$$\sum_{i \in V' \cup \{0\}} x_{ij}^{\omega} = 1 \qquad \forall j \in V', \omega \in \Omega \qquad (3)$$

$$z_{ij}^{\ \omega} + z_{ji}^{\ \omega} = (x_{ij}^{\omega} + x_{ji}^{\omega})Q \qquad \forall (i,j) \in \overline{A}, \mathrm{i} < \mathrm{j}, \omega \in \Omega \qquad (4)$$

$$\sum_{j \in V} (z_{ji}^{\ \omega} - z_{ij}^{\ \omega}) = 2d_i^{\omega} \qquad \forall i \in V', \omega \in \Omega \qquad (5)$$

$$\sum_{j \in V'} z_{0j}^{\ \omega} = \sum_{i \in V'} d_i^{\omega} \qquad \forall \omega \in \Omega \qquad (6)$$

$$\sum_{j \in V'} z_{n+1,j}^{\omega} = \left(\sum_{j \in V'} x_{0j}^{\omega} \right) Q \qquad \forall \omega \in \Omega \qquad (7)$$

$$\sum_{j \in V'} z_{j0}^{\omega} = \left(\sum_{j \in V'} x_{0j}^{\omega} \right) Q - \sum_{i \in V'} d_i^{\omega} \qquad \forall \omega \in \Omega \qquad (8)$$

$$t_j^{\omega} + t_i^{\omega} \geq t_{ij} x_{ij}^{\omega} + (s_j - e_i)(1 - x_{ij}^{\omega}) \qquad \forall i \in V', \forall j \in V', \omega \in \Omega \qquad (9)$$

$$s_0 + t_{0j} \leq t_j^{\omega} \qquad \forall j \in V', \omega \in \Omega \qquad (10)$$

$$t_i^{\omega} + t_{i,n+1} \leq e_{n+1} \qquad \forall i \in V', \omega \in \Omega \qquad (11)$$

$$t_i^{\omega} \geq y_i \qquad \forall i \in V', \omega \in \Omega \qquad (12)$$

$$t_i^{\omega} \leq y_i + w_i + w_i' \qquad \forall i \in V', \omega \in \Omega \qquad (13)$$

$$y_i \in \left[s_i, e_i - w_i - w_i' \right] \qquad \forall i \in V' \qquad (14)$$

$$h_i = t_i^{\omega} - (y_i + w_i) \qquad \forall i \in V', \omega \in \Omega \qquad (15)$$

$$h_i \geq 0 \qquad \forall i \in V' \qquad (16)$$

$$x_{ij}^{\omega} \in \mathbb{C} \qquad \forall (i,j) \in A, \omega \in \Omega \qquad (17)$$

$$z_{ij}^{\omega} \geq 0 \qquad \forall (i,j) \in \bar{A}, \omega \in \Omega \qquad (18)$$

The objective function (1) minimizes the sum of fixed and variable routing and penalty costs. Penalty cost consists of two terms including length of assigned outer layer time window and servicing time violation from the inner layer time window. Constraints (2) and (3) ensure that each customer is visited only by one vehicle. Constraints (4) determines used and left over capacity of a vehicle according toits capacity. Constraint (5) determines the used and left over capacity of a vehicle according to served demand of a visited node. Constraints (6)-(8) determine the required vehicle load and excess capacity when leaving or returning to the depot. Constraint (9) to (14) is related to both inner and outer layers time windows for each customer. Constraint (9) is MTZ inequalities that as subtour elimination constraints. Constraint (10) ensures that vehicle can serve a customer after leaving the depot and traveling to customer node. Constraint (11) schedules vehicles considering the depot closing time. Constraint (12) and (13) guarantees that each customer receive its demand after start endogenous time window and before end of it. Constraint (14) implies that each endogenous time windows must assign within exogenous time window. Constraint (15) calculates the amount of servicing violation from an assigned inner layer time window for each customer.Constraints (16)-(18) define ype of variables.

4. Numerical expamples and results

In this section, some instances includinge 10, 15, 20 and 20 customers are considered and results of last instance is reported in Table 1. All instances are run on Intel (R) core (TM) i7-4500U 1.80GHz with 8GB of RAM computer. The model was coded in GAMS version 24.1.2. It is asumed that stochastic demands follow a uniform distribution. Table 1 shows that the proposed model has a better performance comparing to classic models. The VRPTW was solved with all scenarios and avrage of results is reported. Also exogenous time windows of TWAVRP instances are used as customer's time window in VRPTW. On the other hand, the number of required vehicles decreas in the proposed model in comparison with model that considered one layer time window assignment.

Table 1: A comparison of the proposed Two layer time window model with classic models

		VRPTW			TWAVRP			TL-TWAVRP		
Instance	Customers	Routing cost	Computation time (s)	vehicles fixed cost	Routing cost	Computation time (s)	vehicles fixed cost	Routing cost	Computation time (s)	vehicles fixed cost
1	10	372	0.384	4000	389	5.172	3000	496	12.109	**2000**
2	10	367	0.125	3000	384	2.297	3000	439	2.656	**2000**
3	10	386	0.422	3667	419	8.891	3600	467	768.656	**3300**
4	15	395	0.358	5000	474	3.890	4000	495	5.562	**3000**
5	15	319	0.828	3000	331	2.109	3000	477	7.703	**2000**
6	15	401	0.502	5000	474	5.000	5000	487	6.297	**3500**
7	20	564	3.219	5000	531	9.860	5000	605	5.265	**3300**
8	20	562	4.141	5000	522	5.359	5000	676	2.390	**2000**
9	20	575	6.909	5334	539	30.610	5000	607	17.860	**4000**
10	25	664	20.891	5000	562	12.765	5000	684	20.719	**3000**
11	25	664	129.390	5000	566	25.984	5000	661	13.656	**4000**
12	25	664	20.610	5000	562	5.531	5000	667	5.859	**3000**

Figure2 showed that number of required vehicles to serve the customers in the first layer of assigned time window increaeses by increasing in the penalty cost.

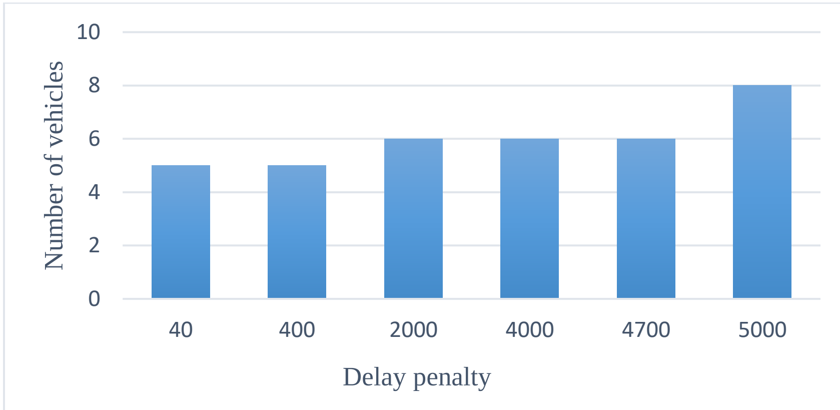

Figure 2: A sensitivity analysis on penalty cost and number of used vehicles

Figure 3 illustrated that paid servicing penalty costs increase by increasing of travel time between pair of customers. It is because of violating the assigned time windows because of considerable travel times.

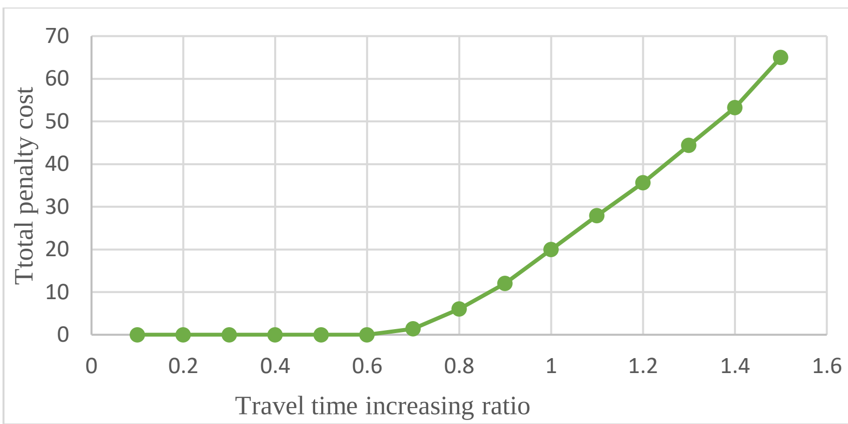

Figure 3: Relation between travel time total penalty cost

In figure 4 it can be seen that by increasing of the travel time from all nodes to a specific customer, its assigned outer layer time window width is increased. A vehicle should arrive at a customer before end of the exogenous time window and a penalty cost for violating the first layer of the time window is paid. Mentioned analyzes confirm the model validity.

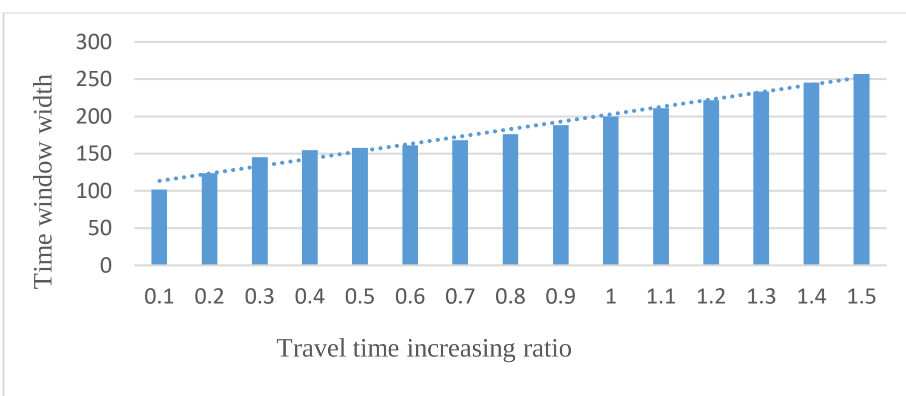

Figure 4: Relevance between travel time and outer layer assigned time window width

5. conclusion

In this paper, a new model of time window assignment VRP was proposed considering two-layer time window assignment. Various sensitivity analysis confirm the proposed model validity. It was shown that more penalty cost should be paid when we face to a problem with more travelling time. Also it was shown that widthof the outer layer assigned time window will be increased in cases with more travel times. By the proposed model, it was concluded that less vehicle will be needed comparing to classic models. As a future study, time window assignment may be considered for other various types of the vehicle routing problem. Moreover, developing heuristic algorithms to solve the largescale instances of the problem may be a s another direction for the future study.

References

Baldacci, R., Hadjiconstantinou, E. and Mingozzi, A. 'An Exact Algorithm for the Capacitated Vehicle Routing Problem Based on a Two-Commodity Network Flow Formulation', *Operations Research*, vol. 52, no. 5, pp. 723–738, 2004.

Dalmeijer, K. and Spliet, R. 'A branch-and-cut algorithm for the Time Window Assignment Vehicle Routing Problem', *Computers and Operations Research*, vol 89, pp. 140-152, 2018.

Jabali, O., Leus, R., van Woensel, T., de Kok, T., 'Self-imposed time windows in vehicle routing problems', *OR Spectrum*, vol. 37, no. 2, pp. 331–352, 2015.

Miller, C. E., Tucker, A. W. and Zemlin, R. A. 'Integer Programming Formulation of Traveling Salesman Problems', *Journal of the ACM*, vol. 7, no. 4, pp. 326–329, 1960.

Mouthuy, S., Massen, F., Deville, Y., & Van Hentenryck, P. 'A Multistage Very Large-Scale Neighborhood Search for the Vehicle Routing Problem with Soft Time Windows', *Transportation Science*, vol. 49, no. 2, pp. 223-238, 2015.

Neves-Moreira, F., Pereira da Silva, D., Guimarães, L., Amorim, P., & Almada-Lobo, B. 'The time window assignment vehicle routing problem with product dependent deliveries', *Transportation Research Part E: Logistics and Transportation Review*, vol. 116, pp. 163–183, 2018.

Oyola, J., Arntzen, H. and Woodruff, D. L. 'The stochastic vehicle routing problem, a literature review, part I: models', *EURO Journal on Transportation and Logistics*, vol 6, no. 4, pp. 349-388, 2016.

Spliet, R., Dabia, S. and Van Woensel, T. 'The Time Window Assignment Vehicle Routing Problem with Time-Dependent Travel Times', *Transportation Science*, no. March, pp. 1–16, 2017.

Spliet, R. and Desaulniers, G. 'The discrete time window assignment vehicle routing problem', *European Journal of Operational Research*, vol. 244, no. 2, pp. 379–391, 2015.

Spliet, R. and Gabor, A. F. 'The Time Window Assignment Vehicle Routing Problem', *Transportation Science*, vol 49, no 4, pp. 721–731, 2014.

Taş, D., Dellaert, N., Van Woensel, T., & De Kok, T .'Vehicle routing problem with stochastic travel times including soft time windows and service costs', *Computers and Operations Research*, vol. 40, no. 1, pp. 214–224, 2013.

Taş, D., Dellaert, N., Van Woensel, T., & De Kok, T 'The time-dependent vehicle routing problem with soft time windows and stochastic travel times', *Transportation Research Part C: Emerging Technologies*, vol. 48, pp. 66–83, 2014.

Taş, D., Jabali, O. and Van Woensel, T. 'A vehicle routing problem with flexible time windows', *Computers and Operations Research*, vol. 52, no. PART A, pp. 39–54, 2014.

Biographies

Mahdi Jalilvand is an M.S. degree student of Industrial Engineering at Shahed University. He received his B.Sc. degree from Bu-Ali Sina University in 2017. His research interests are facility, transportations and supply chain.

Mahdi Bashiri is a Professor of Industrial Engineering at Shahed University. He holds a B.Sc. in Industrial Engineering from Iran University of Science and Technology, M.Sc. and Ph.D. from Tarbiat Modarres University. He is a recipient of the 2013 young national top scientist award from the Academy of Sciences of the Islamic Republic of Iran. He serves as the Editor-in-Chief of Journal of Quality Engineering and Production Optimization published in Iran and the Editorial Board member of some reputable academic journals. His research interests are Facilities Planning, Stochastic Optimization, Meta-heuristics, and Multi-response Optimization. He published about 10 books and more than 190 papers in reputable academic journals and conferences.