) and Sourirajan et al. (2009) develop an integrated network design model that simultaneously considers the operational aspects of lead time (based on queueing analysis) and safety stock. In 2007 they use Lagrangian relaxation and in 2009 they utilize Genetic algorithm and then they compare its’ results to solutions of Lagrangian relaxation. Ozsen et al. (2008) develop a capacitated location model with risk pooling in which they consider capacity constraints based on maximum inventory accumulation. They use Lagrangian relaxation as a solution algorithm. They also present a recent work called a multi-sourcing capacitated location model with risk pooling (Ozsen et al. 2009). Shen (2000) studies multiproduct extension of LMRP and Ghezavati et al. (2009) present a new model for distribution networks considering service level constraint and coverage radius. Ross and Khajehnezhad (2017) develop an integrated multi-echelon location-inventory model under forward and reverse product flows in the used merchandise retail sector.

In our paper, we propose a supply chain design model in which a single plant ships one type of product to a set of retailers, through the capacitated warehouses. The objective of the CLMRPCR is to minimize the sum of the fixed facility location, transportation-safety stock and working inventory costs. This model simultaneously determines warehouse location, assignment of retailers to warehouses, shipment sizes from the plant to warehouses, working inventory and safety stock levels at the warehouses considering the coverage radius each DC could service a retailer. In this model the capacity constraint ensures that the maximum inventory accumulation at a DC does not exceed its capacity. The CLMRPCR is formulated as a nonlinear integer program in which the objective function is neither concave nor convex. According to Ozsen et al. (2009), a Lagrangian relaxation solution algorithm is proposed for CLMRPCR similar to CLMRP. Therefore, we use an efficient algorithm similar to Ozsen et al. (2009). This algorithm provides near-optimal solutions with reasonable computational requirements for large problem instances.

A novel issue that we consider in our paper is coverage radius. This issue is extremely prominent due to many reasons. In many cases we have a deteriorated product cannot be sent to a far place, hence considering coverage radius is inevitable. Also because of increasing in fuel price in recent years, it would be better to reduce the total transportation, so coverage radius would be important. Besides these reasons, the most important matter in considering coverage radius would be environmental conditions, because by coverage radius restriction, we can decrease the transportation and it leads to declining in fuel consumption so naturally, the harmful environmental effects due to air pollution would be less. This phenomenon is much more important than the other reasons.

### 2. A Capacitated Location-Inventory Model with Stochastic Demand and Coverage Radius Constraint

We make the following assumptions: there is a single product, direct shipments from DCs to retailers, single sourcing, we consider coverage radius, Poisson distribution for all demands. Each DC follows an inventory control policy which is an approximation to the $(Q,r)$ model using two stages. Resorting to Ozsen et al. (2009), to minimize the working inventory costs, where the order quantity is the decision variable, we require to solve the capacitated EOQ problem for $DC_j$ (for a general treatment of EOQ models see Choi (2014) and see Minner and Pourghannad (2010) for an example of application). Before formulation, we define all the parameters in table 1 as below:

<table>
<thead>
<tr>
<th>Notation</th>
<th>Parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_j$</td>
<td>Fixed cost of placing an order</td>
</tr>
<tr>
<td>$h$</td>
<td>Annual holding cost per item</td>
</tr>
<tr>
<td>$g_j$</td>
<td>Fixed cost of shipping an order $DC_j$</td>
</tr>
<tr>
<td>$G_j(Q_j)$</td>
<td>Expected annual working inventory cost</td>
</tr>
<tr>
<td>$a_j$</td>
<td>Per-unit shipment cost from the plant to $DC_j$</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Weight factor for transportation costs</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Weight factor for inventory costs</td>
</tr>
</tbody>
</table>
\( \beta \) and \( \theta \) are used in experimental design. Let \( Q_j \) and \( W_j^*(D_j) \) be order quantity and optimal total working inventory cost for DC\( _j \) respectively. So we have

\[
\begin{align*}
\text{Minimize } & G_j(Q_j) = F_j \frac{D_j}{Q_j} + \beta_j \left( \frac{D_j}{Q_j} + a_j D_j \right) + \theta_j \frac{h_Q}{2} \\
\text{subject to } & Q_j + \sqrt{L_j D_j / X_j} + \frac{D_j}{Z_j} \leq C_j \\
& Q_j \geq 0
\end{align*}
\]

Coverage radius parameter:

\[
Z_{ij} = \begin{cases} 
1 & \text{If a DC at site } j \text{ can cover customer } i \\
0 & \text{Otherwise,}
\end{cases}
\]

Decision variables:

\[
X_j = \begin{cases} 
1 & \text{if we locate at candidate site} \\
0 & \text{if not}
\end{cases}
\]

\[
Y_{ij} = \begin{cases} 
1 & \text{if demands at customer } i \text{ are assigned to a DC at candidate site } j, \\
0 & \text{if not}
\end{cases}
\]

Hence, we can formulate CLMRPCR as:

\[
\begin{align*}
\text{Minimize } & \sum_{j=1}^{J} \left[ f_j \frac{X_j}{Q_j} + \beta_j \frac{Y_{ij}}{Q_j} \sqrt{L_j D_j / X_j} + \frac{h_Q}{2} \right] \\
\text{subject to } & \sum_{j=1}^{J} Y_{ij} Z_{ij} = 1, \quad \forall i \in I \\
& Y_{ij} \leq Z_{ij} X_j, \quad \forall i \in I, \ j \in J \\
& Q_j + \sqrt{L_j D_j / X_j} + \frac{D_j}{Z_j} \leq C_j, \quad \forall j \in J \\
& Q_j \geq 0, \quad \forall j \in J \\
& Y_{ij} \in [0,1], \quad \forall i \in I, \ j \in J \\
& X_j \in [0,1], \quad \forall j \in J
\end{align*}
\]
considering coverage radius. Constraints (3) say that a customer can be allocated to a DC if that customer is in the coverage radius of DC and DC is opened. (4) and (5) are constraints of the capacitated EOQ problem discussed before. (6) and (7) are standard binary constraints. To simplify, instead of representing the working inventory cost and the order quantity, we include \( W_j^* (D_j) \) in the formulation above. As you know, for further simplicity, we have:

\[
D_j = \sum_{i \in I} \chi \mu_i Y_{ij}
\]

So CLMRPCR formulation is converted to:

\[
(\text{CLMRPCR}) \quad \text{Minimize} \quad \sum_{j \in J} f_j X_j + \beta \sum_{i \in I} d_i \mu_i Y_{ij} + \sum_{j \in J} \left( \chi \sum_{i \in I} \mu_i Y_{ij} \right)
\]

\[
\text{Subject to} \quad \sum_{j \in J} Y_{ij} Z_i = 1, \quad \forall i \in I \quad (9)
\]

\[
Y_{ij} \leq X_i Z_j, \quad \forall i \in I, \quad j \in J \quad (10)
\]

\[
Y_{ij} \in [0,1], \quad \forall i \in I, \quad j \in J \quad (11)
\]

\[
X_j \in [0,1], \quad \forall j \in J \quad (12)
\]

3. Solution Algorithm

3.1. Obtaining a Lower Bound

Because CLMRPCR model is very close to CLMRP model, so we can corroborate this matter that Lagrangian relaxation would be efficient for CLMRPCR. Now, by relaxing the assignment constraints, we obtain the following Lagrangian Dual problem:

\[
\max_{\pi} \min_{X,Y} \sum_{j \in J} [f_j X_j + \beta \sum_{i \in I} d_i \mu_i Y_{ij} + \sum_{i \in I} (\chi \sum_{j \in J} \mu_i Y_{ij}) + \chi (1 - \sum_{j \in J} Y_{ij})]
\]

\[
= \sum_{j \in J} [f_j X_j + \sum_{i \in I} (\beta d_i \mu_i - \pi_i) Y_{ij} + \sum_{i \in I} \pi_i] + \chi (1 - \sum_{j \in J} Y_{ij}) \quad (13)
\]

\[
Y_{ij} \leq X_i Z_j, \quad \forall i \in I, \quad j \in J \quad (14)
\]

\[
Y_{ij} \in [0,1], \quad \forall i \in I, \quad j \in J \quad (15)
\]

\[
X_j \in [0,1], \quad \forall j \in J \quad (16)
\]

For fixed values of the Lagrangian multipliers, \( \pi \), the aim is minimizing (13) over the location variables, \( X_j \), and the assignment variables, \( Y_{ij} \). For a given \( \pi \) vector, the problem decreases to the following subproblem for each distribution center \( j \):
Resorting to Ozsen et al. (2009), the function $W^*(D)$ is nonlinear and is neither convex nor concave. Also, the properties of $w^*_j(D_i)$ and $W_j(D_i)$ are the same. Hence, we write its integrality relaxation called $(CC-SP_j^{RP})$:

\[
(CC-SP_j^{RP}) = \begin{cases} 
\text{Minimize} & V_j^{RP} = f_j + \sum_{i \in I} A_i Y_i \\
& + \sum_j W_j \left( \sum_{i \in I} Y_i \right) \\
\text{Subject to} & 0 \leq Y_i \leq 1, \quad \forall i \in I
\end{cases}
\]

where $A_i = \beta \xi d_{ij} \mu_i - \pi Y_i$

As formulated CLMRPCR, the structure of model is not different of CLMRP, so we could have a same procedure to solve, and use the solution algorithm stated by Ozsen et al. (2008) for providing a lower bound to CLMRPCR. Also resorting to Ozsen at al. (2008) this algorithm is too efficient for CLMRP, so it can be utilized for our model. They claim two important theorems used in solution algorithm for providing a lower bound.

The solution of $(CC-SP_j^{RP})$ provides a lower bound for $(CC-SP_j^{RP})$ and can be used to find a lower bound solution for the original problem. CLMRPCR The resulting lower bound due to $(CC-SP_j^{RP})$ may not be assured to provide a tight bound for CLMRPCR. So we apply a branch and bound on the assignment variables as same as Ozsen et al. (2008). After solving the Lagrangian Dual Problem, now we have a lower bound on the optimal value of the CLMRPCR over the given values for $\pi_i$. At each step of the Lagrangian heuristic, we use the dual-feasible solution to construct a primal feasible solution to the CLMRPCR, which provides an upper bound. We update the Lagrangian multipliers using subgradient optimization method by Fisher (1985) and proceed for next iteration. We repeat this procedure until some stopping criterion is reached.

3.2. Obtaining an Upper Bound

First, because of two factors, Coverage radius and Capacity constraint, we open all the candidate locations of DCs. So through this, we do not have any non-assigned retailers at the end of finding an upper bound in each iteration. All of the assignments from the lower bound stage remain fixed. We sort the retailers in decreasing order of mean demand, $\mu_i$, and proceed on the retailers based on this order. First we process the retailers with $\sum_{j \in J} Y_{ij} Z_{ij} = 1$; these retailers are divided into two sets; one group is related to the retailers with $\sum_{j \in J} Y_{ij} Z_{ij} = 1$. Because these retailers have been satisfied, so they will be the same and if there are some other assignments for them with violation of the determined coverage radius ($Y_{ij}^{LB}=1$ and $Z_{ij}=0$), so these assignments will be eliminated. The second group is related to the retailers with $\sum_{j \in J} Y_{ij}^{LB} Z_{ij} > 1$. Based on the ordering of retailers, we assign each retailer to a DC in coverage radius that increases the total cost the least, based on the assignments made so far. The rest of assignments will be eliminated.

Next, we process the retailers with $\sum_{j \in J} Y_{ij}^{LB} Z_{ij} = 0$. Also these retailers are divided into two sets; one part is the retailers with $Y_{ij}^{LB}=1$ and $Z_{ij}=0$, because in the lower bound stage, even though there has been a DC or more for that retailer, but none of these DCs has not been in coverage radius of the retailer. So we find the DCs in coverage radius of the retailer and assign that retailer to a DC that increases the total cost the least, based on the assignments made so far. At the end, the rest of assignments for that retailer will be eliminated. The second part is the retailers with $Y_{ij}^{LB}=0$. In this sense we treat to a similar way of previous part. Also in both of the groups we choose retailers based on the ordering we mentioned before. At the end of this process, if there are DCs with no retailers assigned to them, then we close those DCs and if all the retailers are assigned to some DC, then this solution is a feasible one for CLMRPCR and provides an upper bound on the objective function (8). At the end of Lagrangian procedure, we could use a DC-exchange heuristic. The heuristic swaps a DC currently open in the solution with another DC is not currently in the solution, if doing so improves the solution. Teitz and Bart (1968) applied this procedure for the p-median problem. We tested this procedure for our instances and observed about 30%-40% improvement in the objective function.

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We tested our algorithm for the CLMRPCR on a 15-node data set consists of the 15 capitals with the highest demand from the 49 capitals of USA. We use the data which is described in Daskin (2011). Each of nodes represents a retail location and each retail location is a candidate DC location. The mean demand for each node (capital) was obtained by dividing the population of that capital by 1000. Fixed facility location costs were obtained by dividing the facility location costs of those capitals defined in Daskin (2011) by 100. Also, $d_{ij}$ was set to the great circle distance between these locations. Other parameters were set as below:

<table>
<thead>
<tr>
<th>Prob no.</th>
<th>No. of &amp;</th>
<th>DCs opened</th>
<th>DCs closed</th>
<th>No. of</th>
<th>Last capacitated</th>
<th>Open DCs with</th>
<th>Total</th>
<th>Lag</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Open DC</td>
<td>limited capacity</td>
<td>limited capacity</td>
<td>cost</td>
<td>iter</td>
</tr>
<tr>
<td>1</td>
<td>15</td>
<td>1,2,6,7,8,10,11,12,13,15</td>
<td>N/A</td>
<td>10</td>
<td>N/A</td>
<td>N/A</td>
<td>8594</td>
<td>12</td>
</tr>
<tr>
<td>2</td>
<td>15</td>
<td>None</td>
<td>None</td>
<td>10</td>
<td>2</td>
<td>2</td>
<td>9486</td>
<td>13</td>
</tr>
<tr>
<td>3</td>
<td>15</td>
<td>None</td>
<td>None</td>
<td>10</td>
<td>6</td>
<td>2,6</td>
<td>10124</td>
<td>13</td>
</tr>
<tr>
<td>4</td>
<td>15</td>
<td>None</td>
<td>None</td>
<td>10</td>
<td>8</td>
<td>2,6,8,11</td>
<td>10993</td>
<td>15</td>
</tr>
<tr>
<td>5</td>
<td>15</td>
<td>None</td>
<td>None</td>
<td>10</td>
<td>11</td>
<td>2,6,8,11,7</td>
<td>10894</td>
<td>11</td>
</tr>
<tr>
<td>6</td>
<td>15</td>
<td>None</td>
<td>None</td>
<td>10</td>
<td>7</td>
<td>2,6,8,11,7,12</td>
<td>11045</td>
<td>14</td>
</tr>
<tr>
<td>7</td>
<td>15</td>
<td>None</td>
<td>None</td>
<td>10</td>
<td>12</td>
<td>2,6,8,11,7,12,10</td>
<td>11110</td>
<td>12</td>
</tr>
<tr>
<td>8</td>
<td>15</td>
<td>None</td>
<td>None</td>
<td>10</td>
<td>10</td>
<td>2,6,8,11,7,12,10</td>
<td>11453</td>
<td>13</td>
</tr>
<tr>
<td>9</td>
<td>15</td>
<td>None</td>
<td>None</td>
<td>13</td>
<td>9</td>
<td>13,2,6,8,11,7,12,10,15</td>
<td>11873</td>
<td>17</td>
</tr>
<tr>
<td>10</td>
<td>15</td>
<td>None</td>
<td>None</td>
<td>9</td>
<td>15</td>
<td>2,6,8,11,7,12,10,15</td>
<td>12040</td>
<td>14</td>
</tr>
</tbody>
</table>

Table 2. Parameters for numerical studies

<table>
<thead>
<tr>
<th>$F_i$</th>
<th>$g_j$</th>
<th>$a_j$</th>
<th>$h$</th>
<th>$z_{ij}$</th>
<th>$\lambda$</th>
<th>$L$</th>
<th>$\beta$</th>
<th>$\theta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>10</td>
<td>5</td>
<td>1</td>
<td>1.96</td>
<td>1</td>
<td>1</td>
<td>0.0004</td>
<td>0.01</td>
</tr>
</tbody>
</table>

The reason for being $\lambda = L = 1$ is due to $\beta$ and $\theta$ so as transportation and inventory costs should be logical compared to each other and fixed location costs. For Lagrangian relaxation, the parameters of maximum number of iterations and number of iterations in which the objective function not improved are set to 200 and 10 respectively. Also the initial value of “$\alpha$” is set to 2 in which “$\alpha$” is scalar used in computing the step size. The initial value of $\pi$ is set to $10 \mu + 10 f_i$. The notation $\mu$ stands for the average mean demand across all retailers. We terminated the Lagrangian procedure based on the maximum number of iterations at each node or number of iterations in which the objective function of CLMRPCR not improved, whichever occurred first. Although in none of our experiments, the procedure reaches the maximum number of iterations. The program was written in MATLAB 2008. We have tested different values of the DC capacities to vary the difficulty of the instances. (See Ozsen et al., 2008). Now according to the parameters, we set two tables; the first considers coverage radius (R=500), and the second one does not consider it. (R=3000). The column of “Last capacitated DC” refers to Additional DC assigned a limited capacity.

As seen, in the case with coverage radius (R=500), the number of DCs are more than in the case without coverage radius (R=3000). One reason for that is due to the concept of coverage radius. It means that when we consider coverage radius, a number of retailers cannot be allocated to some candidate DCs, hence the number of open DCs increases to serve the total demand of retailers. In this sense, the fixed location costs increase but the transportation costs decrease. Another matter to comparing these two tables is due to the total costs. In the case with coverage radius, because of considering a parameter as coverage radius in CLMRPCR which is a restricting parameter, the total costs are more than the other case without coverage radius. But as you observe, the difference between these costs are not a large value due to the parameters we have set for fixed location, inventory and transportation costs. None of the problem
instances reaches the maximum of Lagrangian iterations, and in all of them the condition about the Number of iterations in which the objective function of CLMRPCR not improved, satisfied. These two tables were set for \((\beta, \theta) = (0.0004, 0.01)\) but for more investigation on the network design and its costs, we can vary these values and then compare the results to previous. Another important issue in this section is related to capacity. In both tables, when we limited the capacity of a DC, in most of the time, it remained open in optimal solution. It means that restricting capacity could not be a main factor for opening or closing a DC.

4. Conclusion and Future Research

CLMRPCR is a more practical model than CLMRP, because besides all of the properties of CLMRP, it considers coverage radius which is a real factor in the business world. There are some reasons for that we mentioned before. In addition, the CLMRPCR has the properties of CLMRP. For example, ordering more in smaller quantities, developing the CFLP and providing a more reasonable measure of capacity for warehouses are some of them.

We want to talk about some extensions for CLMRPCR. A natural extension to the CLMRPCR would be to consider multi products. Another extension would be to incorporate the CLMRPCR with stock-out costs. Also considering multi-sourcing instead of one-sourcing would be a logical extension that would lead to lower logistics costs. Also, in our model we assumed direct shipments from DCs to the assigned retailers. But in reality, the shipments are a traveling-salesman-like tour. Hence considering routing costs instead of direct shipment costs would be more realistic.

As one part of future research, one can focus on developing algorithms to solve the problem we study in this paper for large scale supply chains with thousands of nodes. Examples of such algorithms in supply chain settings are a two-level GA algorithm by Aliabadi et al., 2013, Neural network algorithm by Avşar and Aliabadi, 2015, or agent-based algorithms by Aliabadi et al., 2017.

References


**Biography**

**Milad Khajehnezhad** is currently a fulltime senior credit risk modeler in Realtyshares, a leader in real estate crowdfunding. Mr. Khajehnezhad holds a Master of Science degree in Industrial Engineering from University of Wisconsin Milwaukee. He is a credit and portfolio risk manager with about 4 years of experience in working with large banks and peer-to-peer lending platforms. He has also accomplished various operations research and data analytics projects in supply chain networks design (location-inventory models) and high-volume production lines simulation. He has served as a reviewer for Transportation Research Part E: Logistics and Transportation Review journal.