with the purpose of obtain the minimal total cost solution. This solution satisfies the initial conditions and generates (m+n) – 1 cells. The initial feasible solution is gotten using some popular methods known as the North-west corner method, Total opportunity-cost method (TOM), Least cost method and the Vogel's Approximation method (VAM). (Kulkarni, 2010) developed a heuristic based algorithm to attain an initial feasible solution in obtaining the minimal total cost solution to the modified unbalanced transportation problem. In this paper, the Vogel's approximation method (VAM) is used to arrive at the initial basic feasible solution, this method involves calculating the penalty – i.e., the difference between the lowest cost and the second-lowest cost for each row and column in the matrix, before assigning the maximum number of units possible to the least-cost cell in the row or column with the largest penalty. Modified Distribution (MODI) method is used to test for optimality by enabling us to compute improvement for each unused square without drawing all the closed paths, because of this, it can often provide considerable time savings over other methods for solving transportation problems. The aim of the study is to look at Covenant University cafeterias with the aim of optimizing the cost of purchasing perishables from the various markets patronized.

2 Model Formulation

The transportation problem in this paper is to transport perishables (Tomatoes, Big-red pepper (Tatashey), Fresh pepper (Ata Rodo) and Onions) from multi-origin to multi-destination to determine an optimal solution to minimize the transportation cost.

2.1 Assumptions.

The following assumptions are made in this paper:

i. The movement of the perishables are always from the mentioned origin to the mentioned destination.
ii. The cost of moving items from one origin to a destination is always the same.
iii. The supply from the multi-origin are fixed and only apply to the multi destination.
iv. The supply of tomatoes from origin to destination is measured in basket.
v. The supply of big red pepper (tatashey) is measured in bags.
vi. The supply of fresh pepper (Ata Rodo) is measured in bags.
vii. The supply of onions is measured in bags.

2.2 Initial Tableau

Three (3) origins which represents the markets and Four (4) destinations representing the cafeterias in Covenant University will be considered. Table 1 shows the different origins and destinations. Other subsections show the tableau of each of the perishable foodstuff considered by the destinations, these tableaus show the movement of foodstuff from origin to destination in a month.

Origin		**Destination**	
Ota Market	O_1	Cafeteria One	D_1
Mile 12 Market	O_2	Cafeteria Two	D_2
Ile-Epo Market	O_3	Guest House Restaurant	D_3
		Post-Graduate Cafeteria	D_4

Table 1: Origins and destinations considered.

The supply quantity of the basic perishables (Tomatoes, Big-red pepper (Tatashey), Fresh pepper (Ata Rodo) and Onions) moved from the above-mentioned origins to the to the destinations are shown in Table 2 below:

Tomatoes					Big-red pepper (Tatashey)					Fresh pepper (Ata Rodo)					Onions				
Origin	Destination				Origin	Destination				Origin	Destination				Origin	Destination			
	D_1	D_2	D_3	D_4		D_1	D_2	D_3	D_4		D_1	D_2	D_3	D_4		D_1	D_2	D_3	D_4
O_1	25	10	2	3	O_1	6	5	3	2	O_1	16	8	3	3	O_1	5	3	1	1
O_2	50	20	8	12	O_2	17	7	11	5	O_2	41	20	6	8	O_2	22	17	5	6
O_3	35	20	6	9	O_3	15	4	9	2	O_3	32	16	6	6	O_3	15	13	3	4

Table 2: Supply distribution between the origins and the destinations

2.2.1 Tomatoes Tableau

Table 2 shows the initial tableau of the movement of Tomatoes from different origins to different destinations. Supply and demand are measured in baskets.

Origin/Destination	**D_1**	**D_2**	**D_3**	**D_4**	**Supply**
O_1	50	45	55	47	40
O_2	80	85	73	82	90
O_3	70	60	62	71	70
Demand	80	10	12	5	107/200

Table 3: Initial tomatoes tableau

2.2.2 Big-red pepper (Tatashey) Tableau

Table 3 shows the initial tableau of the movement of big-red pepper (tatashey) from the multi-origin to multi-destination. Supply and demand are measured in bags.

Origin/Destination	**D_1**	**D_2**	**D_3**	**D_4**	**Supply**
O_1	40	39	43	45	15
O_2	85	88	78	82	40
O_3	75	73	68	63	30
Demand	8	4	8	4	24/85

Table 4: Initial Big-red pepper (tatashey) tableau

2.2.3 Fresh pepper (Ata Rodo) Tableau

Table 4 shows the initial tableau of the movement of fresh pepper (ata rodo) from the multi-origin to multi-destination.

Origin/Destination	**D_1**	**D_2**	**D_3**	**D_4**	**Supply**
O_1	25	31	27	35	30
O_2	64	68	72	79	75
O_3	53	59	63	60	60
Demand	64	40	8	8	120/165

Table 5: Initial Fresh pepper (Ata Rodo) tableau

2.2.4 Onions Tableau

Table 5 shows the initial tableau of the movement of onions from the multi-origin to multi-destination.

Origin/Destination	**D_1**	**D_2**	**D_3**	**D_4**	**Supply**
O_1	23	26	25	28	10
O_2	65	66	61	68	50
O_3	51	47	50	53	35
Demand	12	4	2	2	20/95

Table 6: Initial Onions tableau

2.3 Adding dummies to the tableaus.

Because the above tableaus are unbalanced, there is need to balance these tableaus by adding dummies to them, this will enable us to further get the optimal solution. They are reflected in the tableaus below.

2.3.1 Tomatoes Tableau

Origin/Destination	**D_1**	**D_2**	**D_3**	**D_4**	**Dummy**	**Supply**
O_1	50	45	55	47	0	40
O_2	80	85	73	82	0	90
O_3	70	60	62	71	0	70
Demand	80	10	12	5	93	200

Table 7: Balanced Tomatoes tableau.

2.3.2 Big-red pepper (Tatashey) Tableau

Origin/Destination	**D_1**	**D_2**	**D_3**	**D_4**	**Dummy**	**Supply**
O_1	40	39	43	45	0	15
O_2	85	88	78	82	0	40
O_3	75	73	68	63	0	30
Demand	8	4	8	4	61	85

Table 8: Balanced Big-red pepper (Tatashey) Tableau

2.3.3 Fresh pepper (Ata Rodo) Tableau

Origin/Destination	**D_1**	**D_2**	**D_3**	**D_4**	**Dummy**	**Supply**
O_1	25	31	27	35	0	30
O_2	64	68	72	79	0	75
O_3	53	59	63	60	0	60
Demand	64	40	8	8	45	165

Table 9: Balanced Fresh Pepper (Ata Rodo) tableau

2.3.4 Onions Tableau

Origin/Destination	D_1	D_2	D_3	D_4	Dummy	Supply
O_1	23	26	25	28	0	10
O_2	65	66	61	68	0	50
O_3	51	47	50	53	0	35
Demand	12	4	2	2	75	95

Table 10: Balanced Onions tableau.

3. The Transportation Model

From the above tableaus, the transportation model can be formulated as follows:

$$Min \quad \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij} \qquad (1)$$

$$Subject\ to \sum_{j=1}^{n} x_{ij} \leq S_i,\ i = 1,2,\dots,m \qquad (2)$$

$$\sum_{i=1}^{m} x_{ij} \leq D_j,\ \ j = 1,2,\dots,n \qquad (3)$$

$$x_{ij} \geq 0 \ \ \forall i,j.$$

Where,

m =Total number of suppliers
n =Total number buyers
S_i =Supply quantity (in units)
D_j=Demand in units of the jth buyer
C_{ij} =Unit of transportation cost from supply to demand
x_{ij} =Number of units to be transported from i to j in order to attain minimal cost.

4. Problem Solution

4.1 Initial basic feasible solution using the Vogel's Approximation Model (VAM)

The Vogel's Approximation Model (VAM) will be used to determine the initial basic feasible solution for the tableaus of perishable goods moved from multi-origin to multi-destination.

A. Tomatoes tableau

Origin/Destination	D_1	D_2	D_3	D_4	Dummy	Supply
O_1	50 **35**	45	55	47 **5**	0	40
O_2	80	85	73	82	0 **90**	90
O_3	70 **45**	60 **10**	62 **12**	71	0 **3**	70
Demand	80	10	12	5	93	200

Table 12: Initial basic feasible solution for the movement of tomatoes from multi-origin to multi-destination

The total cost of moving tomatoes from the various origin to the various destination is calculated as:

$$35 \times 50 + 5 \times 47 + 90 \times 0 + 3 \times 0 + 12 \times 62 + 10 \times 60 + 45 \times 70 = 6479$$

Before one can proceed to test for optimality, we have to check for degeneracy, that is, if $(m + n) - 1$ is equal to the total number of occupied cells. From Table 12 above, $(m + n) - 1 = 7$ and the total number of occupied cells is 7 which implies that there is no degeneracy. Therefore, we proceed to testing for optimality using the MODI method.

B. Big-red pepper (Tatashey) tableau

Origin/Destination	D_1	D_2	D_3	D_4	Dummy	Supply
O_1	40 **8**	39 **4**	43 **3**	45	0	15
O_2	85	88	78	82	0 **40**	40
O_3	75	73	68 **5**	63 **4**	0 **21**	30
Demand	8	4	8	4	61	85

Table 13: Initial basic feasible solution for the movement of tomatoes from multi-origin to multi-destination.

The total cost of moving big-red pepper (tatashey) from the various origin to the various destination is calculated as:

$$8 \times 40 + 4 \times 39 + 3 \times 43 + 40 \times 0 + 21 \times 0 + 4 \times 63 + 5 \times 68 = 1197$$

Before one can proceed to test for optimality, we have to check for degeneracy, that is, if $(m + n) - 1$ is equal to the total number of occupied cells. From Table 13 above, $(m + n) - 1 = 7$ and the total number of occupied cells is 7 which implies that there is no degeneracy. Therefore, we proceed to testing for optimality using the MODI method.

C. Fresh pepper (Ata Rodo) tableau

Origin/Destination	D_1	D_2	D_3	D_4	Dummy	Supply
O_1	25 **22**	31	27 **8**	35	0	30
O_2	64	68 **30**	72	79	0 **45**	75
O_3	53 **42**	59 **10**	63	60 **8**	0	60
Demand	64	40	8	8	45	165

Table 14: Initial basic feasible solution for the movement of fresh pepper (ata rodo) from multi-origin to multi-destination.

The total cost of moving Fresh pepper (ata rodo) from the various origin to the various destination is calculated as:

$$22 \times 25 + 8 \times 27 + 30 \times 68 + 45 \times 0 + 8 \times 60 + 10 \times 59 + 42 \times 53 = 6102$$

Before one can proceed to test for optimality, we have to check for degeneracy, that is, if $(m + n) - 1$ is equal to the total number of occupied cells. From Table 14 above, $(m + n) - 1 = 7$ and the total number of occupied cells is 7 which implies that there is no degeneracy. Therefore, we proceed to testing for optimality using the MODI method.

D. Onions tableau

Origin/Destination	D_1	D_2	D_3	D_4	Dummy	Supply
O_1	23 **10**	26	25	28	0	10
O_2	65	66	61	68	0 **50**	50
O_3	51 **2**	47 **4**	50 **2**	53 **2**	0 **25**	35
Demand	12	4	2	2	75	95

Table 15: Initial basic feasible solution for the movement of fresh pepper (ata rodo) from multi-origin to multi-destination.

The total cost of moving onions from the various origin to the various destination is calculated as:

$$10 \times 23 + 50 \times 0 + 25 \times 0 + 2 \times 53 + 2 \times 50 + 4 \times 47 + 2 \times 51 = 726$$

Before one can proceed to test for optimality, we must check for degeneracy, that is, if $(m + n) - 1$ is equal to the total number of occupied cells. From Table 15 above, $(m + n) - 1 = 7$ and the total number of occupied cells is 7 which implies that there is no degeneracy. Therefore, we proceed to testing for optimality using the MODI method.

3.1 **Modified Distribution Method (MODI Method)**

In this study, two formulas are used in arriving at an optimal solution.
$u_i + v_j = c_{ij}$, is used to determine the values of u_i and v_j using the values in the occupied cells.
Also, $d_{ij} = c_{ij} - (u_i + v_j)$ is used to determine the best origin to destination route with the least transportation cost. Therefore, the cost with the most negative d_{ij} will be considered.
u_i and v_j are the row and column values to be determined while c_{ij} is the cost per unit and d_{ij} the opportunity cost of all the unoccupied cell.

3.1.1 Tomatoes Tableau

Using initial feasible solution gotten in Table 12 above, we calculate the values of u_i and v_j of the occupied cells, and the result is shown in the table below:

Origin/Destination	**D1**	**D2**	**D3**	**D4**	**Dummy**	**Supply**	u_i
O1	50 **35**	45	55	47 **5**	0	40	0
O2	80	85	73	82	0 **90**	90	20
O3	70 **45**	60 **10**	62 **12**	71	0 **3**	70	20
Demand	80	10	12	5	93	200	
v_j	50	40	42	47	-20		

Table 16: u_i and v_j from all occupied cells.

Calculating opportunity cost d_{ij} for all unoccupied cells where$d_{ij} = c_{ij} - (u_i + v_j)$, we have the table below.

Origin/Destination	**D1**	**D2**	**D3**	**D4**	**Dummy**	**Supply**	u_i
O1	50 **35**	45 [5]	55 [13]	47 **5**	0 [20]	40	0
O2	80 [10]	85 [25]	73 [11]	82 [15]	0 **90**	90	20
O3	70 **45**	60 **10**	62 **12**	71 [4]	0 **3**	70	20
Demand	80	10	12	5	93	200	
v_j	50	40	42	47	-20		

Table 17: d_{ij} of all unoccupied cells

Since all the $d_{ij} \geq 0$, the final optimal solution is achieved. The minimum transportation cost is

$$1750 + 235 + 0 + 0 + 744 + 600 + 3150 = 6479$$

3.1.2 Big Red Pepper (Tatashey) Tableau

Using initial feasible solution gotten in Table 13 above, we calculate the values of u_i and v_j of the occupied cells, and the result is shown in the table below:

Origin/Destination	**D1**	**D2**	**D3**	**D4**	**Dummy**	**Supply**	u_i
O1	40 **8**	39 **4**	43 **3**	45	0	15	0

O_2	85	88	78	82	0 **40**	40	25
O_3	75	73	68 **5**	63 **4**	0 **21**	30	25
Demand	8	4	8	4	61	85	
v_j	40	39	43	38	-25		

Table 18: u_i and v_j from all occupied cells.

Calculating opportunity cost d_{ij} for all unoccupied cells where $d_{ij} = c_{ij} - (u_i + v_j)$, we have the table below.

Origin/Destination	D_1	D_2	D_3	D_4	**Dummy**	**Supply**	u_i
O_1	40 **8**	39 **4**	43 **3**	45 [7]	0 [25]	15	0
O_2	85 [20]	88 [24]	78 [10]	82 [19]	0 **40**	40	25
O_3	75 [10]	73 [9]	68 **5**	63 **4**	0 **21**	30	25
Demand	8	4	8	4	61	85	
v_j	40	39	43	38	-25		

Table 19: d_{ij} of all unoccupied cells

Since all the $d_{ij} \geq 0$, the final optimal solution is achieved. The minimum transportation cost is

$$8 \times 40 + 4 \times 39 + 3 \times 43 + 40 \times 0 + 21 \times 0 + 4 \times 63 + 5 \times 68 = 1197$$

3.1.3 Fresh Pepper (Ata rodo) Tableau

Using initial feasible solution gotten in Table 14 above, we calculate the values of u_i and v_j of the occupied cells, and the result is shown in the table below:

Origin/Destination	D_1	D_2	D_3	D_4	**Dummy**	**Supply**	u_i
O_1	25 **22**	31	27 **8**	35	0	30	0
O_2	64	68 **30**	72	79	0 **45**	75	37
O_3	53 **42**	59 **10**	63	60 **8**	0	60	27
Demand	64	40	8	8	45	165	
v_j	25	31	28	32	-37		

Table 20: u_i and v_j from all occupied cells.

Calculating opportunity cost d_{ij} for all unoccupied cells where $d_{ij} = c_{ij} - (u_i + v_j)$, we have the table below.

Origin/Destination	D1	D2	D3	D4	Dummy	Supply	u_i
O1	25 **22**	31 [0]	27 **8**	35 [3]	0 [72]	30	0
O2	64 [64]	68 **30**	72 [8]	79 [10]	0 **45**	75	37
O3	53 **42**	59 **10**	63 [8]	60 **8**	0 [9]	60	27
Demand	64	40	8	8	45	165	
v_j	25	31	28	32	-37		

Table 21: d_{ij} of all unoccupied cells

Since all the opportunity cost $d_{ij} \geq 0$, the final optimal solution is achieved. The minimum transportation cost is

$$22 \times 25 + 8 \times 27 + 30 \times 68 + 45 \times 0 + 8 \times 60 + 10 \times 59 + 42 \times 53 = 6102$$

3.1.4 Onions Tableau

Using initial feasible solution gotten in Table 15 above, we calculate the values of u_i and v_j of the occupied cells, and the result is shown in the table below:

Origin/Destination	D1	D2	D3	D4	Dummy	Supply	u_i
O1	23 **10**	26	25	28	0	10	0
O2	65	66	61	68	0 **50**	50	-2
O3	51 **2**	47 **4**	50 **2**	53 **2**	0 **25**	35	-2
Demand	12	4	2	2	75	95	
v_j	23	49	52	55	2		

Table 22: u_i and v_j from all occupied cells.

Calculating opportunity cost d_{ij} for all unoccupied cells where $d_{ij} = c_{ij} - (u_i + v_j)$, we have the table below.

Origin/Destination	D1	D2	D3	D4	Dummy	Supply	u_i
O1	23 **10**	26 [7]	25 [3]	28 [3]	0 [28]	10	0
O2	65 [14]	66 [19]	61 [11]	68 [15]	0 **50**	50	-2
O3	51 **2**	47 **4**	50 **2**	53 **2**	0 **25**	35	-2
Demand	12	4	2	2	75	95	
v_j	23	49	52	55	2		

Table 23: d_{ij} of all unoccupied cells

Since all the $d_{ij} \geq 0$, the final optimal solution is achieved. The minimum transportation cost is

$$10 \times 23 + 50 \times 0 + 25 \times 0 + 2 \times 53 + 2 \times 50 + 4 \times 47 + 2 \times 51 = 726$$

4 Results and Discussion

Considering the outcomes of the analysis, Vogel's Approximation Method (VAM) gave the initial basic feasible solution. Each tableau had to be checked for degeneracy. All tableaus showed that the total number of occupied cells equals to $(m + n) - 1$. This ensured that an optimality test should be carried out on the 4 tableaus using MODI method. From table 17, to achieve total minimum cost of moving tomatoes, the following supply routes should be considered, O_1D_4 and O_3D_2. The supply from origin 2 should be ignored as much as possible. From table 19 to achieve total minimum cost of moving tomatoes, the following supply routes should be considered, O_1D_3 and O_3D_4. The supply from origin 2 should be ignored as much as possible. While in table 21, to achieve total minimum cost of moving tomatoes, the following supply routes should be considered, O_1D_3, O_2D_2, O_3D_4. And finally, table 23 shows supply route O_1D_1 can be used to get the minimum cost of moving onions from origin to destination. Routes O_3D_1, O_3D_3 and O_3D_4 can be used arbitrarily to move onions from destination 3 to origins 1, 3 and 4. This analysis can help the school plan and optimize perishable foodstuff delivery from multi-origin to multi-destination.

5 Conclusion

In this paper, cost minimization model was used to determine how the movement of perishable foodstuff (tomatoes, big red pepper (tatashey), fresh pepper (ata rodo) and onions) can be optimized from the various markets (origin) to the cafeterias around campus (destination). Vogel's Approximation method was used to get the initial basic feasible solution, whose result was phenomenally optimal. MODI method was used to obtain an optimal solution for each of the perishable foodstuff. From the result derived, it shows that the total cost of moving foodstuff from origin to destination can be minimized, therefore, improving the quality of food served in cafeterias at a lower rate. Covenant university can use this to manage and plan the purchase of foodstuff as it endeavours to become one of the best universities in the world by the year 2022.

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