

# A multi-product multi-supplier combined inventory and suppliers' selection model

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## Abstract

Figuring out the appropriate procurement model for manufacturing environment is a challenge due to the diversity of procurement models' configurations. A questionnaire was conducted with a number of procurement specialties of the manufacturing sector to characterize the model that fits them the best. Based on the questionnaire result, a constrained mathematical model is developed to optimize the total cost including procurement, holding, ordering and shortage costs. This model includes several factors affect the optimum order quantity and the suppliers' share factor. Finally, a series of simulation experiments are conducted to validate the proposed model under various system configurations.

## Keywords

Procurement models; inventory models; suppliers' selection; payment term; orders quality

## 1. Introduction

Procurement is the cornerstone for the internal success of any industrial organization (Tassabehji & Moorhouse, 2008). It plays a crucial role in improving the organization's efficiency and effectiveness because of their effect on cost reduction, profitability, and flexibility. Procurement is defined as the acquisition of goods or services at the lowest possible total ownership cost with right quantity and quality, at the right time and the right place and from the right source. Handling these multi-criteria concurrently require a structured approach. Configuring the procurement model to fit the business environment is a challenge due to the diversity of the business environments in different countries and different industries (Tatsis, Mena, Wassenhove, & Whicker, 2006).

In section two, a brief literature review of procurement models is introduced. Based on the questionnaire result presented in section three, the proposed procurement and inventory model is mathematically derived in section four. Finally, a case study is introduced to validate the model with real industry data.

## 2. Previous Studies

The literature introduced several procurement models addressing a broad spectrum of business needs and business environments (Bera, Rong, Mahapatra, & Maiti, 2009; H.-C. Chang, Yao, & Ouyang, 2006; H. Chang, Yao, & Ouyang, 2004; Hazra & Mahadevan, 2009; C. Huang, 2010; Kawtummachai & Hop, 2005; Ouyang & Yao, 2002; Qi, 2007; Yang, Ronald, & Chu, 2005; Zhang & Zhang, 2011). The procurement problem has two dimensions, which are the suppliers' selection that answers the "who question" and orders' quantity that answers the "what question" (Hazra & Mahadevan, 2009).

There are various models addressing the suppliers' selection problems (H.-C. Chang, 2011; He, Wen Jiang, & Felix S Chan, 2017; Hlioui, Gharbi, & Hajji, 2017; B. Huang, Bai, Roy, & Ma, 2018; Kawtummachai & Hop, 2005; Keshavarz Ghorabae, Amiri, Zavadskas, Turskis, & Antucheviciene, 2017; Liao & Rittscher, 2007; Mirzaee, Naderi, & Pasandideh, 2018; Qi, 2007; Rezaei & Davoodi, 2008; Rezaei & Salimi, 2012; Wahab & Jaber, 2010;

Zhang & Zhang, 2011). In addition, Minner (2003) gave an extensive review of this issue; he reviewed 92 papers, which were classified based on the supplier's selection criteria. The classification criteria were prices (net prices, discounts, and payment conditions), quality, contracts, and supplier service (delivery time, lead time, flexibility, variability, and reliability).

Kawtummachai & Hop (2005) introduced a suppliers' selection model with order allocation procedures and multiple prices while maintaining a specified service level. The impact of supplier capacity is addressed by several researchers (Manerba, Mansini, & Perboli, 2018; Mohammadivojdan & Geunes, 2018; Qi, 2007). The former procurement models implicitly assume that all receiving items are perfect; although procurement models, which consider the quality of the purchased items, are closer to real-world problems (Rezaei & Salimi, 2012). Liao & Rittscher, (2007) addressed this issue by introducing a model in which the defect units are refunded to their supplier. On the other hand, Rezaei & Davoodi (2007) developed a constrained model with supplier's limited capacities. They assumed that imperfect items could be used at a discounted price; their model objective is to maximize the total profit of sold good and defect items after subtracting them from the total cost of purchasing, transaction and screening cost. Wahab & Jaber (2010) and H.-C. Chang (2011) developed a model involving imperfect items at a discounted price; in addition, Chang's model adopted lot splitting shipment policy and ordering cost variation. Zhang & Zhang (2010) addressed a model like Qi's model while considering suppliers' minimum order quantity (MOQ) constraints and supplier selection cost.

Consequently, there are many models handling the suppliers' selection that consider multiple aspects as order splitting among suppliers, suppliers' capacity, minimum order quantity required by suppliers, handling different unit prices, ordering cost variations, and purchased items quality including multiple aspects as receiving inspection, accepting order with defect units and refusing the defect units. Moreover, the availability of supplier credit facility, which allows the procurement department to delay payment for a certain time, was neglected by the mentioned researchers.

The order's quantity dimension was also heavily investigated by many inventory researchers (Bera et al., 2009; Chandra & Grabis, 2008; H.-C. Chang et al., 2006; H. Chang et al., 2004; Chou, Julian, & Hung, 2009; C. Huang, 2010; Kumar, Kumar, & Maiti, 2005; Lin, 2008; Lo, Liou, Wang, & Tsai, 2018; Ouyang & Yao, 2002; Varnas, Balfors, & Faith-ell, 2009; Yang et al., 2005).

Procurement researchers introduced a mixed inventory model for a single SKU (product) involving ordering, holding, product shortage resulting in backorder (B) and lost sales (L), and lead time crashing cost; they compensated clients in case of shortage (H.-C. Chang et al., 2006; H. Chang et al., 2004; Ouyang & Yao, 2002). Yang et al. (2005) added the reorder point under the effectiveness of time value regarding money to the former model. Lin (2008) formulated a similar model as H.-C. Chang et al. (2006) but with a periodic review. Chou et al. (2009) developed a procurement model involving ordering, holding, shortage with full backorder, and procurement cost to find the optimal solution of lead time, order quantity, and safety factor. Bera et al. (2009) reformulated Yang et al.'s model for multi-item to find the optimal order quantity and the optimal value of safety stock under the budget constraint and random lead time; in addition, full inventory backorder was considered.

The previous models implicitly assumed that there was no delayed payment. However, delayed payment reduces the capital amount invested in inventory during the credit time. Suppliers usually offer payment delay period to encourage buyers to increase their lot size. Huang proposed a procurement model considering payment delay period and the integrated supply chain cost (C. Huang, 2010).

Recently, the environment impact of supplier selections and purchasing decisions has attracted several researchers (Arabsheybani, Paydar, & Safaei, 2018; Babbar & Amin, 2018; Gören, 2018; Hamdan & Cheaitou, 2017b, 2017a; Lamba, Singh, & Mishra, 2018; Moheb-Alizadeh & Handfield, 2017; Mousakhani, Nazari-Shirkouhi, & Bozorgi-Amiri, 2017; Yu, Yang, & Chang, 2018).

Such wide spectrum of procurement models presents a challenge for procurement professionals to pick the appropriate model that utilizes only the data they can estimate, provides the answers they need, and matches the settings of their business. To overcome such challenges, the procurement professionals must be consulted about their business setting, the data that they can estimate, and the decisions that they should take to optimize their business.

### 3. The Questionnaire

Different factors that can be encapsulated into a procurement model. A questionnaire was designed to prioritize those factors for the manufacturing sector. The questionnaire consists of twenty-five classification questions divided into two sections; each section addresses one of the former dimensions. The questionnaire starts by assessing the need for multiple suppliers. This factor is addressed through two questions about the number of SKUs that are supplied by more than one supplier and the maximum number of suppliers for the same SKU. Then, one questions about the top three factors affecting suppliers' selection. The need for order splitting among suppliers is assessed

through two questions about the importance of having a tool to optimize the order size and to split the order among suppliers for the same SKU. Then, MOQ and limited suppliers' capacity are addressed by three questions, followed by three questions about quality addressing receiving inspection, accepting orders with defect units and handling defect units. The Twelfth question is about the order receiving duration. Handling multiple prices is addressed through two questions inquiring about the need to handle different unit prices and multiple order costs.

The first factor of the second dimension, which is the holding cost, is addressed through two questions about having different holding cost for different SKUs and handling limited inventory budget, followed by four questions about product shortage addressing allowing shortage, allowing backorder in case of shortage, compensating customer in case of shortage and replacing the out of stock SKU by another SKU. Lead-time is addressed through two questions about lead-time consistency among different SKUs and different suppliers. Reorder point is addressed through one question. Finally, payment term consistency among different SKUs and different suppliers is addressed by two questions. A pilot test of the questionnaire was performed with one procurement specialist. Based on his feedback, some questions were restated to be more precise and more understandable.

Three hundred industrial companies were contacted to discuss the possibility of conducting a personal visit to fill out the questionnaire. Thirty-two companies responded positively, representing 10.67% response rate. This sample size was considered enough to achieve the goal of the research by getting a casual trend of procurement models that are suitable for the manufacturing sector. However, getting sharp and quantifiable results about the sector procurement preferences could be an interesting point for further research. Fortunately, filling out the questionnaire through direct discussion (interviews) allowed the research team to obtain a deeper understanding of a complicated problem such as the one under investigation. Direct contact questionnaire filling for complex problems was supported by some researchers as Balfors & Faith-ell (2009).

Procurement professionals depend on more than one supplier for sourcing the same SKU, as reported by 81% of the respondents. Concerning the acceptable maximum number of suppliers for the same SKU, half of the respondents stated that it should be three, while the third of them reported that they could accommodate more than four suppliers for the same SKU. It can be concluded that although three is recommended as the maximum number of suppliers for the same SKU, the needed model should have no limitation for the number of suppliers. The respondents were asked to pick the top three factors affecting their suppliers' selection, price (97%) and quality (84%) got the highest rank among all respondents; the lead-time was also highly ranked by 69%. Surprisingly, only 22% of the respondents considered the payment term as a significant factor for supplier selection. Other factors as minimum order quantity, ordering cost and long-term contracts were not highly ranked by the respondents as critical supplier selection criteria. Companies with a limited number of suppliers per SKUs justified it by seeking to build a long-term supplier relationship to improve the service quality.

The need for a tool that optimizes the orders of the same SKU among multiple suppliers was enforced by 91% of respondents, while 86% described such tool "a must have" one. Moreover, 76% of respondents reported that they are used to split the same SKU orders among suppliers based on their quality and price. This result matches the research reported by Qi (2007). Therefore, it can be concluded that there is a need for a procurement model that can be used to optimize passing orders of the same SKU to more than one supplier.

More than three-quarters of the respondents (78%) reported that most of their suppliers have unlimited capacity compared to their order size. Regarding the MOQ, two-thirds of the respondents stated that their suppliers seldom force a minimum order quantity. It should be considered that the majority of the surveyed companies are medium and large enterprises, which usually ask for substantial quantities exceeding the suppliers' MOQ. There is a significant correlation between respondents with limited capacity suppliers and those who reported the need for MOQ ( $r = 0.82$ ). It is concluded that the surveyed companies are not restricted by suppliers' MOQ, because their order size exceeds the suppliers' MOQ. Therefore, there is no need for a procurement model that handles suppliers' MOQ and supplier s' limited capacity.

About three-quarters (72%) of the respondents stated that they receive their orders over a short period. Moreover, 28% of the respondents reported that they receive their order as one shipment. It is obvious that a procurement model with instantaneous replenishment can fit the need of the manufacturing sector.

All the respondents reported that they carry the receiving inspection for some or all their SKUs. Two-third of the respondents reported that they never accept to receive any defect units. Moreover, 60% of the respondents refunded only defect units to their suppliers. Therefore, it is quite beneficial to have a procurement model that considers the effect of returning defect units.

More than three-quarters (78%) of the respondents reported that they source their SKUs with more than one price from more than one supplier at the same time. Moreover, about two-thirds (62%) of the respondents reported that the order cost is not consistent among different SKUs even from the same supplier There is a correlation between the respondents who reported different prices for the same SKU and those who reported different ordering cost of

different SKUs from the same supplier ( $r = 0.71$ ). It can be concluded that the manufacturing sector needs a procurement model that handles multiple prices and multiple ordering costs of the same SKU.

Majority of the respondents (78%) reported having different holding cost; moreover, about the third of them (25%) reported different holding cost for almost all of their SKUs. Therefore, it might be quite beneficial to have a procurement model that considers different holding cost for different SKUs. The survey results showed that 72% of the respondents reported that it is the nature of the business to limit the budget for the inventory value. Although the remaining (28%) Of the respondents reported that they didn't allocate budget for inventory, it can be concluded that the sector needs a procurement model that addresses the limited budget for inventory value.

Procurement professionals usually encounter a shortage in some or even all SKUs, as reported by 72% of the respondents. The remaining 28% of the respondents reported that missing some SKUs might result in stopping their production lines, what would be too costly to afford. Regarding backorders, 87% of the respondents, who encounter a shortage, stated that they have backorders for some and almost all SKUs. The respondents were asked about compensating their clients in case of shortage; this hypothesis was supported by 61% of the respondents for some and all SKUs. It can be concluded that there is a need for a procurement model that implements shortage resulting in both of backorders and lost sales and accommodates shortage penalty cost.

Regarding partial replacement of one SKU by another SKU in case of shortage, only 3% of the respondents reported that almost all of their SKUs are replaceable. About 47% of the respondents reported that none of the SKUs is replaceable. However, 50% of the respondents reported that some SKUs are replaceable, and some are not; therefore, no concrete conclusion can be driven by the importance of accommodating such hypothesis in procurement models. It is suggested that a deeper investigation of the effect of products' irreplaceability across different business sizes and different industries may be an interesting goal for future research.

Suppliers usually have different inventory, production, and shipping policies resulting in different lead-time even for the same SKU; this hypothesis was supported by 72%. In addition, having different payment terms for different SKUs was supported by 84% of the respondents. On the other hand, 10% of the respondents reported that they succeeded to force the supplier to follow their payment conditions. Forcing the suppliers to follow the enterprise's procurement conditions may not be applicable to small and medium ones, because they may not have enough buying power to control the market. Therefore, it might be quite beneficial to have a procurement model that considered different lead-time for different suppliers and different SKUs and delayed payments diversity. About 81% of the respondents reported that they use fixed reorder points for each of their SKUs even if it is ordered from more than one supplier. Therefore, there is a need for a procurement model that considers a fixed reorder point.

#### 4. The proposed model

Based on the results of the questionnaire, a constrained mathematical model is developed to minimize the total cost involving procurement, holding, ordering, and shortage costs while stratifying the given conditions. This model is consisted of a non-linear objective function and several bounding linear and nonlinear constraints and can be used to determine the optimal orders quantity from the available suppliers and determine the suppliers' share factor. The suppliers' share factor is the percentage of supplier participation in the total demand of a single SKU per the cycle time.

The product procurement cost can be calculated using (1) (Varnas et al., 2009).

$$PC = C_p D \quad (1)$$

Where

PC: Total purchasing cost,

$C_p$ : Purchasing cost for one unit,

D: Average demand per cycle.

To accommodate having different purchasing cost from different suppliers for the same SKU, the suppliers' share factor is used to calculate the total weighted purchasing cost as shown in (2).

$$\therefore PC = D \sum_{j=1}^m C_{pj} \beta_j \quad (2)$$

Where:

$C_{pj}$ : Purchasing cost of one unit from supplier (j),

$\beta_j$ : Suppliers' share factor (a decision variable),

m: Number of available suppliers,  $j=1: m$ .

Both of Ouyang & Yao (2002) and (H. Chang et al., 2004) calculated the expected average holding cost in the case of shortages involving back order and lost sales as per (3) and shown in figure (1).

$$HC = c_{hi} \left[ \frac{Q_i}{2} + SS_i + (1 - \alpha_i) E(X_i - r_i)^+ \right] \quad (3)$$

Where:

HC: Total holding cost,

$c_{hi}$ : Inventory holding cost per one unit,

Q: Order quantity (a decision variable),

SS: Safety factor,

$\alpha$ : Fraction of back-ordered demand during the stock-out period

$$0 \leq \alpha_i \leq 1,$$

$E(X - r)^+$ : Expected number of shortage,

i= SKU index.

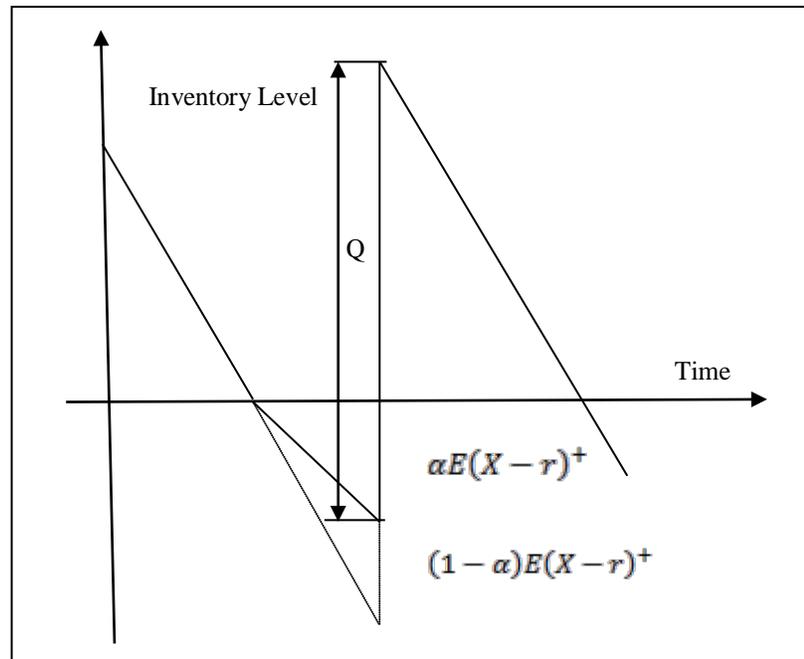


Figure 1. Inventory model with partial back orders

Based on the questionnaire result, the received order is not totally accepted because it may contain a percentage of defects ( $p$ ). The defect items are instantaneously removed from the received order and refunded to their suppliers.

Therefore, the received order quantity is reduced by the factor  $(1 - p)$ . Therefore, the suppliers' share factor is used to calculate the total weighted average holding. Hence, (3) is reformulated to be applicable to the proposed model.

$$HC = c_h \left[ \frac{\sum_{j=1}^n Q_j (1 - p_j)^{\theta_j}}{2} + SS + (1 - \alpha) E(X - r)^+ \right] \quad (4)$$

The safety stock is defined as the difference between the reorder point and the expected demand during the lead time (Bera et al., 2009; H.-C. Chang et al., 2006; H. Chang et al., 2004; Lin, 2008; Ouyang & Yao, 2002; Varnas et al., 2009) and can be calculated as per equation (5).

$$SS_i = r_i - E(x_i) = k_i \sigma_{x_i} \quad (5)$$

Where:

$E(x)$ : Expected demand during lead time,

$r$ : Reorder point,

$k$ : Safety factor,

$\sigma_x$ : Standard deviation of lead time demand.

Chopra & Meindl (2016) calculated the safety stock for a single SKU as per (6).

$$SS = F_z^{-1}(CSL) \sigma_x \quad (6)$$

Where:

CSL: Cycle service level,

$F_S^{-1}$ : The inverse of the cumulative standard normal distribution.

Using equations (5) and (6), it can be concluded that the optimal safety factor should equal the inverse of the cumulative standard normal distribution of optimal cycle service level:

$$\therefore k = F_S^{-1}(CSL) \quad (7)$$

CSL is equal to the probability of having no stock out in a replenishment cycle(Chopra & Meindl, 2016) and the optimal CSL can be calculated as:

$$CSL^* = \frac{C_u}{C_u + C_o} \quad (8)$$

Where:

$C_u$ : Under-stock cost,

$C_o$ : Over-stock cost.

The cost of under-stocking is the margin lost by a firm for each lost sale (lost profit) because there is no inventory on hand (Chopra & Meindl, 2016).

$$\therefore C_u = \text{sales price} - \text{purchasing cost} = S_p - C_p \quad (9)$$

Where:

$S_p$ : Selling price of one sold unit.

As previously mentioned, the purchasing cost should be multiplied with suppliers' share factor to calculate the total weighted average purchasing cost for the available suppliers. Then, the under-stock cost is calculated per (10).

$$\therefore C_u = S_p - \sum_{j=1}^m C_{pj} \vartheta_j \quad (10)$$

The cost of over-stock is the loss occurred by a firm for each unsold unit at the end of the selling cycle (Chopra & Meindl, 2016).

$$\therefore C_o = \text{holding cost on replenishment cycles} = \frac{\text{annual holding}}{\text{number of orders}} = \frac{C_h}{N} \quad (11)$$

Where:

N: Total number of orders per cycle time.

The number of orders from each supplier (j) ( $N_j$ ) of a single SKU per the cycle time can be calculated per (12).

$$N_j = \frac{q_j}{Q_i} \quad (12)$$

The total number of orders for a single SKU from multi suppliers can be calculated per (13).

$$\therefore N = \sum_{j=1}^m N_j \quad (13)$$

From (12) and (13), the total number of orders as a function of D and Q is calculated per (17).

$$\therefore N = \sum_{j=1}^m \frac{q_j}{Q_i} \quad (15)$$

$$\therefore D_j = D * \vartheta_j \quad (16)$$

$$\therefore N = D \sum_{j=1}^m \frac{\vartheta_j}{Q_i} \quad (17)$$

Equation (17) is substituted in (11) to calculate the appropriate over-stock cost for the proposed model

$$\therefore C_o = \frac{C_h \sum_{j=1}^m \frac{\vartheta_j}{Q_i}}{D \sum_{j=1}^m \frac{\vartheta_j}{Q_i}} \quad (18)$$

Equations (10) and (18) are substituted in (8) to calculate the optimal cycle service level.

$$\therefore CSL^* = \frac{C_u}{C_u + C_o} = \frac{S_p - \sum_{j=1}^m C_{pj} \vartheta_j}{S_p - \sum_{j=1}^m C_{pj} \vartheta_j + \frac{C_h \sum_{j=1}^m \frac{\vartheta_j}{Q_i}}{D \sum_{j=1}^m \frac{\vartheta_j}{Q_i}}} \quad (19)$$

Subsequently, the safety factor can be calculated per (20) after substituting by (19) in (7).

$$k = F_S^{-1}(CSL^*) = F_S^{-1} \left( \frac{S_p - \sum_{j=1}^m C_{pj} \vartheta_j}{S_p - \sum_{j=1}^m C_{pj} \vartheta_j + \frac{C_h \sum_{j=1}^m \frac{\vartheta_j}{Q_i}}{D \sum_{j=1}^m \frac{\vartheta_j}{Q_i}}} \right) \quad (20)$$

The standard deviation of lead-time demand was calculated per (21) (Chopra & Meindl, 2016).

$$\sigma_x = \sqrt{L_j \sigma^2 + D^2 \sigma_{L_j}^2} \quad (21)$$

Where:

$L_j$ : Lead time of supplier j,

$\sigma^2$ : Variance of demand,

$\sigma_{L_j}^2$ : Variance of lead time.

The lead-time is multiplied with suppliers' share factor to get the total weighted lead time of all the available suppliers to supply the same SKU.

$$\sigma_x = \sqrt{\sigma^2 \sum_{j=1}^m \theta_j L_j + D^2 \sum_{j=1}^m \sigma_{L_j}^2} \quad (22)$$

Equations (20) and (22) are substituted in (6) to calculate the safety stock for a single SKU from multi suppliers.

$$\therefore SS = F_s^{-1} \left( \frac{s_p - \sum_{j=1}^m c_{p_j} \theta_j}{s_p - \sum_{j=1}^m c_{p_j} \theta_j + \frac{c_h \sum_{j=1}^m \theta_j}{D}} \right) \sqrt{\sigma^2 \sum_{j=1}^m \theta_j L_j + D^2 \sum_{j=1}^m \sigma_{L_j}^2} \quad (23)$$

The expected shortage quantity at the end of the cycle was calculated as a function of safety factor per (24) (Annadurai & Uthayakumar, 2009; C. Chang & Lo, 2009; Chopra & Meindl, 2016; Yang et al., 2005).

$$E(X - r)^+ = \sigma_x [f_s(k)] - SS[1 - F_s(k)] \quad (24)$$

Where:

$f_s$  and  $F_s$ , are the standard normal probability density function (pdf) and the cumulative distribution function (CDF) respectively. Those are originated in the standard normal distribution tables.

Then, equations (22), (20) and (23) are substituted in (24) to be appropriate in the proposed model.

$$E(X - r)^+ = \sqrt{\sigma^2 \sum_{j=1}^m \theta_j L_j + D^2 \sum_{j=1}^m \sigma_{L_j}^2} \left\{ F_s^{-1} \left( \frac{s_p - \sum_{j=1}^m c_{p_j} \theta_j}{s_p - \sum_{j=1}^m c_{p_j} \theta_j + \frac{c_h \sum_{j=1}^m \theta_j}{D}} \right) \right\} - \left\{ F_s^{-1} \left( \frac{s_p - \sum_{j=1}^m c_{p_j} \theta_j}{s_p - \sum_{j=1}^m c_{p_j} \theta_j + \frac{c_h \sum_{j=1}^m \theta_j}{D}} \right) \right\} \sqrt{\sigma^2 \sum_{j=1}^m \theta_j L_j + D^2 \sum_{j=1}^m \sigma_{L_j}^2} \left[ 1 - \left( \frac{s_p - \sum_{j=1}^m c_{p_j} \theta_j}{s_p - \sum_{j=1}^m c_{p_j} \theta_j + \frac{c_h \sum_{j=1}^m \theta_j}{D}} \right) \right] \quad (25)$$

The order cost is the cost of dealing with replenishing the inventory stock. It is typically evaluated as the dollar amount per order. The order cost also depends on the number of orders per cycle (H.-C. Chang et al., 2006; H. Chang et al., 2004; Ouyang & Yao, 2002; Varnas et al., 2009).

$$OC = C_k \frac{D}{Q} \quad (26)$$

Where:

OC: Total ordering cost,

$C_k$ : Ordering cost per order.

To accommodate multi suppliers, equation (26) is reformulated as:

$$\therefore OC = D \sum_{j=1}^m C_{k_j} \theta_j / Q_j \quad (27)$$

Some of the demand will not be satisfied due to insufficient inventory and intolerant customers. Therefore, shortage cost or stock-out cost is the total of all costs associated with shortage units. The effect of backorder, lost sales, and shortage penalty costs are considered by several researchers. They calculated the shortage cost per (30) (H.-C. Chang et al., 2006; H. Chang et al., 2004; Lin, 2008; Ouyang & Yao, 2002).

$$SC = \frac{D}{Q} [C_s + \pi(1 - \alpha)] E(X - r)^+ \quad (28)$$

Where:

SC: Total shortage cost,

$C_s$ : Penalty cost,

$\pi$ : Lost profit.

To consider multi suppliers sourcing, equation (28) is reformulated as follows:

$$SC = \sum_{j=1}^m \frac{D \theta_j}{Q_j} [C_s + \pi(1 - \alpha)] E(X - r)^+ \quad (29)$$

$$\because \pi = S_p - \sum_{j=1}^m c_{p_j} \theta_j$$

$$\therefore SC = D [C_s + (S_p - \sum_{j=1}^m c_{p_j} \theta_j) (1 - \alpha)] \sum_{j=1}^m \frac{\theta_j}{Q_j} E(X - r)^+ \quad (30)$$

The proposed model split the orders between the available suppliers. Moreover, it accumulates the percentage of quantities that are taken from the available suppliers for the same SKU per the cycle time. Therefore, the total suppliers share factor for a single SKU during the cycle time must be equal to 1.

$$\sum_{j=1}^m \theta_j = 1 \quad (31)$$

To keep the supplier active, some orders should be assigned to him during the cycle. Therefore, an arbitrary lower limit should be assigned to suppliers' share factor, which should satisfy (32) to keep a feasible solution.

$$\sum_{j=1}^m \theta_j < 1 \quad (32)$$

Where:

$\theta_j$ : Lower-bound of suppliers' share factor.

According to Bera et al. (2009), the budget constraint was formulated in which the total purchasing cost was less than or equal to the available budget.

$$[BC = \sum_{i=1}^n C_{p_i}(Q_i + r_i - E(x_i))] \leq B \quad (33)$$

Where:

BC: Budget constraint,

B: Available budget.

Equation (33) is reformulated to consider multiple suppliers, defective goods return, sales during credit time, backorder and lost sales as follows:

$$[BC = C_{p_j}[Q_j(1 - p_j) - E(Y(t_j))] - \alpha E(X - r)^+ + SS] \leq B \quad \forall j \quad (34)$$

Therefore, the total cost can be minimized using (3), (4), (23), (25), (27), (30), (31), (32) and (34) as per the below model

$$\text{Min } TC (= PC + HC + OC + SC)$$

S.T.

$$\begin{array}{ll} \sum_{j=1}^m \theta_j = 1 & \forall j \\ \sum_{j=1}^m \theta_j < 1 & \forall j \\ BC \leq B & \forall j \\ Q_j \geq 0 & \forall j \quad \text{and integer} \\ \theta_j \in [\theta_j, 1] & \forall j \end{array}$$

Based on the above model, a three-stage algorithm shown in figures (2) and (3) is used to find the needed decisions variables. The first phase is to address the selected suppliers' list. During the second phase, the selected suppliers' list is queried to assign shares to the selected suppliers. After that, the total cost is calculated. During this phase, the optimum orders quantity and the suppliers' share factor are calculated to minimize the total cost under the given conditions.

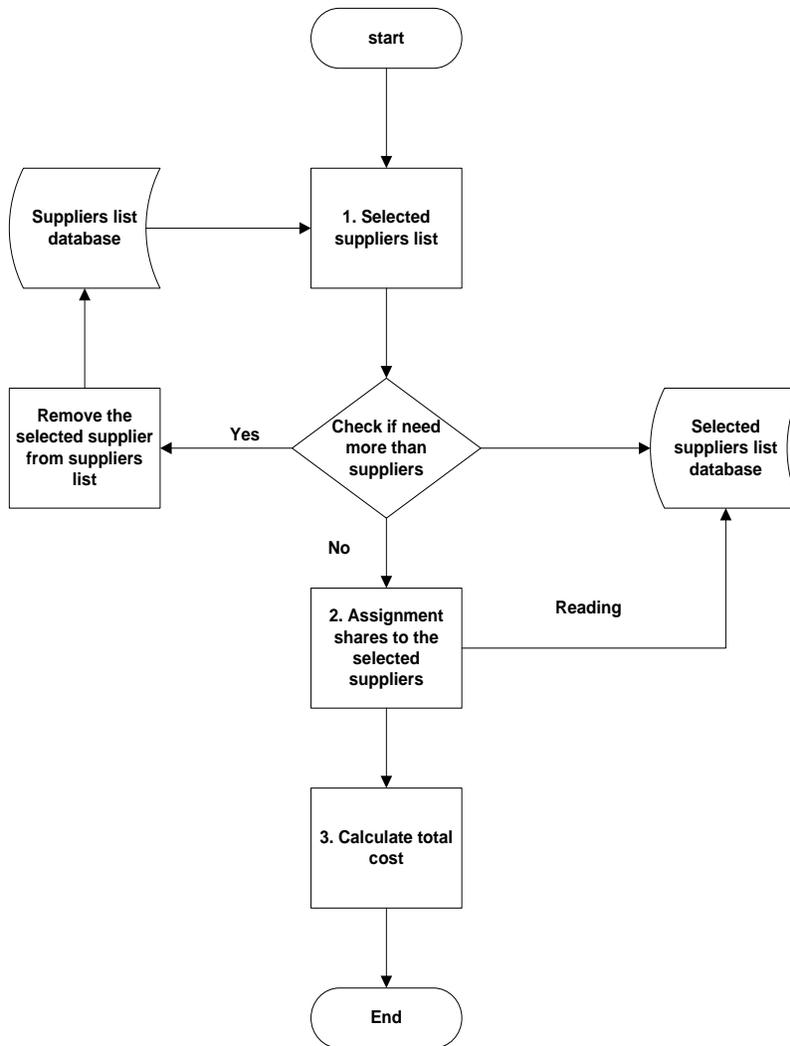


Figure 2: Flowchart of the proposed model algorithm.

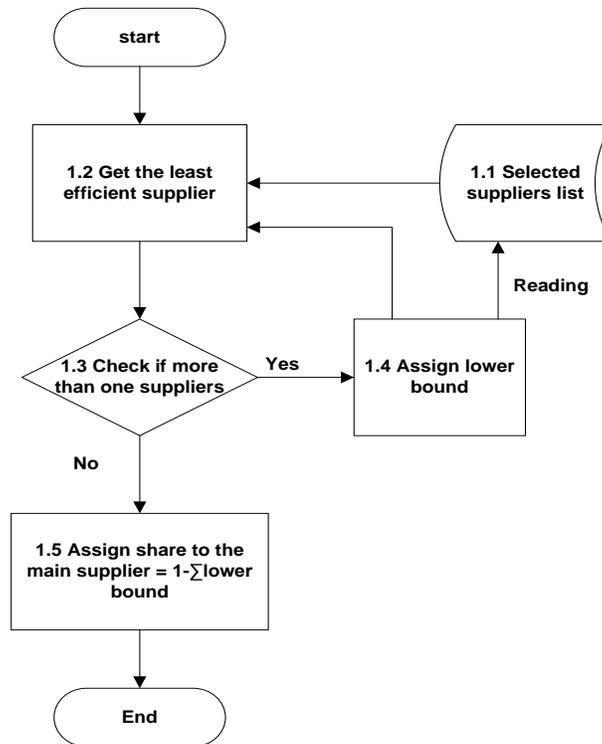


Figure 3: Flowchart of assigning shares to the selected suppliers.

## 5. Case Study

To validate the proposed model, a set of MATLAB functions is developed to simulate the proposed model using MINLP (mixed integer nonlinear programming) and FMINCON solver. The model is tested using data obtained from a food manufacturer in Cairo, Egypt. The company uses 17 main raw material items supplied by 24 suppliers. The company policy is to keep two active suppliers for each of its main raw material items. The company-planning manager chooses 10% as the minimum order share a supplier can have to be considered as an active supplier for an item.

Based on final product bill of material (BOM) (as shown in figure 4), the shortage in any raw material of the company results in a shortage of the final product due to the below two assumptions:

There is no replacement of raw material.

There is no significant stock in the final product due to its short shelf time.

Therefore, the raw material lost profit is calculated based on the lost profit of the final products. Data obtained from the company is used to run the MATLAB functions to get the optimal supplier share and the optimal order cost of each of the 17 items, as shown in table III. The model successfully selected the most economical suppliers and assigned them the largest feasible share of the item demand, while assigning the forced minimum share to the backup supplier. Then the system calculated the optimal order quantity, the optimal reorder point, the optimal safety stock, and the optimal cycle service level. Comparing the actual costs in 2013 to the costs that would occur if the company used the data from the model revealed that the company would save 17.45% of its total inventory cost in 2013 by using the proposed model.

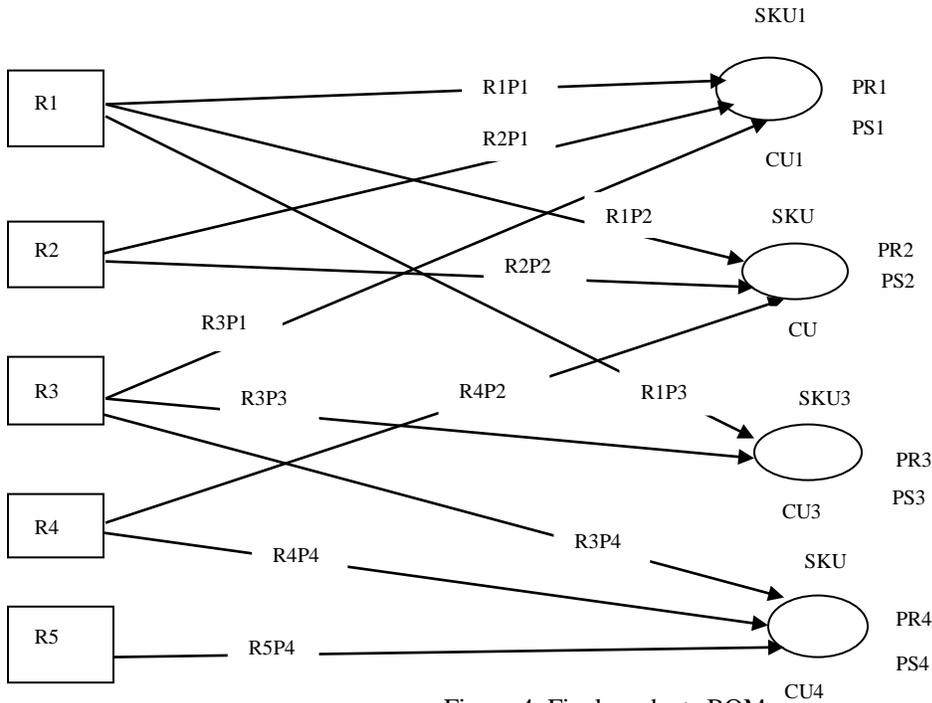


Figure 4: Final products BOM.

A second case study is created by doubling the lead-time of the main supplier while keeping the lead-time of the backup supplier unchanged. The new data is re-used with the MATLAB functions to get the optimal procurement policies for each of the 17 items as shown in table IV. Comparing Table (1) and Table (2) shows that the model responded logically by getting higher optimal total cost when the lead-time lead was increased. Moreover, the model was able to alternate its selection of the main supplier when he became less economical than the backup one (items number 5, 6, and 12).

Table1. The optimal solution of the default data for the system.

	Main supplier			Backup supplier			Optimal total cost
	Code	Q*	0.9	#	Q*	%	
							235,800
1	A	4,791	0.9	B	1,996	0.1	235,800
2	A	4,218	0.9	C	1,821	0.1	81,900
3	A	3,634	0.9	D	1,651	0.1	116,800
4	A	1,470	0.9	C	900	0.1	24,800
5	E	95,380	0.9	F	15,870	0.1	1,307,300
6	G	30,274	0.9	H	6,806	0.1	318,100
7	J	9,042	0.9	M	2,054	0.1	331,800
8	J	9,002	0.9	N	2,031	0.1	58,400
9	J	2,835	0.9	O	1,082	0.1	90,400
10	L	2,516	0.9	O	819	0.1	12,700
11	I	1,774	0.9	Q	940	0.1	90,100
12	I	936	0.9	R	934	0.1	41,900
13	S	15,559	0.9	T	5,555	0.1	650,600
14	U	17,853	0.9	V	4,718	0.1	187,100
15	W	3,613	0.9	X	1,621	0.1	110,600
16	Y	299	0.9	Z	243	0.1	26,900
17	Y	1,025	0.9	Z	656	0.1	9,600

Table 2. The optimal solution after changing the lead time.

	Main supplier	Backup supplier	Optimal total cost
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	Code	Q*	0.9	#	Q*	%	235,800
1	A	5,110	0.9	B	1,660	0.1	236,400
2	A	4,480	0.9	C	1,510	0.1	82,400
3	A	3,840	0.9	D	1,370	0.1	117,100
4	A	1,520	0.9	C	740	0.1	24,900
5	E	15,890	0.1	F	140,460	0.9	1,378,300
6	G	6,880	0.1	H	29,470	0.9	329,900
7	J	12,470	0.9	M	3,050	0.1	336,800
8	J	11,430	0.9	N	2,110	0.1	63,100
9	J	1,850	0.9	O	320	0.1	91,100
10	L	2,820	0.9	O	700	0.1	13,300
11	I	790	0.9	Q	600	0.1	91,100
12	I	230	0.1	R	940	0.9	49,900
13	S	16,900	0.9	T	4,750	0.1	653,700
14	U	20,880	0.9	V	4,450	0.1	196,400
15	W	3,840	0.9	X	1,340	0.1	111,200
16	Y	230	0.9	Z	180	0.1	27,100
17	Y	1,060	0.9	Z	550	0.1	9,700

## 6. Conclusion

Suppliers' selection and orders' quantity models based on continuous inventory review were heavily addressed by procurement literature. Eighteen different factors have been identified to characterize procurement models. A questionnaire with procurement specialists of thirty-two manufacturing companies has been used to select the critical factors for them. The questionnaire has illustrated that procurement specialists need a model that can handle orders splitting of the same item among multi suppliers while considering the quality and the lead-time of those suppliers. Having different holding cost for different items and different prices for the same items at different prices are considered critical aspects of their desired model. On the other hand, payment term, minimum order quantity, ordering cost, and long-term contracts have not been critically requested by the respondents.

The proposed model is tailored to small and medium-sized manufacturing companies. Therefore, characterizing the procurement model needed for non-manufacturing and large enterprises may be research-worthy.

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