A Two-layer Time Window Assignment Vehicle Routing Problem

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Abstract

This paper presents a two-layer time window assignment vehicle routing problem (TL-TWAVRP). It is assumed that there is a predefined time window determined by the customers which is called exogenous time window. Also, there are two-layer endogenous time windows inside the previous one. The outer layer has bigger width and the difference is a violation variable. These new time windows give a flexibility to career companies for visiting customers after the end of assigned time windows to perform services to more customers. If the vehicle arrives at the customer in her/his inner layer assigned time window, no penalty is paid but if vehicle violates inner layer, a penalty will be calculated in the objective function. No extra violation is allowed from the outer layer assigned time window. This problem is modeled as a two-stage stochastic problem. The decisions of first-stage are assigning inner and outer layers time window to each customer. Then in the next stage, routes will be planned for each scenario. Finally, by comparing this model with the TWAVRP, it has been shown that the proposed model has better performance to minimize total cost with serving more customers considering their exogenous time windows.

Keywords  
Vehicle routing, Time window assignment, Two-layer time window, Stochastic programming.

1. Introduction

In daily goods distribution network, career companies face lots of delivery from suppliers to customers that should be cost-effective. In the classic vehicle routing problem, the total traveling cost is minimized as an objective function. One of the most studied subjects in this problem is the vehicle routing problem, considering customer’s time windows which are called VRPTW. In this kind of problems, each client must be visited in a pre-specified time window. The VRPTW is a problem with hard constraints although in some practical applications time window constraint can be considered as a soft constraint and the violations must be calculated as a penalty in the objective function. This attitude converts the problem to vehicle routing problem with soft time window (VRPSTW) which is studied less than VRPTW. The VRPSTW considers time window but the difference is that each customer can be visited at any time and vehicle must pay penalty for time window violations.
In real-world applications, the customer and supplier might agree on a specific time interval for delivery that can consider as an endogenous time window. Also because of some unpredictable conditions, career companies may want to have an extra agreement with customers to have more flexibility with paying a penalty. The amount of flexibility may depend on company equipment like vehicles’ capacity and uncertain distribution conditions like traffic condition and etc., so the agreement with customers is determined by a two-layer assigned time window. It should be noted that mentioned assigned two-layer time windows should be inside of a predefined time window which is called exogenous time window. As illustrated in figure 1, all of definitions can be better illustrated. In the real world, demands are usually unknown, hence TWAVRP model is introduced for vehicle routing problem with stochastic demands.

In this study, more flexibility is considered by assigning two-layer endogenous time windows. Suppose that a career company assigns a time window for each customer before realization of demands and other uncertain parameters. It means that the customer should be visited during its assigned time window while the customer has defined a specific width for its assigned time window. In that case extra agreement with the customer including a penalty will make distribution more efficient, so in the current study inner and outer layer endogenous time windows are assigned in a predefined exogenous time window as the first stage decision. Then routing decisions are made based on the first stage decision after realization of all uncertain demands. In some cases, paying endogenous violation penalty cost is more cost-effective than adding a new vehicle to service fleet according to scenarios. In the proposed model, objective function minimizes the sum of traveling, penalty and fixed vehicle costs. In this paper, a new model based on two commodity flow formulation in Baldacci et al. (2004) and subtour elimination MTZ-inequalities in Miller et al. (1960) is presented.

The sections structured as follows: in the section 2, a literature review is presented. Then the problem is defined and mathematical formulations are presented in section 3. Numerical examples are analyzed in section 4. Finally, conclusion and future research directions are considered in the last section.

2. Literature Review

The researches about vehicle routing problem can be categorized in four main sections: 1-Deterministic capacitated vehicle routing problem(CVRP), 2-Stochastic vehicle routing problem (SVRP), 3-Robust vehicle routing problem(RVPR), 4-Fuzzy vehicle routing problem. Stochastic VRP has been considered more because of its more reality. There are some sources of uncertainty that reviewed by Oyola et al. (2017). For instance, demands, customers, service time, travel time may be stochastic parameters. VRP mainly minimizes total traveling cost but in most of the cases there are some constraints about the main problem such as budget, time, distance and capacity limitations. Another restriction according to real world situations is about the servicing time window. Based on the hardness of time window for servicing customers the VRP can be considered as VRP with hard time windows (violation from customers’ time window is not possible), VRP with soft time window (a penalty cost is paid based on amount of violation), VRP with flexible time window (it is like soft time window with restrictions on the amount of violations).

In the case of vagueness on the time window, the problem will be changed to the VRP with fuzzy time windows. In previous mentioned types of the VRP, time windows are predefined by the customers, however in a new type of
problem, the distributor or supplier may assign the time window according to its limitations instead of customers. In the current study, we will focus on the Time Window Assignment Vehicle Routing Problem (TWAVRP). Among researches in recent years, there are a few studies on the TWAVRP, and it can be seen that it is very common and it is very useful in real distribution cases. In real cases, usually customers prefer to define tighter time windows for their comfort and for their own better planning, but it will make lots of scheduling and resource difficulties for the distributor. As a solution, distributor can assign tighter time window inside a predefined exogenous time window by the customer. This strategy will be more beneficial for both parts (customers and distributor). Because of facing to the uncertainty the distributor may fail to serve some customers in their assigned time windows. More flexibility may be beneficial for distributor to solve the mentioned problem as a solution to encounter uncertainty. It means that the distributor may assign two layers of time windows inside of the exogenous time window. International restaurant deliveries, online market deliveries, school bus scheduling, and etc. are some examples in which may require to schedule their distributions by the proposed two layer time window assignment scheme.

In some cases, a customer time window can be violated considering an appropriate penalty. Tas et al. (2013) presented a model for vehicle routing problem with stochastic travel times. Each customer has a soft time windows and service costs. This model considered transportation and service costs. Transportation cost includes sum of distance traveled, number of vehicles send from depot and all driver’s extra working hours. In this paper, service costs are a penalty for early arrival times and late arrival times. Tas et al. (2014) introduced a VRP with flexible time window. In this model, customers can receive their services with a deviation from their time window. This deviation is determined by a certain tolerance. The main difference in this model comparing with VRPSTW, is the restriction on arrival time of the vehicles. In VRPSTW, vehicle can arrive at each customer at any of time then paying an appropriate penalty for latency but in this model, latency is restricted by a tolerance. Tas et al. (2014) introduced a model for the time-dependent vehicle routing problem with a soft time window and stochastic travel times. Customers have soft time windows and travel times are uncertain and dependent to time and traffic condition. Mothuy et al. (2015) proposed a multistage large scale neighbor search for the vehicle routing problem with soft time windows. This model first intends to minimize the number of routes then minimize the number of early arrival and delay at each customer.

The main contributions of the TWAVRP can be seen in the following researches. The time window assignment vehicle routing problem introduced by Spliet, and Gabor (2014) assigns time window to each customer before demand realization to minimize total transportation cost. For each customer an endogenous time window with predetermined width is selected within the exogenous time window. This problem is a specific kind of vehicle routing problem with consistency conditions studied by Groër et al. (2009). Spliet and Desaulniers (2015) introduced a special case of the TWAVRP, in which time windows should be chosen from the set of defined time windows. Spliet et al (2017) introduced a special kind of the problem in a case of time-dependent travel times. Because of varying of travel times during a day, it is considered in assigning the time window. Dalmeijer and Spliet (2018) presented a branch-and-cut algorithm for the time window assignment vehicle routing problem (TWAVRP). They showed that their algorithm has better performance comparing with the algorithm presented in Spliet and Gabor (2014) and can solve larger instances. Fábio Neves-Moreira et al. (2018) introduced a new version of problem that time windows are dependent to products and split delivery is considered. Jabali et al. (2015) also proposed a model similar to TWAVRP but in this case travel times are uncertain and a heuristic has been used to solve the model.

Based on the presented literature review, it is realized that scheduling of deliveries to customers by the time window assignment in the vehicle routing problem is an important issue which is considered by various researchers. However, it seems that time window assignment in two layers to give more flexibility to distributor and more comfort to customers has not been considered in previous researches. So in this study, it is considered as main characteristic of the current research. Moreover, some career companies may have extra limitations for distribution on the distributing time in a day because of city environmental pollution restrictions. It is considered in this study, too.

3. Problem definition
In this section, a two-layer time window assignment vehicle routing problem (2L-TWAVRP) is introduced. Consider a directed graph $G = (V, A)$ with $n$ customers $V' = \{1, 2, \ldots, n\}$, which denotes the depot and $n + 1$ is the same
depot and acts as a copy of depot. Therefore $V = V' \cup \{0, n+1\}$ denotes all customers and depot spots. Set $A$ is total arcs that is used to connect nodes in network. Arc set $A$ composed total arcs between clients in $V'$ and all arcs leaving and entering to the depot. Set of arcs $A'$ is set of arcs $A$ with two additional arcs $(i, 0)$ and $(n+1, i)$. For every directed arc $(i, j) \in A$. Furthermore $t_{ij}$ is travel time and $c_{ij}$ is travel cost. Note that travel cost must be positive $c_{ij} \geq 0$, also travel time must be non-negative. Each customer, has an endogenous time window that it has two layers, the width of the inner layer is $W_i$ and determined by the customers. The width of the outer layer is $W'_i$ and the model determines its optimal value. The exogenous time window of the customer $i$ is given in an interval $[s_i, e_i]$, also working hours of starting depot and ending depot are given by $[s_o, e_o]$ and $[s_{n+1}, e_{n+1}]$, respectively. It is assumed that there are unlimited identical vehicles with the capacity of $Q$. As mentioned before, there is a demand uncertainty in a finite set of scenarios $\Omega$. Each scenario has related probability $p_\omega$ and demand client $i \in V'$ in $\omega \in \Omega$ is denoted by $d_\omega$ that is $0 \leq d_\omega \leq Q$. Each vehicle has a fixed cost of $C_v$. A violation cost of servicing after assigned time window per unit is $C_v$. Finally, it is assumed that customers disaffect to face to an increased width of outer layer time window, so in the model a dissatisfaction rate of $C_v$ is considered for each unit of the outer layer width. $C_v$ is the set of integral numbers. Our proposed model is a generalized form of the TWAVRP formulation in Dalmeijer and Spliet (2018) with two major differences: first, this new time windows has two layers with different corresponding characteristic, second penalty cost and vehicle fixed cost is considered in the objective function.

Decision variables

$y_i^\omega$: Time window decision variable for $i \in V'$

$x_{ij}^\omega$: A binary variable equals 1 if vehicle travels form $i$ to $j$ under scenario $\omega$.

$t_{ij}^\omega$: Continuous variable determines service receiving time of customer $i \in V'$ under scenario $\omega$.

$z_{ij}^\omega$: Sum of load in vehicle when travels from $i$ to $j$ in scenario $\omega$.

$z_{ji}^\omega$: Remaining capacity when vehicle is traveling from $j$ to $i$ in scenario $\omega$.

$w_i^\omega$: Continuous variable determines the amount of violation in the inner layer.

$h_i^\omega$: Service time violation amount from inner layer of time window for client $i \in V'$.

### 3.1. Mathematical formulation

$$\min \sum_{\omega \in \Omega} p_\omega \sum_{(i,j) \in A} c_{ij} x_{ij}^\omega + C_v \sum_{i \in V'} h_i^\omega + C_v \sum_{i \in V'} \sum_{j \in V'} \sum_{\omega \in \Omega} x_{ij}^\omega + C_v \sum_{i \in V'} w_i^\omega$$

s.t

$$\sum_{j \in V \cup \{n+1\}} x_{ij}^\omega = 1 \quad \forall i \in V', \omega \in \Omega$$

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\[
\sum_{i \in V \setminus \{0\}} x_{ij}^\omega = 1 \quad \forall j \in V', \omega \in \Omega \quad (3)
\]

\[
z_{ij}^\omega + z_{ji}^\omega = (x_{ij}^\omega + x_{ji}^\omega)Q \quad \forall (i, j) \in \tilde{A}, i < j, \omega \in \Omega \quad (4)
\]

\[
\sum_{j \in V} (z_{ji}^\omega - z_{ij}^\omega) = 2d_i^\omega \quad \forall i \in V', \omega \in \Omega \quad (5)
\]

\[
\sum_{j \in V} z_{0j}^\omega = \sum_{i \in V} d_i^\omega \quad \forall \omega \in \Omega \quad (6)
\]

\[
\sum_{j \in V} z_{j+1,j}^\omega = \left(\sum_{i \in V} x_{ij}^\omega\right)Q \quad \forall \omega \in \Omega \quad (7)
\]

\[
\sum_{j \in V} z_{j0}^\omega = \left(\sum_{j \in V} x_{0j}^\omega\right)Q - \sum_{i \in V} d_i^\omega \quad \forall \omega \in \Omega \quad (8)
\]

\[
t_j^\omega + t_i^\omega \geq t_{ij}^\omega + (s_j - e_i)(1 - x_0^\omega) \quad \forall i \in V', \forall j \in V', \omega \in \Omega \quad (9)
\]

\[
s_0 + t_{0j} \leq t_j^\omega \quad \forall j \in V', \omega \in \Omega \quad (10)
\]

\[
t_i^\omega + t_{i,n+1} \leq e_{n+1} \quad \forall i \in V', \omega \in \Omega \quad (11)
\]

\[
t_i^\omega \geq y_i \quad \forall i \in V', \omega \in \Omega \quad (12)
\]

\[
t_i^\omega \leq y_i + w_i + w_i' \quad \forall i \in V', \omega \in \Omega \quad (13)
\]

\[
y_i \in \left[ s_i, e_i - w_i - w_i' \right] \quad \forall i \in V' \quad (14)
\]

\[
h_i = t_i^\omega - (y_i + w_i) \quad \forall i \in V', \omega \in \Omega \quad (15)
\]

\[
h_i \geq 0 \quad \forall i \in V' \quad (16)
\]

\[
x_{ij}^\omega \in \mathbb{C} \quad \forall (i, j) \in A, \omega \in \Omega \quad (17)
\]
The objective function (1) minimizes the sum of fixed and variable routing and penalty costs. Penalty cost consists of two terms including length of assigned outer layer time window and servicing time violation from the inner layer time window. Constraints (2) and (3) ensure that each customer is visited only by one vehicle. Constraints (4) determines used and left over capacity of a vehicle according to its capacity. Constraint (5) determines the used and left over capacity of a vehicle according to served demand of a visited node. Constraints (6)-(8) determine the required vehicle load and excess capacity when leaving or returning to the depot. Constraint (9) is MTZ inequalities that as subtour elimination constraints. Constraint (10) ensures that vehicle can serve a customer after leaving the depot and traveling to customer node. Constraint (11) schedules vehicles considering the depot closing time. Constraint (12) and (13) guarantee that each customer receive its demand after start endogenous time window and before end of it. Constraint (14) implies that each endogenous time windows must assign within exogenous time window. Constraint (15) calculates the amount of servicing violation from an assigned inner layer time window for each customer. Constraints (16)-(18) define type of variables.

4. Numerical examples and results

In this section, some instances including 10, 15, 20 and 20 customers are considered and results of last instance is reported in Table 1. All instances are run on Intel (R) core (TM) i7-4500U 1.80GHz with 8GB of RAM computer. The model was coded in GAMS version 24.1.2. It is assumed that stochastic demands follow a uniform distribution. Table 1 shows that the proposed model has a better performance comparing to classic models. The VRPTW was solved with all scenarios and average of results is reported. Also exogenous time windows of TWAVRP instances are used as customer’s time window in VRPTW. On the other hand, the number of required vehicles decreases in the proposed model in comparison with model that considered one layer time window assignment.

Table 1: A comparison of the proposed Two layer time window model with classic models

<table>
<thead>
<tr>
<th>Instance</th>
<th>Customers</th>
<th>Routing cost</th>
<th>Computation time (s)</th>
<th>Vehicles</th>
<th>Routing cost</th>
<th>Computation time (s)</th>
<th>Vehicles</th>
<th>Routing cost</th>
<th>Computation time (s)</th>
<th>Vehicles</th>
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<td>4000</td>
<td>389</td>
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<td>496</td>
<td>12.109</td>
<td>2000</td>
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<td>0.125</td>
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</table>
Figure 2 showed that the number of required vehicles to serve the customers in the first layer of assigned time window increases by increasing in the penalty cost.

![Figure 2: A sensitivity analysis on penalty cost and number of used vehicles](image)

Figure 3 illustrated that paid servicing penalty costs increase by increasing of travel time between pair of customers. It is because of violating the assigned time windows because of considerable travel times.

![Figure 3: Relation between travel time total penalty cost](image)

In figure 4 it can be seen that by increasing of the travel time from all nodes to a specific customer, its assigned outer layer time window width is increased. A vehicle should arrive at a customer before end of the exogenous time window and a penalty cost for violating the first layer of the time window is paid. Mentioned analyzes confirm the model validity.

![Figure 4: Relevance between travel time and outer layer assigned time window width](image)
5. conclusion
In this paper, a new model of time window assignment VRP was proposed considering two-layer time window assignment. Various sensitivity analysis confirm the proposed model validity. It was shown that more penalty cost should be paid when we face to a problem with more travelling time. Also it was shown that width of the outer layer assigned time window will be increased in cases with more travel times. By the proposed model, it was concluded that less vehicle will be needed comparing to classic models. As a future study, time window assignment may be considered for other various types of the vehicle routing problem. Moreover, developing heuristic algorithms to solve the largescale instances of the problem may be another direction for the future study.

References
Biographies

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