A Modification to Multi Objective NSGA II Optimization Algorithm

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Abstract — This Paper provides a new approach to handle constrained multi objective problems using a modification to the fast elitist multi objective Genetic Algorithm, NSGA II (Non-dominated Sorting Genetic Algorithm II). The modification overcomes shortcomings of some previous approaches to handle constrained multi objective problems like Penalty Function Approach, Jimenez-Verdegay-Gomez-Skarmeta's approach, and Ray-Tai-Seow's Method. Comparative studies are conducted using a group of quality indices [2,19] to compare among three different algorithms: regular NSGA, NSGA-II with penalty function and M-NSGA-II with constraint handling techniques. Results show that the new modification is the best among the three approaches. Due to limited space, only recent studies on the subject of multi objective optimization are reviewed for brevity.

Keywords — Multi-objective Problems, NSGA II Algorithm, Quality Indices.

Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>P</td>
<td>Parent Population</td>
</tr>
<tr>
<td>Pp</td>
<td>Total number of different non-dominated sets in the population</td>
</tr>
<tr>
<td>P</td>
<td>Pareto optimal Front</td>
</tr>
<tr>
<td>Qt</td>
<td>offspring population</td>
</tr>
<tr>
<td>Rt</td>
<td>combined Population of off spring and parent population</td>
</tr>
<tr>
<td>Di</td>
<td>normalized Euclidean distance</td>
</tr>
<tr>
<td>MOEAs</td>
<td>Multi Optimization Evolutionary Algorithms</td>
</tr>
<tr>
<td>CLu (P)</td>
<td>Number of Cluster on the obtained Pareto Frontier</td>
</tr>
<tr>
<td>Do</td>
<td>Scaled Area: Dominant Area</td>
</tr>
<tr>
<td>AC</td>
<td>Accuracy of observed Pareto Frontier</td>
</tr>
<tr>
<td>NDEM</td>
<td>Non-Dominated Evaluation Metric</td>
</tr>
<tr>
<td>MPFE</td>
<td>Maximum Pareto-Optimal Front Error</td>
</tr>
<tr>
<td>NDC</td>
<td>Number of Distinct Choices</td>
</tr>
<tr>
<td>CLu (P)</td>
<td>Number of Cluster on the obtained Pareto Frontier</td>
</tr>
<tr>
<td>S</td>
<td>Spacing</td>
</tr>
<tr>
<td>∆</td>
<td>Spread</td>
</tr>
<tr>
<td>OS</td>
<td>overall Pareto spread</td>
</tr>
</tbody>
</table>

I. INTRODUCTION

Multi-objective optimization is enjoying lots of interests by researchers in the field. Unlike single-objective optimization, finding an optimal trade-off among conflicting objectives in a multi-objective problem is often more complex and challenging [20]. The Pareto Front developed over years and resulting from different techniques adds more to the difficulty of Multi objective optimization compared with the single objective optimization techniques. Several quality indices are developed to assess the resulting solutions. Convergence, diversity of solutions, time consumed to reach a set of solutions, ability to handle the different constraints are some measures. In an earlier paper [19], Various performance indices are used to measure the quality of Pareto-optimal sets to compare...
the performance of three different multi-objective optimization (MOO) algorithms: Regular NSGA, M-NSGA-II, and NSGA with Penalty function approach. In this paper, twelve quality indices are used to compare the different algorithms in a meaningful manner to assist the practicing engineers to interpret different solutions. In addition, this paper develops a selection procedure to develop NSGA II with respect to constraint handling. Four bench mark problems are used for comparative analysis. Past studies are reviewed next.

II. BACKGROUND

Non-Dominated Sorting Genetic Algorithm NSGA, first implemented by Srinivas and Deb [18]. It depends on sorting an initial population into a number of different non-domination based sets. Each set contains non-dominated solutions between each other. It is worth to be mentioned that all solutions in the first set, are the best in the population where any solution dominates all solutions in all other sets. Solutions in the second set are all dominated only by any solution of first set while it dominates all solutions in other sets, and so on, the worst solutions belong to the last rank of solution sets where solutions are dominated by solutions in all other sets. Fitness and niche count assigned to each solution to calculate the shared fitness function. Although this approach provides a tradeoff solution close to true Pareto in one single run, the sharing function approach requires fixing $\sigma$-share which is not practically applicable.

Elitist Non-Dominated Sorting Genetic Algorithm (NSGA-II), was proposed by Deb et al., 2000a, 2000b. it employs an explicit diversity-preserving mechanism. Offspring population Q_t is created by using the parent population P_t. Non-domination sorting is performed on a combined population R_t = Q_t U P_t. to form the first non-dominated front, and continue with solutions of the second non-dominated front, followed by the third non-dominated front and so on. The algorithm depends mainly on five major stages:

1- Combine parent and offspring population in new one population,
2- Perform a non-domination sorting to identify different fronts, F1, F2, …Fn where F1, is the best front, followed by F2, …and Fn.
3- Create an empty $P_{t+1}$ = $\emptyset$ and set 2N is the total number of its solution
4- Add solution of F1 to the population $P_{t+1}$, then F2, F3, and Fn until number of total solution is become more than 2N, then stop adding any more fronts.
5- Apply Crowded Tournament Selection to guarantee the solutions which lie in the last added front to $P_{t+1}$. Solutions residing in a less crowded area with the larger crowding distance wins and the rest are omitted to make population $P_{t+1}$ of 2N only.

Figure 1 depicts the various stages of NSGA II procedure.

![Figure 1 Schematic of the NSGA-II procedure [18].](image)

Adopted from “Multi-Objective Optimization, using Evolutionary Algorithms” by John Wiley

One major disadvantage of NSGA II like any other MOEAs assume that the optimization problem is unconstrained. Since constraints are inevitable in any real-world multi-objective optimization problem, lots of approaches are developed to handle the constrained problems, some of these approaches are concluded here:

- **Penalty Function Approach** [1]: The objective functions are added to the set of constraints by a penalty function. Sum of Constraints violation are multiplied by a penalty parameter and then the product is added to each function to form what so called the penalized function, a consequence any optimization algorithm can be used to solve it, and results in Pareto-optimal set. Penalty Function approach requires several trials and more efforts to get solutions as close to the true Pareto optimal solutions.

- **Jimenez-Verdegay-Gomez-Skarmeta’s Method** [18] deals only with inequality constraints. It requires a careful consideration of feasible and infeasible solutions and the use of niching to maintain diversity in the obtained Pareto-optimal solutions.

- **Ray-Tai-Seow’s Method** [1] A non-domination check of the constraint violations is made. Three different non-dominated rankings of the population are first performed, ranking the solutions of objective functions, the constraints violations, and the third ranking is performed by using a combined objective function and constraint violation values. The major disadvantage of this approach occurs when all population members are feasible and belong to a suboptimal non-dominated front, the algorithm stagnates, due to the all of the population members will have a rank equal to 1, since all solutions are feasible, they fill up the population and no further processing is made. During the crossover operation, three offspring are created.
The fast elitist multi objective Genetic Algorithm, NSGA II is developed by (Kalyanmoy Deb, Amrit Pratap, Sameer Agrawal, T. Meyarivan, 2002). Constraints divide the search space into two domains: feasible and infeasible regions. A constrained multi-objective optimization problem can be stated as:  

\[
\min \ f_m(x), \ m = 1,2,\ldots,M; \\
\text{Subject to:} \\
g_j(x) \geq 0, \ j = 1,2,\ldots,J; \\
h_k(x) = 0, \ k = 1,2,\ldots,k; \\
x_i^{(l)} \leq x_i \leq x_i^{(U)}, \ i = 1,2,\ldots,n;
\]

Where: \( f_m(x), \ m = 1,2,\ldots,M \) is a set of multi objective functions, \( g_j(x) \geq 0, \ j = 1,2,\ldots,J \) is refer to inequality constraint set while \( h_k(x) = 0, \ k = 1,2,\ldots,k \) refers to equality constraint set. \( x_i^{(l)} \leq x_i \leq x_i^{(U)}, \ i = 1,2,\ldots,n \) are the lower and upper decision variable limits. Handling constraints, main shortcomings result from the old methods are taken into consideration in constructing the new approach which depends mainly on predefining the objective functions and coding it to detect constraints in order to enable the optimization algorithm to converge directly to the true optimal solutions and get solutions entirely over the objective space. Better convergence and diversity of solutions are obtained from the developed algorithm. This is the essence of the NSGA II modification given next in Assumption of the New Approach.

Main Advantages of the New Approach:

1- Contrary of ignoring infeasible solution method which only valid to solve a single objective problem, the approach is valid to solve multi objective optimization problems with constraints.
2- No penalty function is needed here which save time trial and error for determining the best penalty parameter in order to have a solution closer to the true Pareto.
3- The approach is valid for both equality and inequality constraints
4- Contrary of Ray-Tai-Seow’s Method [18] only one single run is required to solve the multi objective.

The new approach will use Binh and Korn (1997) 2-variable constrained problem:

\[
\text{Minimize } f(x,y) = \begin{cases} 
4x^2 + 4y^2 \\
(x - 5)^2 + (y - 5)^2 
\end{cases}
\]

Subject to:  
\[
g1(x,y) = (x - 5)^2 + y^2 \leq 25 \\
g2(x,y) = (x - 8)^2 + (y + 3)^2 \geq 7.7
\]

and  
\[0 \leq x \leq 5; \ 0 \leq y \leq 3\]

Figure 2 illustrates the bounds of BNH solutions, the line OCD represents the Pareto Optimal set when no constraints exist, and thus all solutions above this line are feasible, boundary OA(A’EDCB(B’O represents the solutions in the feasible space which result from the interaction between the constraints, part of the dashed line ABC lies under the feasible solution of the objective functions f1, and f2 and this is neglected, as a result the line ABD is a boundary for feasible solution which satisfy all the constraints. All penalty function approaches depend on penalty factor which moves the solution away from constraint violation.

The solutions on the constraint in the objective space are the closer solution to the optimal set when exist, consequently converging closer to the Pareto optimal set. This requires more time and iterations for convergence.

In the Modified NSGA II algorithm, constraints are normalized and converted from inequality to equality form, objective functions are coded and solved only where decision variables are lie on the constraints boundaries which will lead the algorithm to go directly to the solutions that will satisfy all constraints and near to the true Pareto.
Steps of the M-NSGA-II Algorithm is illustrated as:

1: Coding constraints and objective functions then transforming constraints from inequality to equality form.
2: build one code for objective functions and constraints to force algorithm resulting in trade off solution only where the constraints are violated.
3: procedure NSGA-II \((N', g, f_k(x_k))\) \(N'\) members evolved \(g\) generations to solve \(f_k(x)\)
4: Initialize Population \(P'\)
5: If solution does not violate the constraints (go to step 3) other with go to step 7
6: Repeat steps (1-4), to construct the initial population.
7: Generate random population - size \(N'\)
8: Evaluate Objective Values
9: Assign Rank (level) Based on Pareto dominance - sort
10: Generate Child Population
11: Binary Tournament Selection
12: Recombination and Mutation
13: for \(i = 1\) to \(g\) do
14: for each Parent and Child in Population do
15: Assign Rank (level) based on Pareto - sort
16: Generate sets of no dominated vectors along PF known
17: Loop (inside) by adding solutions to next generation starting from the first front until \(N'\) individuals found determine crowding distance between points on each front
18: end for
19: Select points (elitist) on the lower front (with lower rank) and are outside a crowding distance
20: Create next generation
21: Binary Tournament Selection
22: Recombination and Mutation
23: end for
24: end procedure
Several case studies from literature are studied further using the proposed algorithm next.

IV. CASE STUDIES

The regular NSGA, NSGA-II (with penalty approach) and the developed M-NSGA-II are employed to four benchmark problems. In order to measure the quality of the new approach with respect to other algorithms, the following aspects are discussed:

1. Comparison among the three approaches with respect to convergence, diversity and number of iterations to the true Pareto.
2. Comparison among the three approaches with respect to quality indices used as a measure of convergence and diversity of solutions.

Quality indices like Maximum Pareto front error (MPFE), Accuracy of Pareto frontier (AC), Spacing, Overall Spread (OS), Objective Spread, Maximum Spread, and Number of Distinct Choices (NDC), Cluster (CLu (P)), Hyper Area (HV), Dominant Area (Do), Non-dominated Evaluation (NDEM), and Crowding distances are employed to assess the resulting Pareto Front.

Case study 1: CONSTR_EX

Minimize $\begin{cases} f_1(x,y) = x \\ f_2(x,y) = \frac{1+y}{x} \end{cases}$

Subject to $\begin{cases} g_1(x,y) = y + 9x \geq 6 \\ g_2(x,y) = -y + 9x \geq 1 \end{cases}$

Table 1 gives a comparison between the 3 approaches employed and the set of quality indices. Figure 4 gives the Pareto Front of the 3 approaches.

Table 1 Quality indices obtained by M-NSGA II, NSGA + Penalty, and Regular NSGA for CONSTR_EX Problem

<table>
<thead>
<tr>
<th>Quality Indices</th>
<th>Developed NSGA II</th>
<th>NSGA + Penalty</th>
<th>Regular NSGA</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st Obj.</td>
<td>2nd Obj.</td>
<td>1st Obj.</td>
<td>2nd Obj.</td>
</tr>
<tr>
<td>Min</td>
<td>0.36148</td>
<td>1.0393</td>
<td>0.57845</td>
</tr>
<tr>
<td>Max</td>
<td>1.6937</td>
<td>9</td>
<td>1.6571</td>
</tr>
<tr>
<td>Rang</td>
<td>1.3322</td>
<td>7.9607</td>
<td>1.0787</td>
</tr>
<tr>
<td>Std.</td>
<td>0.36866</td>
<td>1.8166</td>
<td>0.32187</td>
</tr>
<tr>
<td>Mean</td>
<td>0.99115</td>
<td>2.3654</td>
<td>1.0331</td>
</tr>
<tr>
<td>Obj. Spread</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>O.S</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Max. Spread</td>
<td>5.7073</td>
<td>1.6336</td>
<td>1.6973</td>
</tr>
<tr>
<td>Crowding distances</td>
<td>0.10281</td>
<td>0.20308</td>
<td>0.11028</td>
</tr>
<tr>
<td>Scaled H.V</td>
<td>0.76815</td>
<td>0.67079</td>
<td>0.67523</td>
</tr>
<tr>
<td>Scaled Dominant Area</td>
<td>0.17065</td>
<td>0.30866</td>
<td>0.30099</td>
</tr>
<tr>
<td>Accuracy of observed Pareto frontier</td>
<td>16.3395</td>
<td>48.656</td>
<td>42.0482</td>
</tr>
<tr>
<td>Non-Dominated Evaluation Metric</td>
<td>50</td>
<td>50</td>
<td>50</td>
</tr>
<tr>
<td>Maximum Pareto-Optimal Front Error</td>
<td>0</td>
<td>0.014149</td>
<td>0.0029519</td>
</tr>
<tr>
<td>NDC(at u= 0.1)</td>
<td>34</td>
<td>34</td>
<td>33</td>
</tr>
<tr>
<td>Spacing</td>
<td>1.4756</td>
<td>0.63307</td>
<td>0.75967</td>
</tr>
<tr>
<td>CLu(P)</td>
<td>1.4706</td>
<td>1.4706</td>
<td>1.5152</td>
</tr>
</tbody>
</table>

Comparing between the three approaches with respect to number of iterations. (CONSTR_EX)
Figure 5 shows the behavior of different algorithms over the entire solution space with respect to number of iterations. The developed M-NSGA II is the fastest among the three algorithms to converge to true Pareto and has the best diversity as well.

Quality indices for NSGA II, M-NSGA II, and NSGA II with Penalty function
The quality indices are plotted on x Axis ordered as follow:
1- OS1, 2- OS, 3- Max Spread, 4- Crowding distances, 5- HV, 6- Do, 7- AC, 8- NDEm, 9- MPFE, 10- NDC, 11- Spacing, and 12- CLu (P).
The values are calculated using the three mentioned algorithm for the problem CONSTR_EX. Figure 6 gives a comparison between different quality indices. The developed M-NSGA II has the advantages of:
- Objective Overall Spread as a measure for Diversity
- Max Spread
- Hyper Volume which is a good measure for both Diversity and Convergence
- Non-Dominated Evaluation Metric and
- Spacing

The NSGA II and NSGA with penalty function have almost the same Crowding Distances and Maximum Pareto-Optimal Front Error (MPFE). Thus, the M-NSGA II is better than Regular NSGA and NSGA II with penalty function.

Figure 6 shows that M-NSGA II wins the other two approaches in the following:
- Objective and Overall Spread which is a measure for Diversity
- Max Spread
- Hyper Volume which is a good measure for both Diversity and Convergence
- Non-Dominated Evaluation Metric
- Spacing

And both of M-NSGA II and NSGA II with penalty almost the same and win Regular NSGA in the following:

- Crowding Distances
- Maximum Pareto-Optimal Front Error (MPFE)

From the above study and analysis, we can conclude that M-NSGA II is better than Regular NSGA and NSGA II with Penalty

**Case Study 2: MOPC1**

Minimize \[
\begin{align*}
&f_1(x, y) = 4x^2 + 4y^2; \\
&f_2(x, y) = (x - 5)^2 + (y - 5)^2;
\end{align*}
\]

Subjected to \[
\begin{align*}
&g_1(x, y) = (x - 5)^2 + 4.1y^2 - 25 \leq 0; \\
&g_2(x, y) = -(x - 8)^2 - (y + 3)^2 + 7.7 \leq 0; \\
&0 \leq x \leq 5, 0 \leq y \leq 5.
\end{align*}
\]

Table 2 gives a comparison between the 3 approaches employed and the set of quality indices. Figure 7 gives the Pareto Front of the 3 approaches.

**Figure 7 M-NSGA II, vs. NSGA II with Penalty and Regular NSGA II for MOPC1**

Table 2 Quality indices obtained by Developed NSGA II, NSGA + Penalty, and Regular NSGA for MOPC1

<table>
<thead>
<tr>
<th>Quality Indices</th>
<th>MOPC1</th>
<th>Developed NSGA II</th>
<th>NSGA + Penalty</th>
<th>Regular NSGA</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>1st Obj.</td>
<td>2nd Obj.</td>
<td>1st Obj.</td>
</tr>
<tr>
<td>Min</td>
<td>256</td>
<td>9</td>
<td>52.9049</td>
<td>6.4185</td>
</tr>
<tr>
<td>Max</td>
<td>355.9447</td>
<td>33.9961</td>
<td>123.5625</td>
<td>12.9019</td>
</tr>
<tr>
<td>Rang</td>
<td>99.9447</td>
<td>24.9961</td>
<td>70.6576</td>
<td>6.4834</td>
</tr>
<tr>
<td>StD.</td>
<td>32.7995</td>
<td>7.861</td>
<td>19.9953</td>
<td>1.9925</td>
</tr>
<tr>
<td>Mean</td>
<td>292.6528</td>
<td>17.0758</td>
<td>83.3958</td>
<td>8.7153</td>
</tr>
<tr>
<td>Obj. Spread</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>O.S</td>
<td></td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Max. Spread</td>
<td>72.8483</td>
<td>50.1724</td>
<td>50.1724</td>
<td>50.1724</td>
</tr>
<tr>
<td>Crowding distances</td>
<td>Inf</td>
<td></td>
<td>2.5165</td>
<td>10.8654</td>
</tr>
<tr>
<td>Scaled HV</td>
<td>0.8243</td>
<td>0.70035</td>
<td>0.76159</td>
<td></td>
</tr>
<tr>
<td>Scaled Dominant Area</td>
<td>0.15839</td>
<td>0.28141</td>
<td>0.21154</td>
<td></td>
</tr>
<tr>
<td>Accuracy of observed Pareto frontier</td>
<td>57.7957</td>
<td>54.8276</td>
<td>37.2242</td>
<td></td>
</tr>
<tr>
<td>Non-Dominated Evaluation Metric</td>
<td>50</td>
<td>50</td>
<td>47</td>
<td></td>
</tr>
<tr>
<td>Maximum Pareto-Optimal Front Error</td>
<td>0</td>
<td>0</td>
<td>0.19187</td>
<td></td>
</tr>
<tr>
<td>NDC(at u= 0.1)</td>
<td>31</td>
<td>31</td>
<td>30</td>
<td></td>
</tr>
<tr>
<td>Spacing</td>
<td>140.5009</td>
<td>40.1027</td>
<td>74.8671</td>
<td></td>
</tr>
<tr>
<td>CLe(P)</td>
<td>1.6129</td>
<td>1.6129</td>
<td>1.5667</td>
<td></td>
</tr>
</tbody>
</table>

Comparing between the three approaches with respect to number of iterations. (MOPC1)

Figure 8 shows the behavior of each algorithm over the entire solution space with respect to diversity and conversion vs. iterations. It is obviously seen that M-NSGA II is the fastest among the three algorithms to converge closer to true Pareto and has the best diversity as well. It goes directly to the optimum solutions set.
Quality indices for NSGA II, M-NSGA II, NSGA II with Penalty function

Figure 9 shows the behavior of different algorithms over the entire solution space with respect to number of iterations. The developed M-NSGA II is the fastest among the three algorithms to converge to true Pareto and has the best diversity as well.

Case Study 3: BinhKorn Function

Minimize  
\[
\begin{align*}
    f1(x, y) &= 4x^2 + 4y^2 \\
    f2(x, y) &= (x - 5)^2 + (y - 5)^2
\end{align*}
\]

Subject to:
\[
\begin{align*}
    g1(x, y) &= (x - 5)^2 + y^2 \leq 25 \\
    g2(x, y) &= (x - 8)^2 + (y + 3)^2 \geq 7.7
\end{align*}
\]

and
\[
0 \leq x \leq 5; \ 0 \leq y \leq 3
\]

Table 3 gives a comparison between the 3 approaches employed and the set of quality indices, While Figure 10 gives the Pareto Front of the 3 approaches.
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Figure 10. M-NSGA II, vs. NSGA II with Penalty and Regular NSGA II for BinhKorn Function

Table 3 Quality indices obtained by Developed NSGA-II, NSGA + Penalty, and regular NSGA for BinhKorn Function

<table>
<thead>
<tr>
<th>Quality Indices</th>
<th>BinhKorn</th>
<th>Developed NSGA II</th>
<th>NSGA + Penalty</th>
<th>Regular NSGA</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>1st Obj.</td>
<td>2nd Obj.</td>
<td>1st Obj.</td>
</tr>
<tr>
<td>Min</td>
<td>0.0032746</td>
<td>7.0687e-006</td>
<td>14.2925</td>
<td>2.0253</td>
</tr>
<tr>
<td>Max</td>
<td>200.0893</td>
<td>49.6888</td>
<td>211.9004</td>
<td>51.9105</td>
</tr>
<tr>
<td>Rang</td>
<td>60.4662</td>
<td>15.3539</td>
<td>60.1117</td>
<td>16.0311</td>
</tr>
<tr>
<td>Mean</td>
<td>63.2803</td>
<td>17.8917</td>
<td>84.8387</td>
<td>18.3387</td>
</tr>
<tr>
<td>Obj. Spread</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>O.S</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Max. Spread</td>
<td>145.7796</td>
<td>144.1135</td>
<td>144.943</td>
<td></td>
</tr>
<tr>
<td>Crowding distances</td>
<td>4.8357</td>
<td>3.004</td>
<td>5.5304</td>
<td></td>
</tr>
<tr>
<td>Scaled H.V</td>
<td>0.82112</td>
<td>0.81879</td>
<td>0.80437</td>
<td></td>
</tr>
<tr>
<td>Scaled Dominant Area</td>
<td>0.16055</td>
<td>0.1618</td>
<td>0.17833</td>
<td></td>
</tr>
<tr>
<td>Accuracy of observed Pareto frontier</td>
<td>54.5631</td>
<td>51.5096</td>
<td>57.808</td>
<td></td>
</tr>
<tr>
<td>Non-Dominated Evaluation Metric</td>
<td>50</td>
<td>50</td>
<td>50</td>
<td></td>
</tr>
<tr>
<td>Maximum Pareto-Optimal Front Error</td>
<td>0.24156</td>
<td>0.69844</td>
<td>0.374</td>
<td></td>
</tr>
<tr>
<td>NDC(at u= 0.1)</td>
<td>33</td>
<td>34</td>
<td>31</td>
<td></td>
</tr>
<tr>
<td>Spacing</td>
<td>49.4608</td>
<td>55.0671</td>
<td>52.5274</td>
<td></td>
</tr>
<tr>
<td>CLu(P)</td>
<td>1.5152</td>
<td>1.4706</td>
<td>1.6129</td>
<td></td>
</tr>
</tbody>
</table>

Comparing between the three approaches with respect to number of iterations. (BinhKorn Function).

Figure 11 shows the behavior of different algorithms over the entire solution space with respect to number of iterations. The developed M-NSGA II is the fastest among the three algorithms to converge to true Pareto and has the best diversity as well.

Again, the M-NSGA II gives better and faster convergence and diversity than the regular NSGAII or with penalty functions.

**Quality indices for NSGA II, M-NSGA II, NSGA II with Penalty function**

Figure 12 gives a comparison between Quality indices obtained by Regular NSGA II, M-NSGA II, and NSGAII+ Penalty function (BinhKorn Function).

![Figure 12. Quality indices plot for BinhKorn Function](image)

Figure 12 shows that M-NSGA II is superior to the other algorithms with respect to maximum spread, crowding distance, spread, and hyper volume. It can also be concluded that NSGA II with penalty approach can miss its search to solution.

**Case Study 4: SRN**

Minimize \( f_1(x, y) = (x - 2)^2 + (y - 1)^2 + 2 \)

Minimize \( f_2(x, y) = 9x + (y - 1)^2 \)

Subject to:

\[ g_1(x, y) = -x - y + 225; \]
\[ g_2(x, y) = -x + 3y - 10; \]

\[ 0 \leq x \leq 6; 0 \leq y \leq 2.25 \]

Table 4 gives a comparison between the 3 approaches employed and the set of quality indices. Figure 13 shows the observed Pareto Front for SRN Test problem using the three different algorithms. The same conclusion holds.

![Figure 13 M-NSGA II vs. NSGA II with Penalty and Regular NSGA II for SRN Function](image)

<table>
<thead>
<tr>
<th>SRN</th>
<th>Quality Indices</th>
<th>Developed NSGA II 1st Obj.</th>
<th>NSGA + Penalty 1st Obj.</th>
<th>Regular NSGA 1st Obj.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Min</td>
<td>14.1001</td>
<td>-54.6041</td>
<td>53.6315</td>
<td>9.6329</td>
</tr>
<tr>
<td>Max</td>
<td>495.3465</td>
<td>38.8549</td>
<td>130.0986</td>
<td>65.2305</td>
</tr>
<tr>
<td>Rang</td>
<td>481.2464</td>
<td>93.459</td>
<td>76.467</td>
<td>55.5976</td>
</tr>
<tr>
<td>StD.</td>
<td>166.8636</td>
<td>30.5476</td>
<td>25.0216</td>
<td>16.6956</td>
</tr>
<tr>
<td>Mean</td>
<td>198.5193</td>
<td>-17.3044</td>
<td>88.3413</td>
<td>33.8199</td>
</tr>
<tr>
<td>Obj. Spread</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>O.S</td>
<td>1</td>
<td></td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

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Comparing between the three approaches with respect to number of iterations.

Figure 14 shows the behavior of different algorithms over the entire solution space with respect to number of iterations. The developed M-NSGA II is the fastest among the three algorithms to converge to true Pareto and has the best diversity as well.

Figure 14 Regular NSGA, NSGA + Penalty, and M-NSGA II with respect to iterations (SRN)

Quality indices for NSGA-II, M-NSGA-II, NSGA-II with Penalty function

Figure 15 gives a comparison between Quality indices for function (SRN).

Figure 15 shows that M-NSGA II is superior to the other algorithms with respect to maximum spread, crowding distance, spread, Pareto accuracy, Spacing, and hyper volume. It can also be concluded that NSGA II with penalty approach can miss its search to solution.

V. Conclusion

A modification to the regular NSGA II algorithm is proposed to deal with the multi objective optimization problems with constraints. A Comparative Analysis between regular NSGA II, NSGA with penalty functions, and M-NSGA II using a set of quality indices. Results indicate that the M-NSGA is the best among the three algorithms in terms of convergence and diversity. Besides, The M-NSGA can handle various multi objective constrained problems in a single run and can reach the optimal set very fast.
VI References


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Contributions: Multi-objective Optimization Indices A comparative analysis

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