Batch Sizing in Sustainable Production Systems with Imperfect Quality

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Abstract

In classic Economic Production Quantity (EPQ), an optimal batch size is determined to minimize total production cost including setup and inventory holding costs, and defective parts are not allowed. In this paper an imperfect EPQ system is studied to minimize the overall cost, where setup cost, scarp rate, batch size, and electrical power demand are determined by the model. In this imperfect production system, a percentage of the batch is defective in each cycle, which will be reworked at an extra charge. In addition, the model considers electrical power demand charge which accounts for a large percentage of industrial utility bills. This framework also determines the optimal level of investment on system design and flexibility which in turn, is a function of setup cost, electrical power requirement (power demand), and scrap rate. The proposed constrained cost minimization problem is formulated as a nonlinear mathematical model, and is solved using a posynomial Geometric Programming (GP) approach to present a closed form solution for the batch size, setup cost, allowable defective rate and power requirements. The model is illustrated through a numerical example and some sensitivity analysis is performed.

Keywords
Economic Production Quantity (EPQ), Energy Cost, Geometric Programming, Power demand

1. Introduction

The main objective of the Economic Production Quantity (EPQ) model is to minimize total cost of setup, holding and production costs, where demand, setup and inventory holding cost components are known parameters. In addition, the original EPQ model does not allow for imperfections in batches, and as a result, imperfect process cost is not included in the model. The EPQ model also considers energy, material and labor cost as a fixed cost per batch, which at the end is removed from the model, as that would be a constant term.

According to the U.S. Energy Information, the industrial sector is responsible for 51% of total energy consumption, and 28% of greenhouse gas emissions in the world (U.S. Energy Information). The increasing trend of the need to energy and the associated charges, along with environmental concerns make energy consideration an influential factor in planning of production systems. In industrial facilities energy requirement reduction can be achieved through either adopting advanced energy efficient equipment with new built-in technologies, or implementing energy-aware scheduling systems using existing technologies. The first approach necessitates some investment in technology and new equipment, whereas the second approach is more cost effective as no additional investment is required to acquire new equipment. The determination of appropriate level of investment in the first approach is a function of various operational parameters such as setup time and cost, reliability/scrap rates, and energy efficiency. Consequently, one cost effective solution is the simultaneous determination of investment level and required production system key parameters, to prevent over-spending in purchasing equipment. This approach has been adopted in this study.

As mentioned above, energy cost is usually ignored in EPQ modeling. Industrial energy cost consists of two components: energy consumption and energy demand charges. Demand charge accounts for the major portion of utility bills, and is to cover the overhead expenses that utility provides bear to satisfy the highest level of required power over a given period of time during the billing period (Módos et al., 2017). Energy demand is measured in kilowatts. Energy consumption charges, on the other hand, are measured by kilowatt-hour (kWh), and represent the actual amount of energy usage over a given period (Nezami et al., 2017). There are a few studies that have considered energy consumption and/or demand charges in modeling EPQ systems.

In this paper, a new EPQ model with imperfect process and rework is studied, where batch size, and setup cost, acceptable defective rate, and maximum level of power requirement are to be determined via the model. Setup cost, defective rate, and power demand are used to determine the investment amount in technological improvement of the facility. In most of the studies in the literature, setup cost, and defective (scrap) rate or reliability ratio are assumed as the known input parameters to the model, and power requirement and cost considerations are not well regarded. For example, Ben-Daya et al. (2008) and Sana (2010) study imperfect EPQ models where production cost depends on production rate, labor and energy costs. Nezami and Heydar (2016) calculate the optimal batch size in an EPQ system with variable energy costs during peak and off-peak hours. Similar studies that consider energy cost in EPQ modeling can be found in Sana (2011) and Sarkar (2012). However, in all these studies, setup cost and failure rate or power requirement are given, and no infrastructure investment modeling is considered. To consider infrastructure investment concept, Leung (2007) presents an EPQ model to minimize the total cost, and determine batch size, reliability level and setup cost such that investment in flexibility improvement could reduce setup cost and improve reliability. The model is solved using Geometric Programming (GP). Similar work has been proposed in (Islam and Roy, 2016), but none of these studies considered power demand and the associated costs.

In summary, the main contribution of this chapter is that in contrary to most of the studies in the literature on cost minimization of imperfect EPQ models, the process failure rate and setup cost are not constant inputs, and are assumed to be decision variables determined by the model. In addition, this model considers energy requirement planning and its charge, as it is a considerable cost factor in industrial facilities. These decision variables provide a basis for investment planning of system infrastructures in a production facility. The proposed model is solved using a posynomial Geometric Programming to obtain the closed from solutions of the variables.

In general, GP has been adopted in various studies on classic EPQ modeling, and is an effective approach for solving these models for optimality (Nezami et al., 2009). For example, Jung and Klein (2005) propose three economic order quantity (EOQ) models to determine batch size, where the unit cost is a decreasing function of demand, and/or batch size. Esmaeili et al. (2007) use GP to solve an EPQ system to determine price, discount amount, production rate and batch size. In another study, Fathian et al. (2009) adopt a GP approach to maximize profit for electronic products, and determine selling price, marketing and service expense per unit of product. In their model, unit cost is a function of product demand, which in turn, could be affected by the above decision variables.

The paper is organized as follows: in the next section the problem statement, mathematical notation, and model assumptions are described, and then the model mathematical formulation is presented. Section 3 discusses the solution approach, and in Section 4 the model is solved for optimality, and closed from solutions are provided. Section 5 gives a numerical experiment to verify the model, provides some insight about model parameters sensitivity. Section 6 concludes the major findings of this study.

2. Problem Statement

This study investigates a cost minimization problem for an imperfect EPQ system, at which defective parts can be reworked and fully recovered at an extra cost. The model also considers power demand level and its cost. As described earlier, energy demand reflects the maximum required power; once the machine is turned on, the system requires *p* kilowatt of power for processing, regardless of the production quantity. The notation utilized in this paper is summarized as follows:

- D: Demand rate
- *H*: Inventory holding cost per unit of product per unit of time
- C_e: Power demand charges per kW
- *R*: Rework cost per unit of defective item
- *A*: Scaling constant in technology investment
- F_{tres}: Maximum acceptable defective rate
- β: Failure rate elasticity of the technology investment cost
- α: Setup cost elasticity of technology investment cost
- γ: power demand elasticity of technology investment cost
- *p*: Power demand (decision variable)
- *q*: Production lot size (decision variable)
- f: Defective rate (decision variable)
- *s*: Setup cost (decision variable)

The investment function has been defined as a function of *p*, *f*, and *s*, such that $I(s,f,p)=As^{-\alpha}f^{-\beta}p^{-\gamma}\ for\ A,\alpha,\beta,\gamma\geq 0$ (Leung, 2007; Islam and Roy, 2007). In other words, one can assume that more advanced technologies are entitled to less setup time and cost, and failure rates, and are more energy efficient.

The objective function is to minimize the overall cost of the production system. The cost per cycle is determined by:

$$Min(Cost/cycle)=s+\frac{Hq^2}{2D}+Rfq+As^{-\alpha}f^{-\beta}p^{-\gamma}+C_p p+cq \tag{1}$$

The above function aims at minimizing setup, inventory holding, rework, investment, power demand, and material and labor costs per cycle, respectively. Multiplying Equation (1) by the number of periods (D/q) yields the overall cost as follows, given the constraint on failure rate to be less than a desirable threshold (F_{tres}):

$$Min\ \ TC(s,q,f,p)=s\frac{D}{q}+\frac{Hq}{2}+RD\,f+AD\,s^{-\alpha}f^{-\beta}p^{-\gamma}q^{-1}+C_pD\,p\,q^{-1}\ ,\ \ s.t.\ \ f\leq F_{tres} \tag{2}$$

For solving the above constrained nonlinear model, the necessary conditions of optimality are given by the Karush-Kuhn-Tucker (KKT) theorem, and the Lagrangian is as follows:

$$L(s,q,f,p,\lambda)=TC(s,q,f,p)+\lambda(F_{tres}-f)\qquad s,q,f,p,\lambda\geq 0 \tag{3}$$

Therefore

$$\begin{bmatrix} \frac{\partial TC}{\partial s} = Dq^{-1} - AD\alpha\, s^{-\alpha-1} f^{-\beta} p^{-\gamma} q^{-1} \\ \frac{\partial TC}{\partial q} = \frac{H}{2} - Dsq^{-2} - C_p Dq^{-2} + AD\, s^{-\alpha} f^{-\beta} p^{-\gamma} q^{-2} \\ \frac{\partial TC}{\partial p} = -\gamma AD\, s^{-\alpha} f^{-\beta} p^{-\gamma-1} q^{-1} + C_p Dq^{-1} \\ \frac{\partial TC}{\partial f} = RD - \beta AD\, s^{-\alpha} f^{-\beta-1} p^{-\gamma} q^{-1} \end{bmatrix} + \lambda \begin{bmatrix} 0 \\ 0 \\ 0 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow \lambda(f - F_{tres}) = 0 \ \& \ f - F_{tres} \le 0 \quad (4)$$

Obtaining a closed form solution using Equation set (4) is a very complicated task. As a result, a geometric programming approach is adopted to obtain the optimal closed form solutions.

3. Geometric Programming

Geometric programming (GP) introduced by Duffin, Peterson, and Zener (1967) is a powerful mathematical technique for many real-world engineering problems, and is based on the inequality between arithmetic and geometric means:

$$u_1 + u_2 + \ldots + u_T \ge \left(\frac{u_1}{w_1}\right)^{w_1} \left(\frac{u_2}{w_2}\right)^{w_2} \ldots \left(\frac{u_T}{w_T}\right)^{w_T} \quad (5)$$

Where $w_1+w_2+\ldots+w_T=1$; $w_i \ge 0$. A general standard geometric programming model (GP) is presented as follows:

$$\begin{aligned} &\min \quad f(X) = \sum_{K=1}^{l_0} \theta_{0k} C_{0k} \prod_{j=1}^{n} (X_j)^{a_{0KJ}} \\ &\quad \text{s.t:} \quad g_i \equiv \sum_{K=1}^{l_i} \theta_{ik} \cdot C_{ik} \prod_{j=1}^{n} (X_j)^{aijk} \le \theta \qquad ; i = 1, \ldots m;\ j = 1, \ldots n;\ k = 1, \ldots l_i \\ &\qquad\qquad \mathrm{X_j} > 0 \qquad (6) \\ &\theta_{0k} = \pm 1, \theta_{ik} = \pm 1, \theta_i = \pm 1; C_{0k}, C_{ik} > 0 \\ &a_{0kj,} a_{ikj} : real\ number \end{aligned}$$

In this setting, when all θ_{0k} and θ_{ik} are positive, the model is a posynomial, and if some of these two parameters are negative, the model is a signomial GP (Beightler and Phillips, 1976). One of the most important issues in GP is the degree of difficulty of the model, which is defined as the total number of terms minus the number of variables plus one. As the degree of difficulty of the model increases, the model gets more difficult to solve (Duffin et al. ,1967; Beightler and Phillips, 1976). To solve the GP models which are not generally convex, the dual program is defined over a convex region with linear dual constraints through the following mathematical program (Boyd, 2004):

$$Max\ d(\alpha)=\prod_{i=0}^{m}\prod_{k=1}^{l_i}\left(\frac{c_{ik}.\beta_i}{w_{ik}}\right)^{w_{ik}}$$

Subject to:

$$\sum_{k=1}^{l.}\alpha_{0k}=\beta_0=1$$

$$\sum_{i=0}^{m}\sum_{k=1}^{l_i}a_{ikj}w_{ik}=0;\ j=1,\ldots n$$

$$\sum_{k=1}^{l_i}w_{ik}=\beta_i;i=1,\ldots,m \tag{7}$$

$$w_{ik}\geq 0;i\neq 0$$

$$\beta_i\geq 0;i\neq 0$$

Using the dual constraints, the values of dual variables (w_{ik}), which are the indicators of the relative weights of each term in the primal model, are calculated and the dual objective function value is obtained accordingly. Since the objective function values of the primal and dual are equal at optimality (Duffin et al. ,1967; Beightler and Phillips, 1976; Boyd, 2004), the values of the variables in the primal model can be determined.

4. Problem Solution

To minimize the total cost function of the proposed model, which is a posynomial GP with one degree of difficulty (6-(4+1)=5), the dual problem is formulated as follows:

$$\max(DTC)=\left(\frac{D}{w_1}\right)^{w_1}\left(\frac{0.5H}{w_2}\right)^{w_2}\left(\frac{RD}{w_3}\right)^{w_3}\left(\frac{AD}{w_4}\right)^{w_4}\left(\frac{C_pD}{w_5}\right)^{w_5}\left(\frac{\Delta}{F_{tres}w_6}\right)^{w_6}$$

$$\begin{aligned}&subject\ to:\\ &\quad w_1+w_2+w_3+w_4+w_5=1\\ &\quad w_3-\beta w_4+w_6=0\\ &\quad -\gamma w_4+w_5=0\\ &\quad w_1-\alpha w_4=0\\ &\quad -w_1+w_2-w_4-w_5=0\\ &\quad w_6=\Delta\end{aligned} \tag{8}$$

The above equation set has five equations and six variables. In order to obtain the value of the unknown variables, the variables can be represented as a function of one variable, let assume w_4. Therefore, one can conclude:

$$\begin{aligned}&w_1=\alpha w_4\ \ ,\ \ w_2=(\alpha+\gamma+1)w_4\ \ ,\ \ w_3=1-2(\alpha+\gamma+1)w_4\ \ ,\ \ w_5=\gamma w_4\\ &w_6=(\beta+2(\alpha+\gamma+1))w_4-1\end{aligned} \tag{9}$$

This gives the value of the dual objective function as follows:

$$\max(DTC(w_4)) = (\frac{D}{\alpha w_4})^{\alpha w_4} (\frac{0.5H}{(\alpha+\gamma+1)w_4})^{(\alpha+\gamma+1)w_4} (\frac{RD}{1-2(\alpha+\gamma+1)w_4})^{1-2(\alpha+\gamma+1)w_4} \times (\frac{AD}{w_4})^{w_4} (\frac{C_p D}{\gamma w_4})^{\gamma w_4} (\frac{1}{F_{tres}})^{(\beta+2(\alpha+\gamma+1))w_4 - 1} \quad (10)$$

To solve the above indefinite function for w_4, we take the natural logarithm of equation (10). This yields the equation (11), which is to maximize a concave function with a convex feasible solution area. Equation (11) is concave because it is a negative summation of several convex functions, i.e., $w_i \ln(\frac{w_i}{c_i})$.

$$\begin{aligned} \max \ \ln(DTC(w_4)) &= -\alpha w_4 \ln(\frac{\alpha w_4}{D}) - (\alpha+\gamma+1)w_4 \ln(\frac{(\alpha+\gamma+1)w_4}{0.5H}) \\ &- (1-2(\alpha+\gamma+1)w_4)\ln(\frac{1-2(\alpha+\gamma+1)w_4}{RD}) - w_4 \ln(\frac{w_4}{AD}) - \gamma w_4 \ln(\frac{\gamma w_4}{C_p D}) \\ &- (\beta + 2(\alpha+\gamma+1))w_4 - 1)\ln(F_{tres}) \end{aligned} \quad (11)$$

To determine the optimal value of w_4, the first derivative of equation (11) is calculated and set to zero:

$$\begin{aligned} &\frac{\partial \ln(DTC(w_4))}{\partial w_4} = 0 \\ &\Rightarrow \alpha \ln(\frac{\alpha w_4}{D}) + (\alpha+\gamma+1)\ln(\frac{(\alpha+\gamma+1)w_4}{0.5H}) - 2(\alpha+\gamma+1)\ln(\frac{1-2(\alpha+\gamma+1)w_4}{RD}) \\ &+ \ln(\frac{w_4}{AD}) + \gamma \ln(\frac{\gamma w_4}{C_p D}) + (\beta + 2(\alpha+\gamma+1))\ln(F_{tres}) = 0 \end{aligned} \quad (12)$$

Logarithmic calculations conclude:

$$\begin{aligned} &\Rightarrow \ln(\gamma^{\gamma} \alpha^{\alpha} (\alpha+\gamma+1)^{(\alpha+\gamma+1)} {w_4}^{2(\alpha+\gamma+1)} (1-2(\alpha+\gamma+1)w_4)^{-2(\alpha+\gamma+1)} {F_{tres}}^{\beta+2(\alpha+\gamma+1)}) \\ &= \ln(A {C_p}^{\gamma} R^{-2(\alpha+\gamma+1)} (0.5H)^{(\alpha+\gamma+1)} D^{-(\alpha+\gamma+1)}) \end{aligned} \quad (13)$$

As a result,

$$\begin{aligned} &\gamma^{\gamma} \alpha^{\alpha} (\alpha+\gamma+1)^{(\alpha+\gamma+1)} {w_4}^{2(\alpha+\gamma+1)} (1-2(\alpha+\gamma+1)w_4)^{-2(\alpha+\gamma+1)} {F_{tres}}^{\beta+2(\alpha+\gamma+1)} \\ &= A {C_p}^{\gamma} R^{-2(\alpha+\gamma+1)} (0.5H)^{(\alpha+\gamma+1)} D^{-(\alpha+\gamma+1)} \end{aligned} \quad (14)$$

The optimal value of w_4 can be easily obtained numerically using equation (14). Once w_4^* is determined, the values of w_1, w_2, w_3, w_5, and w_6 variables can be calculated accordingly, using the equation set (9). Substituting the values of w_i into objective function (8) gives the optimal value of DTC^*. Since DTC is the dual function of TC, at optimality $DTC^*=TC^*$. To find the optimal values of s, q, p, and f, assume:

$$u_1 = s\frac{D}{q},\ \ u_2 = \frac{Hq}{2},\ \ u_3 = RD\,f\,,\ \ u_4 = AD\,s^{-\alpha}\,f^{-\beta}\,p^{-\gamma}q^{-1},\ \ u_5 = C_p D\,p\,q^{-1},\ u_6 = f \qquad (15)$$

Consequently, at optimality we have ((Duffin et al. ,1967; Beightler and Phillips, 1976):

$$\frac{u_1^*}{w_1^*} = \frac{u_2^*}{w_2^*} = \frac{u_3^*}{w_3^*} = \frac{u_4^*}{w_4^*} = \frac{u_5^*}{w_5^*} = DTC^* \qquad (16)$$

Combining equations (15), (16) and (9) concludes:

$$q^* = \frac{(\alpha+\gamma+1)w_4^*\,DTC^*}{0.5H} \Rightarrow s^* = \frac{\alpha w_4^* DTC^* q^*}{D},\ p^* = \frac{\gamma w_4^* DTC^* q^*}{C_p D}$$
$$f^* = \frac{(1-2(\alpha+\gamma+1)w_4^*)\,DTC^*}{RD} \qquad (17)$$

5. Numerical Case Study and Sensitivity Analysis

To validate the proposed model, an EPQ system with following parameters is assumed: D=10,000 units/year, H= \$5 per unit per year, R= \$10 per part, C_p= \$5/kW, the maximum acceptable defective rate F_{tres}=0.1, and A=5, $\alpha = 0.1, \beta = 2, \gamma = 0.9$. Substituting the parameters in equation (14) determines the optimum value of w_4^*. As a result, the optimal values of the remaining dual variables can be computed using equation set (9), accordingly. This yields: $w_1^* = 0.0052, w_2^* = .1038, w_3^* = 0.7924, w_4^* = 0.0519, w_5^* = 0.0467, w_6^* = -0.6886$. The objective function (8) or equation (10) calculated the optimal value of the objective function DTC^*=\$12,620 as the total cost of the production system. Using the objective function value and the calculated dual variables, the optimal quantity of the decision variables are obtained using equation set (17), which yields q^*=524 units, s^*= \$3.43, p^*=6.18 kW, and f^*=10%. It can be easily seen that by substituting the values of w_i^* into the *DTC* objective function from equation (8), and then replacing its value (DTC^*) in f^* formulation in equation (17), the optimal value of f*=F_{tres} is obtained. As a result, at optimality, the model constraint is always binding, and the problem can be reduced modified from *TC(s,q,p,f)* to $TC(s,q,p,F_{tres})$.

In general, in GP, when the degree of difficulty is not zero, the sensitivity analysis will not be an easy task; Any changes in model parameters requires solving equation (14), and following the described process to resolve the model to determine the output values. The analysis of the F_{tres} parameter shows that when the acceptable level of defective rate increases, the overall cost increases, while the batch size (*q)* and total cost of technology decreases. Increasing the price of energy demand (C_p) leads to an increase in overall system cost, and concludes a less energy-consuming (more energy-efficient) technology. As a result, the total investment in technology term increases. Increasing demand (*D*) and holding cost (*H*) leads to a growth and decline in batch sized (*q*), respectively. Both scenarios increase the total cost of the system.

6. Conclusion

This study analyzes a new imperfect EPQ system considering rework, technology investment, and energy demand costs. In contrary to traditional EPQ models, this model assumes set up cost, power demand, and defective rate as model decision variables. The proposed constrained nonlinear model is solved using a posynomial geometric programming approach to minimize the cost, and determine the optimal values of setup cost, batch size, defective rate, and power requirement of the equipment. The analysis shows that model constraint is binding at optimality. In addition, the sensitivity analysis proves that more investment on infrastructure enhances energy efficiency, and reduces defective rate. From managerial perspective, knowing about the power requirement of the system is beneficial for desigining proper utility contracts, and prevent over-spending in power planning. Including energy consumption costs, shortage cost, or imperfect production systems with total loss (no rework process) can be considered as directions of future studies.

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