Modelling of A36 Steel Plate Dynamic Response to Uniform Partially Distributed Moving Iron Load using Differential Transform Method

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Abstract

In the present paper the authors focus on dynamic response of A36 steel plate supported by a simple subgrade, under a moving iron load. The fourth order partial differential equations governing the dynamic behavior of the plate were transformed into their algebraic forms using differential transform method. The new set of algebraic equations were solved analytically. Computer software - Maple was deployed to plot the three-dimensional (3D) graphs. The results obtained revealed that the absolute value of the deflection of the steel was very high when the mass of the iron load is highest at constant velocity. Also, the A36 steel plate deflected more under a moving iron load especially with a high velocity and less foundation rigidity.
Keywords: A36 steel plate, Uniform Partially Distributed Moving Iron Load, Differential Transform Method, Dynamic Response

1. Introduction

A36 steel, usually referred to as referred to as ASTM A36 steel, is a common structural steel whose standard was established by the ASTM International[1]. ASTM A36 steel is one of the most widely used low carbon structural steels. Its plate can be used for general structural purposes. For instance, it can be welded in the construction of building and bridges. It is equivalent to EN S275 steel plate [1]. It exhibits good strength and can be galvanized to provide increased corrosion resistance.

A36 plate application depends on the thickness and corrosion resistance of the alloy. Some of the products manufactured using A36 structural steel plate are: Buildings, Cabinets, Pipe and tubing [2]. Apart from the fact it can be securely welded, it is easy to machine and fabricate. It can have fabricated into different forms including Plates, Structural Shapes, Bars, Sheets, Girders, Angle iron, T iron. Some properties of A36 steel are represented in table 1[2,3]. During production, sometimes loads made of iron traverses through the ASTM A36 steel plate which causes vibration of the plate. The traversed load is assumed to uniformly partially distributed [4].

Differential transform method (DTM) is a semi analytical method. It involves the transformation of differential equations to their algebraic forms. DTM is relatively simple to apply compared to other methods used in solving such problem, which computationally intensive.

Many researchers [5,6,7,8,9] have worked in this area. Some of the authors include: Y. Kumar [10], who studied the free vibration analysis of isotropic rectangular plates on Winkler foundation using differential transform method. He obtained the Characteristic equations in the form of infinite series and solved them numerically using a computer program developed in C++ to obtain

natural frequencies. The results obtained show reliability and fast convergence of the method for rectangular plates.

Agarana et al also studied application of differential transform method to vibration analysis of damped railway bridge on Pasternak foundation under moving train. He applied differential transform method to analysis the vibration of orthotropic damped rectangular railway bridge supported by Pasternak foundation. The results obtained revealed that both the foundation modulus and damping have effects on the deflection of the railway bridges which was modelled as the plate in this work [11,12,13]

None of the authors have considered specifically the dynamic response of A36 steel plate, supported by a subgrade under a moving iron load using DTM, without the effect of damping and considering simple foundation. This is what this paper attempt to address, because of the importance of this type of steel.

Mechanical Properties of A36 steel plate	Values
Density	7,800 kg/m^3 (0.28 lb/cu in).
Young's modulus	200 GPa (29,000,000 psi)
Poisson's ratio	0.26
shear modulus	75 GPa (10,900,000 psi).
minimum yield strength (thickness of plate <8in)	36,000 psi (250,000 kPa)
ultimate tensile strength (thickness of plate <8in)	58,000–80,000 psi (400–550 MPa)
minimum yield strength (thickness of plate >8in)	32,000 psi (220 MPa)
ultimate tensile strength (thickness of plate >8in)	58,000–80,000 psi (400–550 MPa)

Table 1: Mechanical properties and their values for A36 steel plate.

2. Formulation of Problem

Following Agarana et al, the equation governing the vibration of damped simply supported orthotropic plate resting on Pasternak foundation under a moving load can be written as [14,15]:

$$\alpha_1 \frac{\partial^4 w}{\partial x^4} + 2\alpha_2 \frac{\partial^4 w}{\partial x^2 \partial y^2} + \alpha_3 \frac{\partial^4 w}{\partial y^4} + Kw + m\frac{\partial^2 w}{\partial t^2} + 2m\gamma \frac{\partial w}{\partial t} + G_1\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right) = 0 \tag{1}$$

Where,

w = w(x,y,t) is the deflection of the plate at the point (x, y), m is the mass density per unit area, K, G1 is the foundation stiffness, t is the time in seconds, E is the Young's modulus, v is the velocity, H is the thickness of plate, γ is the viscous damping coefficient, α_1 is the flexural rigidity in the x direction, α_2 is the flexural rigidity in the y direction, and α_3 is the effective torsional rigidity.

For this paper, the simplest form of foundation was considered was considered. The effect of damping was neglected also. Therefore equation (1) becomes:

$$\alpha_1 \frac{\partial^4 w}{\partial x^4} + 2\alpha_2 \frac{\partial^4 w}{\partial x^2 \partial y^2} + \alpha_3 \frac{\partial^4 w}{\partial y^4} + Kw + m\frac{\partial^2 w}{\partial t^2} = 0 \tag{2}$$

With initial and boundary conditions [16]:
Following Kumar, the deflection function w, can be written as [17]

$$w = \bar{w}\sin(my\pi), \quad -\infty \leq m \leq +\infty \tag{3}$$

The last term in equation (2), $m\dfrac{\partial^2 w}{\partial t^2}$, is a force. It can be written as $\Omega\bar{w}$.

Where Ω is the frequency parameter, given as []:

$$\Omega^2 = \frac{12(1-v^2)}{Eh^3} \tag{4}$$

Substituting equation (3) into (2) leads to;

$$\alpha_1\frac{d^4\bar{w}}{dx^4} - 2\alpha_2\pi^2 m^2\frac{d^2 w}{dx^2} - (\Omega^2\alpha_3\pi^4 m^4 - k)\bar{w} = 0 \tag{5}$$

3. Problem Solution

Equation (5) can be solved using Differential transform method (DTM). Before solving the equation, let the following values of the parameters be substituted.

Mechanical properties of A36 steel	Notation	Formula	Value
Young modulus	E		200
Shear modulus	G		75
Flexural rigidity	D	$Gh^3/6(1\text{-}v)$	0.0021
Density			7800
Poisson ratio	v		0.26
Frequency parameter	Ω	$\sqrt{\dfrac{12(1-v^2)}{Eh^3}}$	21.16
mass	m	$-\infty \leq m \leq +\infty$	5
Foundation stiffness	k		100
Height of the plate	h		0.05
Torsional constant	J	$2.25l^4$	36

Table 1: Mechanical properties of A36 steel and parameters values

Substituting specific A36 steel plate values for flexural rigidity in x and y directions, effective torsional rigidity, mass density per unit area, and the foundation stiffness, into equation (5) after dividing by π^4 gives:

$$\alpha_1 / \pi^4 \frac{d^4 \overline{w}}{dx^4} - 2\alpha_2 m^2 / \pi^2 \frac{d^2 \overline{w}}{dx^2} - (\Omega^2 \alpha_3 m^4 - k)\overline{w} = 0 \qquad (6)$$

$$\alpha_1 / 180^4 \frac{d^4 \overline{w}}{dx^4} - 0.0015\alpha_2 \frac{d^2 \overline{w}}{dx^2} - (279841\alpha_3 - 100)\overline{w} = 0 \qquad (7)$$

Following the works of Kumar and Agarana;

$$\frac{d^4 \overline{w}}{dx^4} = a \frac{d^2 \overline{w}}{dx^2} + b\overline{w} \qquad (8)$$

where

$$a = \frac{0.0015\alpha_2}{\alpha_1 / 180^4}, b = \frac{279841\alpha_3}{\alpha_1 / 180^4}$$

$$DT[\overline{w}^4] = a\overline{w}^2 + b\overline{w} \qquad (9)$$

$$\Rightarrow (k+1)(k+2)(k+3)(k+4)Y(k+4) = a(k+1)(k+2)Y(k+2) + bY(k)$$

$$\therefore Y(k+4) = \frac{1}{(k+1)(k+2)(k+3)(k+4)}[a(k+1)(k+2)Y(k+2) + bY(k)] \qquad (10)$$

With initial conditions;
Y(0) = 0, Y(1) = -1, Y(2) = 1, Y(3) = -2

For k = 0:
$$Y(4) = \frac{1}{24}[2a] \qquad (11)$$

For k= 1:
$$Y(5) = \frac{1}{120}[-12a - b] \qquad (12)$$

For k = 2:
$$Y(6) = \frac{1}{360}[a^2 + b] \qquad (13)$$

For k = 3:
$$Y(7) = \frac{a}{6}[-12a - b] \qquad (14)$$

And so on.

Generally, for k+4 = n, (k = 0,1,2,3…) we have

$$Y(n) = \frac{a}{12}, \frac{-12a-b}{120}, \frac{a^2+b}{360}, a\left(\frac{-12a-b}{6}\right), ... \tag{15}$$

It was assumed in this paper that the flexural rigidity in both the x and y directions are of he plates have the same value of 0.0021. It was also assumed that the plate is a square with length of 4 units. Therefore, the values of a and b can be evaluated as follows:

$$a = \frac{0.0015\alpha_2}{\alpha_1 / 180^4} = 1575000$$

$$b = \frac{279841\alpha_3}{\alpha_1 / 180^4} \approx 0.$$

Therefore equation (15) becomes:

$$Y(n) = 131250, -157500, 6890625000, -4961250000000, ... \tag{16}$$

Following Edeki's work [], le

$$y(x) = \sum_{n=0}^{n=\infty} Y(n)x^n \tag{17}$$

$$\therefore y(x) = Y(0) + Y(1)x + Y(2)x^2 + ...$$

$$= 0 + 131250x - 157500x^2 + 6890625000x^3 - 4961250000000x^4 + ... \tag{18}$$

$$\therefore w(t) = 0 + 131250t - 157500t^2 + 6890625000t^3 - 4961250000000t^4 + ... \tag{19}$$

For t = 0,1,2,3,... respectively gives;

$$w(0) = 0$$

$$w(1) = -4954359401000$$

$$w(2) = -79324875630000$$

$$w(3) = -401861250000000$$

For velocity, equation (19) is differentiated to give:

$$w'(t) = 0 + 131250 - 315000t + 20671875000t^2 - 19845000000000t^3 + ... \tag{20}$$

For t = 0,1,2,3,..., respectively gives;

$w^{/}(0) = 131250$

$w^{/}(1) = -19824328310000$

$w^{/}(2) = -158677313000000$

$w^{/}(3) = -535628953900000$

For acceleration, equation (20) is differentiated to give:

$$w^{//}(t) = 0 + 131250 - 315000 + 41343750000t - 59535000000000t^2 + ... \qquad (21)$$

For t = 0,1,2,3,..., respectively gives;

$w^{//}(0) = -183750$

$w^{//}(1) = -59493656430000$

$w^{//}(2) = -238222687300000$

$w^{//}(3) = -535690968900000$

The above shows the Truncated values of the deflection, velocity of the deflection and acceleration of the deflection after the fourth terms.

4. RESULT DISCUSSIONS

Putting the assumptions into consideration, the results obtained show that the absolute values of the deflection, the velocity of the deflection and the acceleration of the deflection increase as time increases, within the range of time considered (t = 0,1,2, 3). Equations (19), (20) and (21) show the huge dynamic response after time zero. The deflection, the velocity of the deflection and the acceleration of the deflection are well pronounced. However, comparing the increase in angular velocity with the increase in angular acceleration at any given time, the increase in angular acceleration is more than the increase in angular velocity. Similarly comparing values of the deflection, velocity of the deflection and acceleration of the deflection, at a time, shows that it is most pronounced in the acceleration, followed by the velocity, then the deflection itself. The reason for the huge deflection and its rate with time can be attributed to the iron load and the type of plate, the A36 steel plate, bearing in mind that damping was neglected in this study.

5. CONCLUSIONS

A36 steel plate's dynamic response to uniform partially distributed moving iron load was modelled and analyzed using differential transform method. The results obtained are put in table form and agree with the ones in literature. Deductions are made from the results. The differential transformation method is an effective method for solving the type of differential equation problem being considered in this paper. It is also easy to handle, because the differential equations are transformed into their algebraic form. The mechanical properties of A36 steel plates have effects on the dynamics of the system.

REFERENCES

[1] https://www.leecosteel.com/a36-steel.html

[2] http://www.onealsteel.com/carbon-steel-plate-a36.html

[3] https://www.leecosteel.com/a36-steel.html

[4] Agarana M.C. and Ede A.N., (2016), Application of Differential Transform Method to Vibration Analysis of Damped Railway Bridge on Pasternak Foundation under Moving Train. Lecture notes in engineering and computer science: proceedings of the world congress on engineering 2016, June 29 – July 1, 2016, London UK. Pp ….

[5] Z. Odibat. 2008. Differential Transform Methods for Solving Volferra Integral Equations with Separable, Kernels, Mathematical and Computer Modeling. 48: 1144-1146.

[6] M. Thongmoon and S. Pusjuso. 2010. The Numerical Solutions of Differential Transform Method and the Laplace Transform Methods for a System of Differential Equations. Nonlinear Analysis: Hybrid Systems. 4: 425-431.

[7] 4. A. Arikoglu and I. Ozkol. 2005. Solution of Boundary Value Problems for Integro-Differential Equations by Using Transform Method, Applied Mathematics and Computation. 168: 1145-1158

[8] H. Abdel - Halim Hassan. 2008. Application to differential transformation method for solving systems of differential equations. Applied Mathematical Modeling. 32: 2552-2559.

[9] S. O. Edeki, *et al.* 2014. A Semi-Analytical Method for Solutions of a Certain Class of Second Order Ordinary Differential Equations, Applied Mathematics. 5: 2034-2041.

[10] Y, Kumar, differential transform method to study free transverse vibration of monoclinic rectangular plates restimg on Wimkler foundation, Applied and Computational mechanics, 2013, 7, 145-154

[11] M.C Agarana and M.E Emetere, (2016), Solving Non-Linear Damped Driven Simple Pendulum With Small Amplitude Using A Semi Analytical Method, ARPN Journal of Engineering and Applied Science, Vol 11, No 7, Pp 4478-4484

[12] M. C. Agarana and O. O. Agboola. 2015. Dynamic Analysis of Damped Driven Pendulum using Laplace Transform Method, International journal of Mathematics and Computation, 26(3): 98-109.

[13] M. C. Agarana and S. A. Iyase. 2015. Analysis of Hermite's Equation Governing the Motion of Damped Pendulum with Small Displacement. International Journal of Physical Sciences. 10(2): 364-370

[14] L. E. Millet. 2003. The Large – angle Pendulum Period. Phys. Teach. 41: 162-163.

[15] M. C. Agarana, and J. A. Gbadeyan, "Finite Difference Dynamic Analysis of Railway Bridges Supported by Pasternak Foundation under Uniform PartiallyDistributed Moving Railway Vehicle," Lecture Notes in Engineering and Computer Science: Proceedings of The World Congress on Engineering and Computer Science 2015, 21-23 October, 2015, San Francisco, USA,pp996-1000

[16] M. C. Agarana, J. A. Gbadeyan, O. O. Agboola, T. A. Anake, O. J. Adeleke, '' Dynamic Response of an inclined Railway bridge supported by Winkler Foundation under a Moving Railway Vehicle.'' Australian Journal of Basic and Applied Sciences, 2015, Vol. 9 No.11, pp 355-361.

[17] Y. Kumar, Free Vibration analysis of Isotropic Rectangular plates on Winkler foundation using differential transform method, International Journal of Applied Mechanics and Engineering (2013) Vol. 18, no.2, pp 589-597