Reliable Capacitated Single Allocation Hub Network Design under Hub Failure: A Scenario Based Approach

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Abstract

Protecting air hub nodes against failure, which may exist because of some unforeseen events like severe weather, terrorism threats, etc., is a significant concern of managers in a strategic level, as reactivation of disrupted hubs would be too costly and time consuming. In this paper, we address a mixed integer linear programming model based on disruption scenarios, for reliable capacitated single allocation hub network design (RCSAHND). For each disruption scenario, our model sets potential backup hubs among available nodes. The objective here is to minimize the nominal cost, when no disruption occurs, as well as reducing the disruption risk using the p-robustness criterion. p-robustness is one of the robust optimization measures in the literature, trying to bound the cost in disruption scenarios. To show effectiveness of our reliable design, the failure of some hub nodes through number of scenarios are studied on Iranian Aviation Dataset (IAD). Results show that by imposing trivial change in cost for designing the reliable hub network from the beginning, we will have a more reliable hub network which charges us less when a disruption occurs.

Keywords
Reliable hub network, hub failure, scenario based optimization, p-robustness, backup hub.

1. Introduction

Hub-and-spoke networks have been used in many areas such as transportation, logistic, and telecommunication systems. Hubs in such networks are consolidation, switching, or transshipment points, applied to decrease the number of transportation links between origin and destination nodes. A problem related to determination of hub locations and allocation of non-hub nodes (spokes) to them, is called hub location problem (HLP). One important feature in a HLP is the assignment pattern of the non-hub nodes to the hubs. There are two ways of assignment: single allocation (SA) and multiple allocation (MA). In the SA case, used in this study too, each spoke is connected to a single hub; while in the MA case, there is no restriction on the number of hubs to which a spoke might be connected (Ernst and Krishnamoorthy, 1998). Traditional HLPs often solve this problem under certain environment, assuming all parameters are deterministic and components of the network are always available. However, uncertainty is an inevitable part of each network, and hub and spoke networks are not an exception too. In general, we are dealing with two kinds of uncertainty in designing a hub network: uncertainty in input parameters such as time, cost, capacity, etc. and uncertainty in network components, when some parts of network fail and become unavailable due to disruption. The former recently has been noticed by many researchers. For instance, Sim et al. (2009) deal with variability of travel time using stochastic model, and Alumur et al., (2012) address several aspects of the hub location problems under uncertainty. Two sources of uncertainty are generally considered: setup costs for the hubs and the demands to be transported between the nodes. Boukani et al., (2014) present scenario-based approach to tackle with HLP under uncertainty in setup costs and capacity of nodes. Ghaffari-Nasab et al., (2015) apply a robust optimization approach for modeling capacitated HLP, in which the quantity of the flow between every pair of the nodes have stochastic nature. But, there are only few studies that addressed the later one, i.e., the vulnerability and uncertainty of network component such as hubs. Since efficiency of hub networks heavily relies on the location and availability of these hubs, it is clearly obvious that if any of these hubs being exposed to some unexpected disruptions like severe weather, labor strikes, or terrorist attacks, it would endanger the whole system function. To come up with hub disruption some strategies, such as rerouting, cancelling, or delaying the flights often have been considered. But these are not so effective, i.e., they are very expensive and time consuming. Thus, designing a reliable network, in which these crucial central points are working even under the disruption, would be so desirable for managers specially in strategic level.

Similar situations for facilities in distribution centers have been investigated in facility location problems in the context of supply chain and logistics systems. Some of the most important studies that handle facility disruption by designing reliable facility location models are Snyder and Daskin (2007), Lim et al., (2010), Shen et al., (2011). As far as we know, in contrast to reliable facility location problems that have attracted the attention of many researchers up to now, designing reliable hub networks have been addressed rarely in the literature. From these few studies, a p-hub protection model based on SA structure with primary and secondary routes is discussed in Kim et al., (2009); where, a tabu search heuristic is also proposed to solve the real instances with up to 20 nodes. In An et al., (2015), authors propose a non-linear mixed integer model in order to designing the reliable hub network. They handle hub unavailability by means of back up hubs and alternative routes in the designing stage. The proposed model minimizes the operating cost in both normal and disrupted situation. Azizi et al., (2012) also use back up hubs to deal with hub disruption. They assume that once a hub becomes unavailable, the flow which initially passed through this facility is rerouted via one of the operating hubs in the network. Their work has some weaknesses in real situations; specifically, when a hub become disrupted, it is not likely that other remaining hubs could respond to the flows going through that hub. Moreover, once a hub is removed from the network, we are deprived from the advantage of hubs as a consolidate center, and it imposes huge cost to the network. There are also few other studies which address kind of backup hubs by virtual hubs in dynamic transportation networks like Teymourian et al., (2011) and Baher et al., (2012), where these reserve hubs can switch on or off for one time during their planning horizon.

In the most of aforementioned studies, it has been assumed that all nodes including hubs are uncapacitated. Despite the fact that in real world each hub has definite capacity which allows it to only accept limited flows from other nodes. In the most similar study to our work (Mohammadi et al., 2011), authors present a mixed integer non-linear programming (MINLP) model for designing a reliable hub network using backup hubs. They consider special form of non-fully interconnected hub network with a hierarchal structure. They assume that once a hub is being disrupted, the nodes connected to that hub should be reallocated to a predefined backup hub in the higher level; also, the hub in the highest level is a non-failable hub, i.e., it would never be disrupted. In this paper, we are offering a preventive approach to tackle the vulnerability of hub under uncertain conditions. Since disruptions are rare events, it is difficult to collect historical data so their probabilities can be hard to estimate too. That is why, we opt to use robust optimization, which does not require probabilistic information, as an alternative way to cope with uncertainty. Like Peng et al., (2011), we use scenario-based robust optimization approach to deal with the uncertainty. Our model objective is to minimizing the nominal cost -when there is no disruption occur-, and reducing the risk when some disruptions are happened. This helps us find a reasonable network design under any disruption scenario. We use one of the most efficient criteria in robust optimization called p-robustness measure, in which the relative regret in each scenario should be no more than a constant p (Snyder, 2006). The efficiency of this measure compared to other measures like minimax cost and minimax regret is then discussed. The main contributions of this paper include:

- Proposing a reliable hub network design using potential back up hubs through the network when some of these facilities are being unavailable.
- Modeling the problem based on scenario-based robust optimization approach considering hub disruptions, and capacity limitation for each node in a fully interconnected network.
- Iranian aviation data set is studied as a real case to illustrate the effectiveness of this approach.

The rest of the paper is organized as follows. Our reliable capacitated single allocation hub-and-spoke network (RCSAHND) is formulated in Section 2. Section 3 presents some scenarios and discusses our computational results on Iranian Aviation Dataset (IAD) as a case study to show the performance of our approach. Finally, we provide our conclusion and future remarks in section 4.

2. Reliable Capacitated SA Model

2.1 Nomenclature

Sets

N set of nodes $\{1,2,3,\ldots,n\}$

S set of scenarios $\{0,1,\ldots,l\}$; where 0 represents the nominal scenario with no disruption

Parameters

f_k	$k \in N$	fixed cost of locating a hub at node k
C_k	$k \in N$	capacity of a hub installed at node k
U_s^*	$s \in S$	optimal cost of scenario s
d_{ij}	$i,j \in N$	distance from node i to j
w_{ij}	$i,j \in N$	flow originated at node i that is destined to node j
a_{ks}	$k \in N, s \in S$	1 if hub k is disrupted in scenario s, 0 otherwise
$O_i = \sum_{j\in N} w_{ij}$	$i \in N$	total flow originated from node i
$D_i = \sum_{j\in N} w_{ji}$	$i \in N$	total flow destined to node i
p	desired robustness level, $p \geq 0$	
χ	(spoke to hub) collection cost per unit flow per unit distance	
α	(hub to spoke) distribution cost per unit flow per unit distance	
δ	(hub to hub) transfer cost per unit flow per unit distance	

Decision variables

y_{kl}^{is}	$i,k,l \in N, s \in S$	amount of flow originated from node i and is routed through hubs (k,l) in scenario s
z_{iks}	$i,k \in N, s \in S$	1 if node i is allocated to hub k under scenario s, 0 otherwise
x_{ik}	$i,k \in N$	1 if node i is allocated to hub k in RCSAHND model, 0 otherwise

2.2 p-robustness

We use the same definition for p-robustness in Snyder and Daskin (2006) as follows. For a given set S of scenarios, suppose the deterministic minimization problem for each scenario s is an HLP, and its optimal objective function value is U_s^*. Moreover, considering x and y as location and allocation variables, we can define $U_s(x,y)$ as an objective value of (x,y) for each scenario s. Then (x,y) is called p-robust if $\forall s \in S$:

$$\frac{U_s(x,y) - U_s^*}{U_s^*} \leq p \tag{1}$$

where p is a given constant, indicating the desired robustness level. The left-hand side of equation (1) is the relative regret for scenario s. The p-robust measure sets upper bounds on the maximum allowable relative regret for each scenario.

2.3 Model formulation

The classical SA hub and spoke model developed by Ernst and Krishnamoorthy (1999) is extended to **RCSAHND** using p-robust measure, as follows.

$$\min \quad \sum_{k\in N} f_k\, x_{kk} + \sum_{i\in N}\sum_{k\in N} \chi\, d_{ik} O_i z_{ik0} + \sum_{i\in N}\sum_{k\in N}\sum_{l\in N} \alpha\, d_{kl} y_{kl}^{i0} + \sum_{i\in N}\sum_{k\in N} \delta\, d_{ki} D_i z_{ik0} \tag{2}$$

Subject to:

$$\sum_{k\in N} f_k\, x_{kk} + \sum_{i\in N}\sum_{k\in N} \chi\, d_{ik} O_i z_{iks} + \sum_{i\in N}\sum_{k\in N}\sum_{l\in N} \alpha\, d_{kl} y_{kl}^{is} + \sum_{i\in N}\sum_{k\in N} \delta\, d_{ki} D_i z_{iks} \le (1+p) U_s^* \quad \forall s \in S \tag{3}$$

$$\sum_{k\in N} z_{iks} = 1 \quad \forall i \in N, s \in S \tag{4}$$

$$z_{iks} \le x_{kk} \quad \forall i,k \in N, s \in S \tag{5}$$

$$\sum_{l\in N} y_{kl}^{is} - \sum_{l\in N} y_{lk}^{is} = O_i z_{iks} - \sum_{k\in N} w_{ij}\, z_{jks} \quad \forall i,k \in N, s \in S \tag{6}$$

$$\sum_{l\in N, l\neq k} y_{kl}^{is} \le O_i z_{iks} \quad \forall i,k \in N, s \in S \tag{7}$$

$$\sum_{i\in N} O_i\, z_{iks} \le (1-a_{ks}) C_k x_{kk} \quad \forall k \in N, s \in S \tag{8}$$

$$z_{iks} \in \{0,1\} \quad \forall i,k \in N, s \in S \tag{9}$$

$$x_{ik} \in \{0,1\} \quad \forall i,k \in N \tag{10}$$

$$y_{kl}^{is} \ge 0 \quad \forall i,k,l \in N, s \in S \tag{11}$$

Objective function (2) minimizes the fixed cost of establishing hubs as well as the whole transportation costs. Constraints (3) enforce the p-robust criteria, i.e., each scenario cost can not be more than $100(1+p)\%$ of the optimal scenario cost U_s^* . Constraints (4) and (5) respectively guarantee that each node is a hub or is allocated to a single hub, and also no node can be assigned to a site unless a hub is opened at that site. Flow balance limitation between nodes is defined in constraints (6) and (7); where, variable y_{kl}^{is} cannot get a non-zero value when z_{iks} is equal to zero. Constraints (8) ensure that the total flow through a hub does not exceed its capacity; when it is opened and fully functional in that scenario. These constraints prevent any flow when hub is closed or disrupted. The domain of decision variables z and x is defined binary in constraints (9) and (10). Finally, constraints (11) ensure that y variables are positive real values.

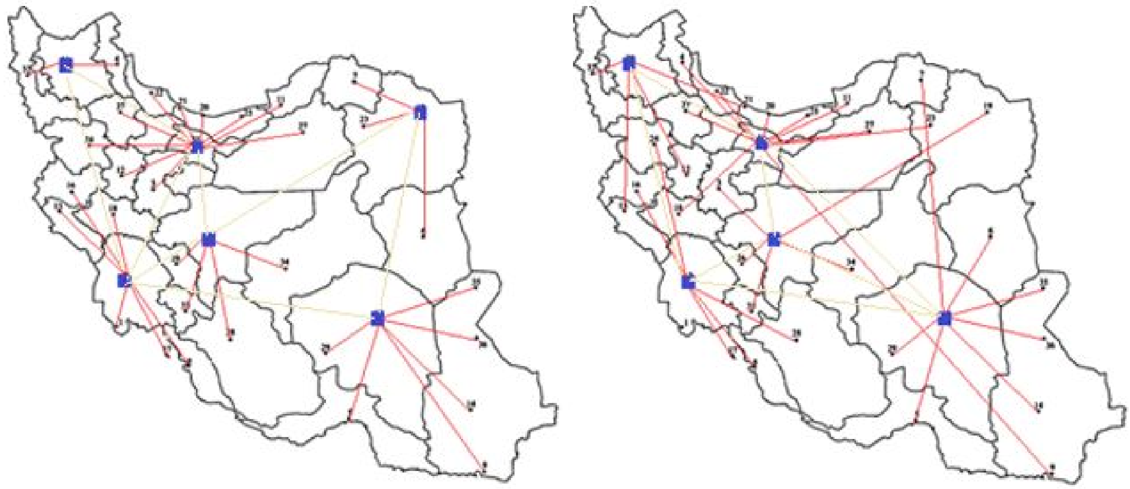
The optimal scenario cost U_s^* for each of the scenarios $s \in S$ are calculated by solving the **CSAHND** model as follows. Objective function is revised as equation (12) below. Moreover, variable x_{ik} or constraints (10), as well as constraints (3) in model RCSAHND are removed; constraints (5) and (8) is revised by replacement of z_{kks} instead of x_{kk}, while other constraints (4), (6), (7), (9), and (11) are the same as before.

$$U_s^* = \quad \min \sum_{k\in N} f_k\, z_{kks} + \sum_{i\in N}\sum_{k\in N} \chi\, d_{ik} O_i z_{iks} + \sum_{i\in N}\sum_{k\in N}\sum_{l\in N} \alpha\, d_{kl} y_{kl}^{is} + \sum_{i\in N}\sum_{k\in N} \delta\, d_{ki} D_i z_{iks} \tag{12}$$

In RCSAHND model, the objective function seeks into minimizing costs under normal condition, while the p-robustness constraints are interested in protecting the network against the future hub failure by investing more in infrastructure at designing stage.

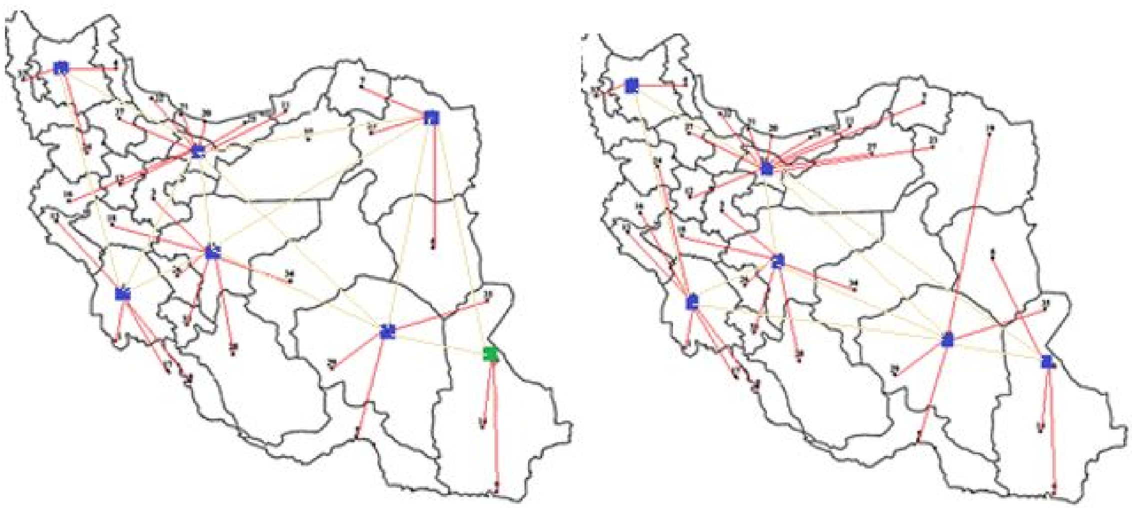
3. Experimental Result

In this section, we demonstrate the results of reliable capacitated single allocation hub location problems in small sizes considering some scenarios. Our test bed based on IAD dataset, initially prepared by Karimi and Bashiri (2011), includes demands, fixed setup costs and distances based on airline passenger interactions among the 37 cities of Iran. To study different disruption scenarios, we define four scenarios in which one hub is disrupted at a time. Our model tries to locate a backup hub among the potential non-hub nodes considering some factors like distance, capacity and fixed cost to compensate these failures. Value of demands $D_{(i,k)}$ for each node as well as fixed cost f_k for each hub are based on data set IAD. To set the capacity of each node, we calculate $\overline{C} = \sum_{i,k\in N} D_{(i,k)}$, and then take the capacity C_i uniformly within the interval $[1.5\overline{C}, 3.5\overline{C}]$. Other parameters are tuned as follows: $\alpha = 0.4, \chi = 1,$ and $\delta = 1$.



Left: cost (nominal) = $134,791,483 Right: cost (hub 19 disrupted) = $164,924,667

Figure 1. Network design without considering disruption



Left: cost (nominal) = $136,532,350 Right: cost (hub 19 disrupted) = $148,172,956

Figure 1. Network design considering disruption

3.1 Computational Analysis

Figure 1 shows that in normal condition when there is no disruption, six hubs including 2,10,15,19,30, and 31 are located in the network, and has a cost of $134,791,483. Assuming one of these hubs being disrupted due to some unexpected reasons. For instance, if hub 19 is disrupted, nodes connected to this hub should satisfy their demand via other available hubs. It will impose noticeable cost near 23% to the network. Suppose that we can predict the possibility of such disruption (i.e., in a scenario) before it happens, in this case, our reliable model considers the potential back up hub 36 (green square node) in the network. As depicted out in Figure 2, this back up hub will help to fulfil demands in a network with much less cost. In these figures (Figure 1 and 2), it has been showed that by protecting our network against failure, however, a little extra investment is needed but we have much less cost when a disruption occurs in the future. Moreover, as we said before all the hubs in our network have limited capacity, so it is likely other hubs are not capable of supplying whole demand when a hub is disrupted. That is, we will face an unfeasible solution if there are no back up hubs to respond the situation. Below, we represent four scenarios including nominal scenario 0, and compare their costs in "with" and "without back up hub’s approaches.

3.2 Comparison with other robustness criteria

As mentioned before, p-robust is the more efficient and suitable measure for managers than other robustness criteria. The two most widely studied robustness measures, *minimax cost* and *minimax regret*, have been questioned for being too conservative from the managerial view; since they protect against only the worst-case scenario, that may happen with a small probability (Peng et al., 2011). Designing a reliable network with p-robust criterion not only benefits manager in a long period of time by minimizing nominal cost, but also it has short term effect as it reduces the cost of each scenario. In order to compare the effectiveness of our proposed approach with other two criteria, we solve RCSAHND with new objective functions that are formulated below along with all previous constraints. Result is presented in Table 1.

$$Minimax\ Cost\text{: } \min\max_{s} U_s(X,Y) \qquad Minimax\ Regret\text{: } \min\max_{s} \frac{U_s(x,y) - U_s^*}{U_s^*}$$

Table 1. Result of disruption scenarios

		Scenario 0	Scenario 1	Scenario 2	Scenario 3
Hubs or Cost	Disrupted hub	-	10	19	2
	Costs without backup hub	134,791,479	infeasible	164,924,667	infeasible
	Located hubs in each scenario	2,10,15,19,30,31	2,15,19,28,30,31	2,10,15,30,31,36	10,15,19,28,30,31
	U_s^*	134,876,483	142,114,138	148,172,956	140,197,613
	p-robust cost	134,876,483	147,172,173	148,913,040	148,913,040

Table 2 shows the optimal scenario costs U_s^*, scenario cost and relative regret of each model, i.e., p-robust, minimax cost, and minimax regret, for each scenario. The percentage differences between the scenarios cost of p-robust comparing with the minimax cost and minimax regret models are also demonstrated as " DIFF(%)" column. The weakness of minimax cost and minimax regret models is that they just protect against worst-case scenario (i.e., scenario 2). One can see that the relative regrets in minimax cost model are very large comparing with other criteria. Besides, p-robust model has less relative regret than minimax regret model. It reveals that p-robust model is less conservative than other models. Another advantage of p-robust model is that it allows decision maker to achieve desired level of reliability by adjusting the value of p, called *price of robustness*. It means that Manager are able to gain each level of reliability based on their budget. A little increase in cost brings us much more reliable network. If we put p=1, the p-robust constraints become inactive and the problem will change to a deterministic hub location problem. The more the value of p reduces, the more the objective function increases while the more the relative regret decreases. We can reduce p value until no feasible solution can be found, i.e., no more robustness is achievable.

Table 2. Comparison with other robust criteria

		Optimal	p-Robust		Minmax Cost			Minmax Regret		
			Cost	Regret	Cost	DIFF (%)	Regret	Cost	DIFF (%)	Regret
Scenario	0	134,876,483	134,876,483	1.000	142,852,033	5.91	1.059	138,645,672	2.8	2.79
	1	142,114,138	147,172,173	1.036	154,213,546	4.9	1.085	144,543,278	1.7	1.7
	2	148,172,956	148,913,040	1.004	166,876,901	12.2	1.126	155,654,760	4.6	0.32
	3	140,197,613	148,913,040	1.062	149,050,211	0.11	1.063	148,934,674	0.03	1.31

4. Conclusion and Future remarks

In this paper, we proposed a reliable capacitated single allocation hub network design model using p-robust optimization criterion to minimize the nominal cost in a long horizon time along with controlling risk of disruption when it happens in a short time. Our model seeks to design a reliable hub network with the help of backup hubs that performs well against disrupted situations. We also defined some disruption scenarios to show the performance of our model in small size. The results confirmed that by boosting hub network infrastructure in a design stage, less costs would be imposed to the network when a disruption occurred, and the desired level of robustness depends on p factor. Comparing p-robust criterion with traditional measures like minimax cost and minimax regret revealed that this criterion is less conservative than other two criteria. Since the hub and spoke network design is a NP-hard problem, proposing efficient algorithm to solve RCSAHND model in real and large size problems could be a good direction for future studies. Other future remarks could be formulating the expected cost by assigning weights to scenarios based on their importance for the decision maker.

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