An Enhanced VIKOR MCDM Model to Solve Complex and Multi-Objective Optimization Problems

Seyedmohammad Salehi
Computer and Information Sciences, University of Delaware, Newark, DE 19716, USA
salehi@udel.edu

Maghsoud Amiri
Department of Industrial Management, Allameh Tabataba'i University, Tehran, Iran
amiri@atu.ac.ir

Saeed Ramezanzadeh
Department of Basic Sciences, Faculty of logistics, Amin Olum-e-entezami University, Tehran, Iran
saeeed.ramezan@gmail.com

Mohammadali Abedini
Australian Maritime College, University of Tasmania, Launceston, Tasmania 7250, Australia
mohammadali.abedinisanigy@utas.edu.au

Abstract

This paper has developed an extension to VIKOR algorithm in order to solve multi-objective decision-making problems. The extended algorithm introduces a multi-criteria ranking index based on measuring of ''closeness'' to the ideal solution where the distance is measured by the L_P-metric for rectilinear distance or p = 1 and maximum norm or p = ∞. Utilizing two L_P-metrics, a k-dimension objective space can be reduced to a two-dimensional objective. Finally, a single-objective programming problem is derived and solved using the positive and negative ideal solutions of the two-dimensional problem. The effectiveness of the proposed algorithm is illustrated by solving a numerical example and the results are compared with the extended TOPSIS for MODM method.

Keywords

VIKOR, Multi-Criteria Decision-Making, Multi-Objective Decision-Making, Compromise Solution, Ideal Solution, LP-Metric.

1. Introduction

Multiple criteria decision-making (MCDM) methods have shown promising solutions to the complex problems in management sciences. These problems are often characterized by several conflicting criteria and there may be no solution that satisfies all criteria simultaneously (Ou Yang, Shieh et al. 2009). For instance, (Vahdat, Griffin et al. 2018) showed that there is a trade-off between benefits of implementing new technologies in healthcare and the extra workload to care-givers that ultimately decrease quality of care. Shahraki and Noorossana (2014) proposed a methodology to find the reliable and robust Pareto-optimal solutions in the design stage of product development.

MCDM methods build a structure that allow decision makers to prioritize preferences and differentiate between possible solutions. Unlike other Operations Research methods such as simulation (e.g., (Vahdatzad and Griffin 2016)) and optimization (e.g. (Rabiei, Droguett et al. 2018)) that are widely data-driven models, MCDM methods can handle problems with non-commensurable and qualitative criteria. Examples of MCDM methods are Analytic Hierarchical Process (AHP), Data Envelopment Analysis (DEA), PROMETHEE, and ELECTRE. MCDM methods have been widely used in different problems such as sustainable and renewable energy (Nigim, Munier et al. 2004), transportation systems (Celik, Bilisik et al. 2013), service industry (Talib and Rahman 2015), reliability optimization (Shahraki and Yadav 2015) and maintenance (Tajadod, Abedini et al. 2016).

VIKOR is one of the well-known classical MCDM methods, based upon a particular measure of ‘‘closeness to the ideal/aspired level’’. This method focuses on the ranking of a set of choices in the presence of conflicting criteria. It helps decision-makers to select the ‘‘best’’ compromise choice (Ou Yang, Shieh et al. 2009). VIKOR method was developed as a multiple attribute decision-making (MADM) method to solve the discrete decision problems with non-commensurable and conflicting criteria (Tzeng, Teng et al. 2002, Opricovic and Tzeng 2004, Tzeng, Lin et al. 2005, Opricovic and Tzeng 2007, Ou Yang, Shieh et al. 2009, Vahdani, Hadipour et al. 2010, Vinodh, Varadharajan et al. 2013, Anvari, Zulkifli et al. 2014). It was first proposed by (Opricovic and Tzeng 2004, and Opricovic and Tzeng 2007) as an alternative method to TOPSIS, another common MCDM method. VIKOR method is based on the work of (Yu 1973) in which the L_P-metric for a distance function is first proposed. VIKOR method provides a tradeoff between the maximum ‘‘group utility’’ of the ‘‘majority’’ and the minimum of the individual regret of the ‘‘opponent’’. While the method structure and calculation is allegedly simple and straightforward, corresponding compromise solution has shown to be not only a feasible solution but also the closest to the ideal solution, and a compromise means an agreement established by mutual concessions.

While MADM methods provide promising solution for single objective decision making problems, these methods are unable to tackle multi-objective problems. For this purpose, the majority of literature have looked into extension of MADM methods to solve multiple objective decision making (MODM) problems. For instance, TOPSIS has been extended by several researchers (Lai, Liu et al. 1994). Abo-Sinna (2000) extended TOPSIS approach to solve multi-objective dynamics programming (MODP) problems. Further extensions to TOPSIS are presented by (Abo-Sinna and Amer 2005, Abo-Sinna, Amer et al. 2008, Chen 2000, and Deng, Yeh et al. 2000). However, we have not noticed any contribution that may have developed VIKOR method for MODM. Our proposed VIKOR for MODM method transfers m-objectives into two objectives where both are the shortest distances from the Positive Ideal Solution (PIS) and the longest distance from Negative Ideal Solution (NIS). This is equivalent to measuring the distance by two L_P-metrics where only p = 1 and p = ∞ are taken into account. With this method, a k-dimensional objective space is reduced to a two-dimensional objective with minimization objectives namely *S* and *R*. Then, by using the PIS and NIS of the two-dimensional problem, a single-objective programming problem is constructed. Final solution can be obtained by solving the latter problem.

The remainder of the paper is organized as follows. In the following section, details pertaining to proposed VIKOR approach for MODM are presented. To demonstrate the process and applicability of proposed framework, a numerical example is given in section 3. Finally, in Section 4, we conclude by reviewing contributions of this research and proposing suggestions for future research.

2. VIKOR for MODM Problems

In MODM problems, instead of dealing with single objective function, a vector of objective functions $F(x) = (f_1(x), f_2(x),..., f_k(x))$ exists. A general MODM problem can be formulated as follows:

$$\begin{aligned} &\max \; F(x) = (f_1(x), f_2(x),..., f_k(x)) \\ &\text{s.t}: \; x \in X = \{x = (x_1, x_2,..., x_n) \in R^n \,|\; g_i(x) \le 0, \; i = 1,2,..., m\} \end{aligned} \tag{1}$$

We denote the positive ideal and negative ideal solutions (PIS and NIS) of objective functions vector by $F^* = (f_1^*, f_2^*,..., f_k^*)$ and $F^- = (f_1^-, f_2^-,..., f_k^-)$ in which $f_i^* = \max_{x \in X} f_i(x)$ and $f_i^- = \min_{x \in X} f_i(x)$, i=1,2,…,k

respectively. It is self-evident that in case of a minimization model $f_i^* = \min_{x \in X} f_i(x)$ and $f_i^- = \max_{x \in X} f_i(x)$. The L_P-metric is used for the measure of ''closeness''. The L_P-metric defines the normal distance between two points $F(x)$ and F^* as:

$$d_p = \left\{ \sum_{i=1}^{m} w_i \left(\frac{f_i^* - f_i}{f_i^* - f_i^-} \right)^p \right\}^{\frac{1}{p}}, \; p = 1, 2, \ldots, \infty. \tag{2}$$

where w_i, $i = 1, \ldots, m$ are the weights of objectives.

To obtain a compromise solution of MODM problem (1), the ideal vector of objective functions $F(x) = (f_1^*, f_2^*, \ldots, f_k^*)$ is considered to be the reference point and the distance to the reference point is calculated by using distance function presented in (2). One may easily show that this problem can be transformed to solve the following auxiliary problem (Lai, Liu et al. 1994):

$$\min_{x \in X} \; Z = d_p^p = \sum_{i=1}^{m} w_i \left(\frac{f_i^* - f_i}{f_i^* - f_i^-} \right)^p, \; p = 1, 2, \ldots, \infty. \tag{3}$$

In the VIKOR method for MADM, the distance is measured by Eq. (2) for p = 1 and p = ∞. So, to develop the VIKOR method for solving MODM problems, we rewrite (3) as the following two-objective programming model:

$$\begin{aligned} &\min_{x \in X} \; S = \sum_{i=1}^{m} w_i \left(\frac{f_i^* - f_i(x)}{f_i^* - f_i^-} \right) \\ &\min_{x \in X} R = \max_i \left\{ w_i \left(\frac{f_i^* - f_i(x)}{f_i^* - f_i^-} \right) : i = 1, 2, \ldots, m \right\} \end{aligned} \tag{4}$$

The model (4) converts to the following problem:

$$\begin{aligned} &\min S = \sum_{i=1}^{m} w_i \left(\frac{f_i^* - f_i(x)}{f_i^* - f_i^-} \right) \\ &\min R \\ &\text{s.t}: \quad R \geq w_i \left(\frac{f_i^* - f_i(x)}{f_i^* - f_i^-} \right) \quad i = 1, 2, \ldots, m \\ &\qquad\qquad x \in X \end{aligned} \tag{5}$$

Remark: In model (4), we can calculate the distance with formula (2) for $p \geq 2$. Therefore, model (5) can be rewritten as follows:

$$\begin{aligned} &\min S = \sum_{i=1}^{m} w_i \left(\frac{f_i^* - f_i(x)}{f_i^* - f_i^-} \right)^p \\ &\min R \\ &\text{s.t}: \quad R \geq w_i \left(\frac{f_i^* - f_i(x)}{f_i^* - f_i^-} \right) \quad i = 1, 2, \ldots, m \\ &\qquad\qquad x \in X, \; p = 1, 2, \ldots \end{aligned} \tag{6}$$

By solving problem (5), the positive ideal (S^*, R^*) and negative ideal (S^-, R^-) solutions of objective functions can be obtained, in which $S^* = \min_{x \in X} S$, $S^- = \max_{x \in X} S$, $R^* = \min_{x \in X} R$ and $R^- = \max_{x \in X} R$. Therefore, we can construct the following model by the VIKOR methodology:

$$\min Q = \lambda\left(\frac{S^* - S(x)}{S^* - S^-}\right) + (1-\lambda)\left(\frac{R^* - R(x)}{R^* - R^-}\right) \quad (7)$$
$$\text{s.t}: \quad x \in X, \lambda \in (0,1].$$

λ is introduced as a weight function that acts as a preference model between ‘‘the majority of criteria” and ‘‘the maximum group utility”. Here, for the ease of interpretation, we assumed that $\lambda = 0.5$, or there is no preference between two factors. By solving the model (7) compromise solutions of MODM problem (1) can be attained.

3. Numerical example

To illustrate the applicability of proposed VIKOR algorithm for solving the MODM problems, the following numerical example is adopted from (Lai, Liu et al. 1994), solved and the results are compared with extended TOPSIS method. The illustrated problem is a multi-objective non-linear problem with four objective functions and set of linear and non-linear constraints.

Lai, Liu et al. (1994) solved the above problem with a proposed extension to TOPSIS method. They obtained PIS and NIS values, as shown in Tables 1 and 2, to be utilized in their proposed TOPSIS framework.

Table 1. PIS payoff

	f_1	f_2	f_3	f_4	x_1	x_2	x_3
max: f_1	6.7922	0.3413	9.6824	11.375	1	1	1
max: f_2	5.2821	6.2752	2.6863	1.9311	0.8507	0.9095	0.8496
min: f_3	4.8569	0.4288	1.0412	2.1365	0.9995	0.7816	0.8638
min: f_4	6.7558	1.3161	9.6082	0.9593	0.9936	0.9997	0.7418

PIS: F^* =(6.7922, 6.2752, 1.0412, 0.9593)

Table 2. NIS payoff

	f_1	f_2	f_3	f_4	x_1	x_2	x_3
min: f_1	4.7797	6.2737	1.5632	2.0750	0.8509	0.8427	0.8593
min: f_2	5.3930	0.3431	2.0919	1.9789	0.9999	0.8809	0.8530
max: f_3	6.4778	5.4838	9.6803	1.6066	0.9111	0.9999	0.8235
max: f_4	5.3277	5.4192	2.4319	11.375	0.9144	0.8986	1

NIS: F^- = (4.7797, 0.3431, 9.6803, 11.3750)

Next, model (5) is constructed as follows:

$$\min S = w_1\left(\frac{6.7922 - f_1(x)}{2.0125}\right) + w_2\left(\frac{6.2752 - f_2(x)}{5.9321}\right) + w_3\left(\frac{1.0412 - f_3(x)}{-8.6391}\right) + w_4\left(\frac{0.9593 - f_4(x)}{-10.4157}\right)$$

$$\begin{aligned} &\min R \\ \text{s.t}: \quad & R \geq w_1\left(\frac{6.7922 - f_1(x)}{2.0125}\right) \\ & R \geq w_2\left(\frac{6.2752 - f_2(x)}{5.9321}\right) \\ & R \geq w_3\left(\frac{1.0412 - f_3(x)}{-8.6391}\right) \\ & R \geq w_4\left(\frac{0.9593 - f_4(x)}{-10.4157}\right) \\ & x \in X \end{aligned} \tag{8}$$

In order to analyze the numerical solutions, two sets of weight values ($w_1, w_2, w_3, and\ w_4$) are considered. In the first set, the weights are assumed to be equal and the results are shown in Table 3. In the second set, the weights are assumed to be unequal but normalized with the following values $w_1 = 0.4, w_2 = 0.3, w_3 = 0.2, w_4 = 0.1.$ Payoff tables with unequal weights are shown in Table 4.

Table 3. Solutions for equal weights.

	S	R	x_1	x_2	x_3
min S	0.2469	0.1527	0.9404	0.8601	0.7966
min R	0.2858	0.0939	0.9591	0.9526	0.7690

$$(S^*, R^*) = (0.2469, 0.0939)$$

$$(S^-, R^-) = (0.2858, 0.1527)$$

Table 4. Solutions for unequal weights $(w_1 = 0.4, w_2 = 0.3, w_3 = 0.2, w_4 = 0.1)$

	S	R	x_1	x_2	x_3
min S	0.3274	1.0039	0.9621	0.9086	0.7845
min R	0.3725	0.1239	0.9591	0.9796	0.7555

$$(S^*, R^*) = (0.3274, 0.1239)$$

$$(S^-, R^-) = (0.3725, 1.0039)$$

Using model (7) for the numerical example, we conclude the following linear programing model:

$$\begin{aligned} &\min Q = \lambda\left(\frac{0.3274 - S(x)}{-0.0451}\right) + (1-\lambda)\left(\frac{0.1239 - R(x)}{-0.88}\right) \\ &s.t: \quad x \in X \end{aligned} \tag{9}$$

By solving the model (9), solutions of VIKOR method for MODM are calculated and presented in table 5. Table 6 shows the responses that are obtained by Lai, Liu et al. (1994) using their proposed TOPSIS method. The obtained responses from proposed VIKOR method have significant difference with solutions taken from TOPSIS.

Table 5. VIKOR for MODM Solutions for λ=0.5

Weights	Q	f_1	f_2	f_3	f_4	x_1	x_2	x_3
$w_1 = w_2 = w_3 = w_4 = 0.25$	-0.8781	6.0031	4.6242	1.8345	1.3375	0.9447	0.8643	0.7957
$w_1 = 0.4, w_2 = 0.3$ $w_3 = 0.2, w_4 = 0.1$	0.2234	6.2011	3.9381	2.7475	1.1377	0.9652	0.9119	0.7835

Table 6. TOPSIS for MODM Solutions

Weights	P	f_1	f_2	f_3	f_4	x_1	x_2	x_3
$w_1 = w_2 = w_3 = w_4 = 0.25$	*1*	5.7500	5.6993	4.0187	1.2339	0.8988	0.9474	0.7830
	2	5.3553	6.2761	2.9074	1.2358	0.8506	0.9177	0.7832
	∞	6.0097	3.9650	4.4057	1.5669	0.9608	0.9547	0.8198
$w_1 = 0.4, w_2 = 0.3,$ $w_3 = 0.2, w_4 = 0.1$	*1*	6.1150	6.1479	6.7830	1.0417	0.8639	0.9830	0.7555
	2	6.1420	6.2669	7.2419	1.0262	0.8516	0.9865	0.7530
	∞	6.2000	3.9255	5.4408	1.1788	0.9616	0.9697	0.7757

A comparison between the proposed VIKOR approach and TOPSIS method by Lai, Liu et al. (1994), tables 5 and 6, shows that VIKOR solutions have a better performance compared to TOPSIS solutions for p=∞. However, for other cases of P, VIKOR and TOPSIS, may or may not outperform each other. Therefore, there is a lack of confidence to comment on the superiority of the VIKOR or TOPSIS while p is not equal to infinite ($p \neq \infty$).

4. Conclusion

The decision making process has become more complex than ever. One of the underlying reasons is today's decisions require to simultaneously satisfy several objectives. Recent studies show that in order to obtain better performance in a firm or service industry, the decision maker should implement multifaceted policies with the objectives that are most of the time conflicting (Vahdat, Griffin et al. 2017). In this paper, we proposed a methodology to extend the VIKOR algorithm that can be utilized for multi-objective decision making. The proposed algorithm transfers k objectives, into two objectives, which are commensurable. After that, a new problem namely *Q* by PIS and NIS of the two-objective problem is constructed. Solving the latter problem provide solutions for the initial multi-objective problem. Using a numerical example, the proposed extended VIKOR algorithm is compared with extended TOPSIS and effectiveness of the proposed algorithm for different values are shown. For the future research, the proposed solution can be compared with other methods that solve multi-objective problems.

5. References

Abo-Sinna, M. (2000). "Extensions of the TOPSIS for multi-objective dynamic programming problems under fuzziness." Advances in Modelling and Analysis **43**(4): 1-24.

Abo-Sinna, M. A. and A. H. Amer (2005). "Extensions of TOPSIS for multi-objective large-scale nonlinear programming problems." Applied Mathematics and Computation **162**(1): 243-256.

Abo-Sinna, M. A., A. H. Amer and A. S. Ibrahim (2008). "Extensions of TOPSIS for large scale multi-objective non-linear programming problems with block angular structure." Applied Mathematical Modelling **32**(3): 292-302.

Anvari, A., N. Zulkifli and O. Arghish (2014). "Application of a modified VIKOR method for decision-making problems in lean tool selection." The International Journal of Advanced Manufacturing Technology **71**(5-8): 829-841.

Celik, E., O. N. Bilisik, M. Erdogan, A. T. Gumus and H. Baracli (2013). "An integrated novel interval type-2 fuzzy MCDM method to improve customer satisfaction in public transportation for Istanbul." Transportation Research Part E: Logistics and Transportation Review **58**: 28-51.

Chen, C.-T. (2000). "Extensions of the TOPSIS for group decision-making under fuzzy environment." Fuzzy sets and systems **114**(1): 1-9.

Deng, H., C.-H. Yeh and R. J. Willis (2000). "Inter-company comparison using modified TOPSIS with objective weights." Computers & Operations Research **27**(10): 963-973.

Lai, Y.-J., T.-Y. Liu and C.-L. Hwang (1994). "Topsis for MODM." European Journal of Operational Research **76**(3): 486-500.

Nigim, K., N. Munier and J. Green (2004). "Pre-feasibility MCDM tools to aid communities in prioritizing local viable renewable energy sources." Renewable energy **29**(11): 1775-1791.

Opricovic, S. and G.-H. Tzeng (2004). "Compromise solution by MCDM methods: A comparative analysis of VIKOR and TOPSIS." European journal of operational research **156**(2): 445-455.

Opricovic, S. and G.-H. Tzeng (2007). "Extended VIKOR method in comparison with outranking methods." European journal of operational research **178**(2): 514-529.

Ou Yang, Y.-P., H.-M. Shieh, J.-D. Leu and G.-H. Tzeng (2009). "A VIKOR-based multiple criteria decision method for improving information security risk." International Journal of Information Technology & Decision Making **8**(02): 267-287.

Rabiei, E., E. L. Droguett and M. Modarres (2018). "Fully Adaptive Particle Filtering Algorithm for Damage Diagnosis and Prognosis." Entropy **20**(2): 100.

Shahraki, A. F. and R. Noorossana (2014). "Reliability-based robust design optimization: a general methodology using genetic algorithm." Computers & Industrial Engineering **74**: 199-207.

Shahraki, A. F. and O. P. Yadav (2015). System reliability optimization considering probabilistic common-cause failures and uncertainty. IIE Annual Conference. Proceedings, Institute of Industrial and Systems Engineers (IISE).

Tajadod, M., M. Abedini, A. Rategari and M. Mobin (2016). "A comparison of multi-criteria decision making approaches for maintenance strategy selection (a case study)." International Journal of Strategic Decision Sciences (IJSDS) **7**(3): 51-69.

Talib, F. and Z. Rahman (2015). "Identification and prioritization of barriers to total quality management implementation in service industry: an analytic hierarchy process approach." The TQM Journal **27**(5): 591-615.

Tzeng, G.-H., C.-W. Lin and S. Opricovic (2005). "Multi-criteria analysis of alternative-fuel buses for public transportation." Energy Policy **33**(11): 1373-1383.

Tzeng, G.-H., M.-H. Teng, J.-J. Chen and S. Opricovic (2002). "Multicriteria selection for a restaurant location in Taipei." International journal of hospitality management **21**(2): 171-187.

Vahdani, B., H. Hadipour, J. S. Sadaghiani and M. Amiri (2010). "Extension of VIKOR method based on interval-valued fuzzy sets." The International Journal of Advanced Manufacturing Technology **47**(9-12): 1231-1239.

Vahdat, V., J. Griffin and J. E. Stahl (2017). "Decreasing patient length of stay via new flexible exam room allocation policies in ambulatory care clinics." Health care management science: 1-25.

Vahdat, V., J. A. Griffin, J. E. Stahl and F. C. Yang (2018). "Analysis of the effects of EHR implementation on timeliness of care in a dermatology clinic: a simulation study." Journal of the American Medical Informatics Association.

Vahdatzad, V. and J. Griffin (2016). Outpatient clinic layout design accounting for flexible policies. Proceedings of the 2016 Winter Simulation Conference, IEEE Press.

Vinodh, S., A. R. Varadharajan and A. Subramanian (2013). "Application of fuzzy VIKOR for concept selection in an agile environment." The International Journal of Advanced Manufacturing Technology **65**(5-8): 825-832.

Yu, P.-L. (1973). "A class of solutions for group decision problems." Management Science **19**(8): 936-946.

Biographies

Seyedmohammad Salehi is currently a PhD student in computer and information sciences department of University of Delaware, Newark, DE, USA. He has received his B.Sc. in Software Engineering from Yazd University, Iran, and M.Sc. in Telecommunications and Networked Systems from University of New Haven, CT, USA. His research interests span from quality of service in wireless networks, and game theory to optimization techniques. He enjoys multi-disciplinary and inter-disciplinary research.

Maghsoud Amiri is a Professor at the Department of Industrial Management of the Allameh Tabataba'i University (Tehran). He has a PhD in industrial engineering from the Sharif University of Technology (Tehran). He has published many papers in leading international journals like Expert Systems with Applications, The International Journal of Advanced Manufacturing Technology, Reliability Engineering & System Safety, International Journal of Production Research, Journal of Cleaner Production, Applied Mathematics and Computation, Applied Mathematical Modelling, Applied Soft Computing, and Technological and Economic Development of Economy. He is the Editor-in-Chief of the Industrial Management Studies (An open-access, peer-reviewed journal published by Allameh Tabataba'i University). His research interests include multi-criteria decision-making (MCDM), data envelopment analysis (DEA), design of experiments (DOE), response surface methodology (RSM), fuzzy MCDM, inventory control, supply chain management, simulation and reliability engineering.

Saeed Ramazanzadeh is an Assistant Professor with the Department of Basic Sciences of Amin University, Tehran, Iran. He received his PhD degree in industrial management from Allameh Tabataba'i University, Tehran, Iran and MS degree in applied mathematics from Shahid Bahonar University, Kerman, Iran. His research interests include fuzzy stochastic programming, fuzzy logic control (FLC), fuzzy inventory control, data envelopment analysis (DEA), and fuzzy MCDM.

Mohammadali Abedini is currently a PhD candidate at the University of Tasmania and he is focusing on agricultural produce supply chain planning. He received his master's degree in industrial engineering from Iran University of Science and Technology. He also received his bachelor's degree in industrial engineering from Yazd University. His academic research interests include mathematical modelling, optimization, data mining, decision-making and decision-making support systems. Apart from academia, he has been working with different industries including pharmaceutical production industry, supplement production, tile production and so forth. Throughout his collaborations with these industries, he has focused on different areas such as system analysis, process improvement, quality management, inventory planning, and data analysis. At the moment he is working on optimization models under uncertainty in order to improve farmers' profitability in fresh produce supply chains.