System Approach of Production System Reliability and Maintainability

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Abstract

History proves the permanent and incessant change of the competitive environment of the company, putting it in a stifling field of competitiveness, which forces the companies around the world to commit to the permanent optimization of their economic performance based on the continuous improvement of their production resources, in particular their material manufacturing systems, and this is linked to the need for continuous renewal and development of approaches to improve their dependability, aiming to manage the manufacturing systems with a multidimensional, flexible and crossroads approach of control and optimization, making the optimal dependability and the minimum cost of maintenance of the integrity of a material manufacturing system two faces of a same species. In this perimeter, our research work aims to improve the reliability and maintainability attributes of a system in all its life phases and within a field of maintenance cost constraints of its sub-systems, by establishing a system approach applied to reliability and maintainability, making the bridge from the science of failures and breakdowns to the discipline of mathematical programming that is part of the science of operations research. It is in view of our dynamic vision allowing us to perceive the holistic character of production system.

Keywords
Manage the manufacturing systems, field of maintenance cost constraints, holistic character of production system, system approach applied to reliability the maintainability, from the science of failures and breakdowns to the discipline of mathematical programming.

1. Introduction

As early as 1951, the Austrian biologist Ludwig von Bertalanffy, in a major article published in the journal Human Biology under the title "General Systems Theory", formulated an innovative concept of complex interacting elements (Institut Numérique, 2014). He said "A system can be defined as complex of interacting elements (Bertalanffy, 1968)", which developed and popularized the concept of system (Hallioui and Herrou, 2020). The Systemic constitutes, according to the Bertalanffy's own words, "a new philosophy of nature" (Turchany, 2020). For a system, the complexity and criticality of the tasks to be accomplished and the environmental realities are constraints that justify the need for irreproachable and permanent improvement in terms of reliability and maintainability. According to the literature, the 1960s and 1970s were marked by attempts to generalize the probabilistic approach "reliability theory", which was so successful, to other components such as mechanical, hydraulic, electrical, then software and human components, after electronic components in aeronautics, defense and nuclear during the 1940s and 1950s. The 1960s and 1970s were also characterized by the extension of the approach to the return to normal, to reliability is added Maintainability (Leroy and Signoret, 1992).

Nowadays, the production systems must be highly competitive (Díaz-Reza and García-Alcaraz, 2019), it is the aim for which the system reliability optimization is a living problem, with solutions methodologies that have evolved with the advancements of mathematics, development of new engineering technology, and changes in management perspectives (Coit and Zio, 2019). Indeed, the optimization of production system reliability, maintainability and maintenance cost is one of main objectives for the industrial managers and the researchers since maintainability has just been added to reliability, in fact, since the extension of reliability theory. The methodologies of this optimization
have evolved with the research and development in the areas of mathematics, engineering technology and industrial management.

In this paper, we aim to improve the reliability and the maintainability of a production system confronting its very requiring environment in terms of multidimensional control, flexibility and optimization of the functional performances and maintenance cost aspects which have opposite directions of improvement. To improve the reliability and the maintainability attached to the maintenance cost of a production system, we propose a system approach (systemic approach) applied to the two attributes of dependability, it is called " The Production System Reliability and Maintainability Approach". It is based on our hypothesis of reliability and maintainability of classes of interacting subsystems. In this context, we have been able to make the bridge from the dependability to the operational research science by introducing the sub-systems' optimal maintenance costs.

2. General hierarchical diagram of a production system

This is a part extracted and developed from our research work entitled System Approach for Improving the Dependability of Production Systems, State of the Art - Proceedings of the 5th North American International Conference on Industrial Engineering and Operations Management, Detroit, Michigan, USA, August 10-14, 2020, it is a reprise in the context of improving the reliability and maintainability attributes of production activities, in fact, in the context of establishing methods allowing the efficiency combining production and dependability.

Rosnay (1975) said that system analysis as a method and one of the tools of the system approach leads to the reduction of the system into its elements and elementary interactions. So, it is opposed to the cartesian reductionism, which reduces the system to its elements (Hallioui and Herrou, 2020). Through our dynamic vision of the system approach attached to the systemic thinking, indeed, through the system analysis, we carried out the basic diagram (fig. 1) of our hypothesis for improving the production systems reliability and maintainability:

![General hierarchical diagram of a system with the classes of its subsystems and the interactions](image)

*Figure 1. General hierarchical diagram of a system with the classes of its subsystems and the interactions (Hallioui and Herrou, 2020).*

This is the basic diagram of our hypotheses for improving dependability through the system approach. We carried it out as part of our research work, to the extent of putting it the first stone of our critical thinking leading to the taking into account of the system approach in the optimization of the attributes Safety, Reliability, Maintainability and
System availability. So, through our dynamic vision of the system approach attached to systems thinking (Hallioui and Herrou, 2020):

- The system includes classes of subsystems and internal and external interactions;
- Each class has subsystems and their interactions;
- Each subsystem of class k (with \( k \geq 1 \)) can be composed by one or more subsystems of class \( k + 1 \). Example: according to the general hierarchical diagram of the system in the figure above, class 1 is composed of the subsystems i and j. And:
  - In class 2:
    - \( i_1, i_2, \ldots i_N \) are Sub-Subsystems of the Whole System, because they are subsystems of the subsystem \( i \) of class 1;
    - \( j_1, \ldots j_M \) are Sub-Subsystems of the Whole System, because they are subsystems of the subsystem \( j \) of class 1;
  - In class 3:
    - \( i_{11}, i_{12}, \ldots i_{13} \) are Sub-Sub-Subsystems of the Whole System, because they are subsystems of the SubSubsystem \( i_1 \) of class 2;
    - \( i_{21}, i_{22}, \ldots i_{23} \) are Sub-Sub-Subsystems of the Whole System, since they are subsystems of the Sub-Subsystem \( i_2 \) of class 2;
    - \( i_{N1} \) and \( i_{N2} \) are Sub-Sub-Subsystems of the Whole System, since they are subsystems of the Sub-Subsystem \( i_N \) of class 2;
    - \( j_{11} \) and \( j_{12} \) are Sub-Sub-Subsystems of the Whole System, because they are subsystems of the Sub-Subsystem \( j_1 \) of class 2;
    - \( j_{M1}, j_{M2}, \ldots j_{M3} \) are Sub-Sub-Subsystems of the Whole System, because they are subsystems of the Sub-Subsystem \( j_M \) of class 2;
  - In the same way up to class L;
- For each class of the hierarchical diagram of a system, the number of SUBS associated with the word system (for the subsystems of all the classes) is equal to the level of the class of the subsystem compared to the Target System or whole, making the top of the hierarchical diagram, therefore, it must take the name “Subsystem X of class \( k \geq 1 \)”; for example, in Figure 1:
  - The subsystem \( i_{12} \) of class 3: is a SUB-sub-system of the whole system;
- The internal interactions are:
  - Those between the classes of the subsystems (see Figures 1). They are called the interactions between two successive classes. Example: class 1 and class 2 are linked by:
    - The interaction between the subsystem \( i \) of class 1 and its subsystems \( i_1, i_2, \ldots i_N \) of class 2;
    - The interaction between the subsystem \( j \) of class 1 and its subsystems \( j_1, \ldots j_M \) of class 2;
  - The interactions between subsystems of the same class (see Figure 1).
- The external interactions are those between the system and its environment.

3. What is common for the system reliability

3.1. Reliability and system concepts

In this section, we summarize the main definitions derived from the systemic thinking in the literature, indeed, these are definitions framing the concepts of system and reliability in the context of our research work. It is only an introduction to the optimization of the reliability of a material production system.

According to our work entitled System Analysis: A Literature Review, we have proven that "the systemic fixed the principles characterizing the natural character that ensures the equilibrium for any system, whatever its nature and structure. It has given to the concept of system its deserved value, by taking it out of the cage from the closure designed by the reductionists (the Cartesians) towards the opening on its environment (Hallioui and Herrou, 2020)". In literature, we found that the system is a set of interdependent elements oriented towards the realization of a function (Soro, 2011). While, the reliability is one of the main attributes of dependability. The implementation of reliability aims to study, characterize and measure system failure in order to improve the operational use of systems (Ebeling, 1996). So, the reliability is the ability of a device to perform a required function under given operating conditions and for a specified period of time (NFX60-500, 1988). Indeed, any system is subdivided into subsystems of 1st class, then into components and elements which are subsystems of the class \( k > 1 \) and make subsystems to subsystems of the class \( K-1 > 1 \), until the class L of subsystems at the basis of the hierarchical diagram of the target system (Fig. 1).
3.2. Failure data analysis

This section is a presentation of an overview on the empirical and parametric methods for failure data analysis, in the light of which the production system's dependability managers can base themselves to optimize its functional performance in terms of reliability and maintainability. According to (Verma et al., 2015), the credibility of any reliability/safety studies depend upon the quality of the data used. This section deals with the treatment of failure data and subsequent usage in reliability/safety studies. The derivation of system reliability models and various reliability measures is an application of probability theory, whereas the analysis of failure data is primarily an application of statistics. The objective of failure data analysis is to obtain reliability and hazard rate functions. This is achieved by two general approaches: Nonparametric methods or empirical methods which are deriving empirical reliability and hazard functions directly from failure data. The nonparametric method is useful for preliminary data analysis to select appropriate theoretical distribution. This method finds application when no parametric distribution adequately fits the failure data; Parametric method which is dedicated to identify an approximate theoretical distribution, estimate the parameter(s) of distribution, and perform a goodness of fit test. Indeed, this second and usually preferred approach, is to fit a theoretical distribution, such as the exponential, Weibull, or normal distributions. As theoretical distributions are characterized with parameters, these methods are known as parametric method. Nonparametric methods have certain practical limitations compared with parametric methods.

3.2.1. Reliability analysis by the Weibull method

The Weibull distribution is one of the most widely used life distributions in reliability analysis. The distribution is named after the Swedish professor Waloddi Weibull (1887-1979) who developed the distribution for modeling the strength of materials. The Weibull distribution is very flexible, and can, through an appropriate choice of parameters, model many types of failure rate behaviors (Rausand and Hoyland, 2003). The mathematician Waloddi Weibull has given to failure rate a general formula that depends on three parameters \( \eta, \beta \) and \( \gamma \) (Sidi Mohamed Ben Abdellah University: Faculty of Sciences and Techniques of Fez, 2016):

- \( \eta \): Scale parameter (expressed in hours at most, \( \eta > 0 \));
- \( \beta \): Shape parameter, it is a number \( \geq 0 \), without unit, which defines the distribution pattern of Weibull making the reliability model of the diagnosed equipment;
- \( \gamma \): Position parameter, it is also called the parameter of offset or localization of the curve with respect to time \( (-\infty > \gamma > +\infty, \text{ and usually } \gamma = \text{zero}) \).

The Weibull model of the equipment should normally be made on the ALLAN PLAI paper (or the Weibull paper) which is a special logarithmic paper, or with a software tool, following the standard method of the inventor of this reliability analysis approach:

![Weibull paper or ALLAN PLAI paper](image)

(Sidi Mohamed Ben Abdellah University: Faculty of Sciences and Techniques of Fez, 2016).
The scales used on the WEIBULL paper in the Fig. 2 are: the high abscissa which is a natural scale in X=Ln(t); the middle abscissa that is a logarithmic scale, it is a reading of the parameter η; the lower abscissa which is a logarithmic scale, where each value of "t" is made to correspond to its natural logarithm Ln(t); the left ordinate where we place the values of F(t) in percentage in scale Ln (Ln(1/(1-F(t))); the ordinate on the axis X = -1 which present the reading of the parameter β, in fact, these are the values Ln(Ln(1/(1-F(t)))); this is the Y axis (Bellaouar and Beleulmi, 2013). Indeed, this paper has two families of coordinates:

- **Raw coordinates**: t=TBF, F(t);
- **Weibull coordinates**: Ln(t)=Ln(TBF), Ln(1/(1-F(t))).

Where TBF is the Time Between Failure, it designates the time between two consecutive failures. On the basis of the Fig. 2 and by taking the parameter γ=0, we can describe in detail the methodology for building the Weibull model on the ALLAN PLAI paper as follows:

1. Consult the history of system failures and determine the Time Between Failures (set of TBFs).
2. Rank the TBFs in ascending order.
3. Assign a rank "i" to each TBF.
4. Calculate the F(i) (the probabilities of failure at each instant "t" or at each rank "i") according to the number N of failures (approximation of F(t)).
5. Establish the point cloud by placing the TBFs on the abscissa and the F(i) on the ordinate.
6. Draw the line that best passes through the points obtained, if the points are aligned on a straight line: γ=0 (most often γ=0).
7. The scale parameter (µ): is obtained by the intersection of the line drawn with the µ axis, the scale used for the reading must be the same as the one chosen for the axis of "t" (or "TBF").
8. The shape parameter (β): is obtained by drawing a parallel to the previous line and passing through the value 1 of the µ axis. The value of β is read on the β axis, at the intersection with the parallel line.
9. Interpretation of results.

Figure 3. Methodology for building the Weibull model on the ALLAN PLAI paper by taking γ=0.

In order to provide more precision when plotting the reliability model for a material production system as a Weibull distribution, it is necessary to work with the coordinates (X = Ln(t) = Ln(TBF), Y (X) = Ln(1/ (1-F(t)))). In order to determine the function Y(X) of the regression line which models the reliability of the studied system, we use the distribution function F(t) (or the probability of failure) associated to the Weibull law of parameters β, γ and η, it is expressed by the formula F(t)=1-R(t), where R(t) is the symbol of reliability at an instant "t" of system operation:

\[
F(t) = 1 - e^{-\left(\frac{t-\gamma}{\eta}\right)^\beta}
\]

When obtaining a cloud of points aligned on a straight line (regression line): γ=0, so:

\[
1 - F(t) = e^{-\left(\frac{t}{\eta}\right)^\beta}
\]

\[
\frac{1}{1-F(t)} = \left(\frac{1}{\eta}\right)^\beta ;
\]
\[ 1 - F(t) = e^{\eta \beta} \]
\[ \ln \left( 1 - F(t) \right) = \frac{i}{N} \beta \]
\[ \ln \left( \ln \left( 1 - F(t) \right) \right) = \ln \left( \frac{i}{N} \right)^\beta ; \]
\[ \ln \left( \ln \left( 1 - F(t) \right) \right) = \beta \ln \left( \frac{i}{N} \right) \]
\[ Y = \beta \cdot X - \beta \cdot \ln(\eta) \]
\[ Y = A \cdot X - B \]

For building the Weibull model on the ALLAN PLAI (Fig. 3), the calculation of \( F(i \) or \( t) \) which is the distribution function (expressing the probability of failure at each instant "\( t\)" or at each row "\( i\)" of TBF of the system) is done following the number of samples (\( N\)) (Tab. 1), which is the number of TBF, it is equal to the number of failures in the Weibull distribution coordinate calculation table:

<table>
<thead>
<tr>
<th>( N \leq 20 )</th>
<th>( 20 &lt; N &lt; 50 )</th>
<th>( N \geq 50 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( F(i) = \frac{i - 0.3}{N + 0.4} = F(t) )</td>
<td>( F(i) = \frac{i}{N + 1} = F(t) )</td>
<td>( F(i) = \frac{i}{N} = \frac{\sum R(i)}{N} = F(t) )</td>
</tr>
</tbody>
</table>

The case where \( \gamma \) is non-zero is a bit more tricky. Computer tools allow us to adjust the law to three parameters (this is a case of non-linear regression), but we must ask ourselves the relevance of this model:

- In case of \( \gamma > 0 \), this means that the probability of failure in the first uses of the system is null (Bellaouar and Beleulmi, 2013). Graphically, the curve on the Weibull paper is convex (rounded down) (Sidi Mohamed Ben Abdellah University: Faculty of Sciences and Techniques of Fez, 2016);
- In case of \( \gamma = 0 \), this means that a probability of failure will be present as soon as the System is commissioned (Bellaouar and Beleulmi, 2013);
- In case of \( \gamma < 0 \), this means that these are failures prior to commissioning (Sidi Mohamed Ben Abdellah University: Faculty of Sciences and Techniques of Fez, 2016), in fact, a probability of failure is already present at the time of installation of the system. Graphically, the curve on the Weibull paper is concave (rounded up) (Bellaouar and Beleulmi, 2013).

If the point cloud corresponds to a curve (Fig. 4), we straighten it by a translation of all the points by adding or subtracting from the abscissa "\( t\)", the same value (gamma) in order to obtain a straight line (Bellaouar and Beleulmi, 2013). By experience, it is necessary to have at least 20 points to be able to estimate that the curve is not linear, or even more if \( \gamma \) is close to zero. This is shown in the figure below:

Figure 4. Search for the parameter \( \gamma \neq 0 \) (Bellaouar and Beleulmi, 2013).

According to Rausand and Hoyland (2003): When \( \beta = 1 \), the failure rate is constant; When \( \beta > 1 \), the failure rate function \( \lambda(t) \) increases; When \( 0 < \beta < 1 \), \( \lambda(t) \) decreases; When \( \beta = 2 \), the resulting distribution is known as the Rayleigh distribution. The Weibull distribution is considered flexible and can be used to model life-cycle distributions, where the failure rate function decreases, remains constant, or increases.

### 3.3. Calculation of system reliability on the basis of its hierarchical structure

#### 3.3.1. Reliability structure model of a serial system
Until this part of the paper, we are still in line to describe what is common in matters of system reliability in the context of our research work. The reliability $R_s$ of a system (machine or production line) constituted of several components (interacting subsystems of the 1st class, see figure 1) interacting in series is equal to the product of the respective reliability $R_{ss1}, R_{ss2}, R_{ss3}, ..., R_{ssN}$ of each component:

$$R_s = \prod_{i=1}^{N} R_{ssi}$$

### 3.3.2. Reliability structure model of a parallel system

The reliability of a system can be increased by placing the components in parallel (Verma et al., 2015). A device constituted of $N$ components in parallel (Fig. 6) will fail only if all the $N$ components fail at the same time. If $F_i$ is the failure probability of a component, the associated reliability $R_i$ is its complementary $F_i = 1 - R_i$ (Bellaouar and Beleulmi, 2013). In the perimeter of this research work, it is essential to indicate that the dynamically interacting components connected in parallel in the system $S$ and for which the determination of $F_i$ is important, are only its interactive subsystems of the 1st class according to the diagram established in the Fig. 1. It is also important to note that each subsystem of class $K>=1$ is a system for its interacting components of class $K+1>1$.

For a system $(S)$ with $N$ subsystems of 1st class connected in parallel, the failure probability $F_S$ can be expressed by:

$$F_S = \prod_{i=1}^{N} F_i$$
So: \[ F_S = \prod_{i=1}^{N}(1 - R_i) \]
But: \[ R_S = 1 - F_S \]
Therefore, in this case, the reliability of the system \( (S) \) is as follows:
\[ R_S = 1 - \prod_{i=1}^{N}(1 - R_i) \]

4. What is common for the system maintainability and overview on its optimization in a distributed context on the company

This part is the result of our research in the literature on system maintainability. In this perspective, we aim to anchor what is common for the maintainability of a system, to present thereafter, an overview on the optimization of system maintainability in a context distributed on the company, indeed, the integration of the system approach in the sphere of the company for optimizing the industrial performance areas (Productivity, Quality, Costs, Delays, Safety and Motivation), this is what is known by the process approach oriented towards the optimization of the maintenance and the productivity of production systems within the company or the Total Productive Maintenance. In the Fig. 7, we have included the implementation of the TPM in the bottleneck field of system maintainability optimization, in view that it is a process approach applied to the maintenance and the productivity of production systems, a global management issue and an indispensable element in the company's philosophy. In addition to the implementation of the TPM, in the framework of the system maintainability optimization, we have focused on: the plant management philosophy in general, requiring the application of the augmented reality, the openness on the scientific research, the fluidity of the company's procedures, the mastery of the psychological state of the staff and the work climate in addition to the management by the system approach in general; the design; the quality of the supplier's after-sales service; the logistics and maintenance management.

4.1. Maintainability position for a system

According to the NASA (2008), the United States Army Materiel Systems Analysis Activity has proved that the maintainability is emphasized because an estimated 38% of life-cycle cost is directed toward maintainability issues. The term maintainability has the following meanings: A characteristic of design, construction, and installation, expressed as the probability that an item will be retained in or restored to a specified condition within a given period of time. This probability is based on the assumption that maintenance is performed in accordance with prescribed procedures and resources; The ease with which maintenance of a functional unit can be performed in accordance with prescribed requirements. Maintainability should be considered during original design, during renovations, and during equipment and system replacement. Inadequate detailing for access, inadequate detailing for installation, and improper specifications (indoor equipment being used outside) are common problems.

4.2. Pillars of the maintainability component

An Inaccessible equipment is a common maintainability problem. Examples of poor accessibility in this regard include, but are not limited to: Equipment installed above inaccessible ceilings; Hard-wired electrical switches mounted on access panels; Access panels on the wrong side of equipment (no space for access); Air-handling units with no coil access; Electrical panels with clearance not in compliance with the National Electric Code; Equipment with electrical (control) panels which do not have clearance in compliance with the manufacturer’s recommendations and National Electric Code (NASA, 2008). Optimizing the maintainability of the system relies on several pillars that can act as bottlenecks in the case of a deficiency of control from the company (fig. 7). Thanks to our systemic thinking considering the system creative evolution, indeed, the holistic behavior of the system as the whole of its parts in their synthesis, in this context we are aiming at any material system operated for production, we were able to carry out a system analysis applied to the optimization of maintainability, in order to obtain the bottleneck diagram for the production material system maintainability optimization:
Figure 7. Bottleneck diagram for the production material system maintainability optimization.
4.3. Requirement for a system maintainability program

In view that that maintainability is concentric for the improvement of the systems’ dependability, in the case of requirements for a high level of mastery of the maintenance function, the companies outsource the works of the latter on behalf of manufacturers or service providers specialized in the maintenance business and judged capable to perform it at all its levels. According to Smith (2011) the detailed activities during design, manufacturing and installation are sometimes spelt out contractually. In a development contract this enables the customer to monitor the reliability and maintainability design activities and to measure progress against agreed milestones. Sometimes standard program requirements are used, for example the US Military Standard 470, Maintainability Program Requirements. The typical activities specified are:

- **Prediction**: data sources, mathematical models;
- **Testing**: methods and scheduling of design, environmental and other tests;
- **Design review**: details of participation in design reviews;
- **Failure mode and effect analysis**: details of method and timing;
- **Failure reporting**: failure reporting documents and reporting procedures.

4.4. Estimation of maintainability by the Exponential method

By this parametric method of failure data analysis, the maintainability of a repairable system can be estimated as a probability by the following expression (Sidi Mohamed Ben Abdellah University: Faculty of Sciences and Techniques of Fez, 2016):

\[ M(t) = 1 - e^{-\mu t} \]

With:
- \( \mu = \) constant: It is the repair rate of the equipment, which has an expression as follows (Bellaouar and Beleulmi, 2013):
  \[ \mu = \frac{1}{\text{Mean Time To Repair}} = \frac{1}{\text{MTTR}} \]
- \( t \) (in the expression of maintainability): It is the permissible time constraint for the maintenance action (Sidi Mohamed Ben Abdellah University: Faculty of Sciences and Techniques of Fez, 2016).

5. The Operational Research, a Production System Reliability and Maintainability Approach

5.1. Overview on the operational research science and its discipline mathematical programming

After the remarkable success of Operational Research in the military field during World War II, it was the turn of industry to take an interest in this new science. Since, the operational research has become more and more present in the daily life of managers: production planning, personnel allocation, inventory management, transportation problems, scheduling, etc. All these problems explicitly or implicitly call for operational research techniques. The interest for this science, which was initially hampered by the complexity of the calculations and the slowness of first-generation computers, has grown considerably during recent decades with the remarkable power of computers (Alaoui and Benadada, 2012). The operational research tackles problems of management and decision making in economic organizations and the consideration of combinatorics and uncertainty (Nakhla and Moisdon, 2010).

One of the most exploited disciplines of Operational Research is that of Mathematical Programming, which has as its objective the theoretical study of optimization problems and the design and use of algorithms for solving problems in the world concerned (Alaoui and Benadada, 2012), which is the industrial world, and particularly the perimeter of reliability, maintainability and therefore availability of production material systems. According to Elhilali Alaoui and Bendada (2012), the mathematical programming includes several classes of problems, we quote in the context of this work: The Linear Programming, for the study of optimization problems of linear functions under linear constraints (Dantzig, 1949); The Non-Linear Programming, for the study of non-linear optimization problems with or without constraints (Kuhn and Tucker, 1951); The Integer Programming, for the study of optimization problems where variables are constrained to take only integer values (Gomory, 1958) and The Dynamic Programming, which is a general approach to the optimization of dynamic problems (Bellman, 1957).
5.2. Deployment of the Production System Reliability and Maintainability Approach

5.2.1. Hypothesis of reliability and maintainability of classes of interacting subsystems

This is the hypothesis of our Production System Reliability and Maintainability Approach, which take into account the holistic character of production system. The objective is to put any production material system in a mathematical formalism modeling its behavior in terms of its functional performances characterizing its dependability as well as in terms of the maintenance costs of its evolving subsystems thanks to their dynamic interactions and forming the classes discovered in the Fig. 1 of this paper. Indeed, in context of the systems theory or the general systems theory attributed to the Austrian biologist Ludwig von Bertalanffy, and thanks to: Our dynamic and global vision of the system allowed by the systemic, in fact, attached to a systemic thinking opposite to the analytical thinking (classical or Cartesian-analytical); An exhaustive definition of the new paradigm of system defined by the systemic, which was given by Rosnay (1975), who wrote in his treatise "The macroscope: towards a global vision" that a system is a set of elements in dynamic interaction, organized according to a goal; Our general hierarchical diagram of a system showing the classes of its subsystems and the different interactions (Fig. 1).

We said that (Halliou and Herrou, 2020) (extracted part from our paper entitled "System approach for improving the dependability of production systems, state of the art"):

- The level of importance or criticality of interactions follows an ascending arrow (from bottom to top) on the general hierarchical diagram of a system (Fig. 1), indeed, the interactions as well as the classes of the subsystems closest to the top of a system hierarchy are the most critical in terms of dependability in general and reliability and maintainability in particular;
- The system includes several classes of subsystems in dynamic interaction and according to an order guaranteeing the balance and the characteristics of the normal functioning of the system, under the system immediately there is the 1st class, it is the set of subsystems and internal and external interactions making the test body (the element in direct contact) for their system, and therefore the optimization of their reliability, maintainability and maintenance costs immediately generates optimization of the system;
- The optimization of these dependability components (reliability and maintainability) of a system can be drained attached to the optimization of the maintenance costs of its subsystems in dynamic interaction and organized according to the purpose of the whole which is the system. This is what formulates:
  - The main objective of our new approach which is "the Production System Reliability and Maintainability Approach" based on mathematical programming;
  - The problem of optimizing two families of aspects (dependability performance & maintenance costs) with opposite directions of improvement.
- For any system (machine or production line) studied with N interacting subsystems in the 1st class, each of these has:
  - Reliability and maintainability (therefore an availability) to optimize (to maximize);
  - An optimal (minimum) maintenance cost, estimated and objective for the responsible service during a well-defined time interval.

**Question asked!** What are the optimal (maximum) values of reliability and maintainability that must be available for each interacting subsystem of 1st class of the studied system? Under the constraints:

- Objectives of the responsible service in terms of the costs of maintaining the subsystems to be minimized;
- Objectives set by the management responsible for the reliability and maintainability of the system studied.

5.2.2. Mathematical programming

Thanks to our new "Reliability and Maintainability Production System Approach" based on the discipline of Mathematical Programming of the Operational Research Science that we have deployed to the dependability of production material systems, it is possible to solve the complex problem of optimization. To treat the problematic that we have formulated in our hypothesis, the approach followed is as follows:

- Modeling the problem by one or two Linear Mathematical Programs (LMPs) (since it concerns the optimization of the two components "Reliability" and "Maintainability" of each subsystem of the 1st class of the system);
• Solving the mathematical problem (or program) of maximizing the function called "objective or economic function" under a set of constraints to be extracted from the problem described in the section of our hypothesis.

5.2.2.1. Modeling the 1st problem: Minimizing the maintenance costs of subsystems of 1st class of a system and optimizing their reliabilities

In this first step, we start by optimizing the reliability and minimizing the maintenance costs of each sub-system of 1st class. According to the problematic formulated in the hypothesis, the mathematical program is composed of:

- \( x_{ssi} \): Reliability of the interacting subsystem n°1 in the 1st class of the system;
- \( x_{ss2} \): Reliability of the interacting subsystem n°2 in the 1st class of the system;
- \( x_{ssN} \): Reliability of the interacting subsystem n°N in the 1st class of the system;
- \( C_{ssi}=\text{constant} \): Minimum maintenance cost with known value (already calculated or planned) for the interacting subsystem n°1 in the 1st class of the system;
- \( C_{ss2}=\text{constant} \): Minimum maintenance cost with known value (already calculated or planned) for the interacting subsystem n°2 in the 1st class of the system;
- \( C_{ssN}=\text{constant} \): Minimum maintenance cost with known value (already calculated or planned) for the interacting subsystem n°N in the 1st class of the system.

The mathematical model of the 1st problem is the following Linear Program (LMP written in its standard form):

\[
\begin{align*}
\text{Min} & \quad z = \sum_{i=1}^{N} C_{ssi} \cdot x_{ssi} \\
\text{Subject to (under the following constraints):} & \\
\quad x_{ss1} \geq \text{minimum of the objective to be achieved in reliability for the } S \text{ of n°1 of the 1st class;} \\
\quad x_{ss2} \geq \text{minimum of the objective to be achieved in reliability for the } S \text{ of n°2 of the 1st class;} \\
\quad x_{ssN} \geq \text{minimum of the objective to be achieved in reliability for the } S \text{ of n°N of the 1st class;} \\
\quad \text{All } x_{ssi} \geq 0 \text{ and integers (integrity constraint).}
\end{align*}
\]

5.2.2.2. Modeling the 2nd problem: Minimizing the maintenance costs of subsystems of 1st class of a system and optimizing their maintainabilities

According to the problematic formulated in our hypothesis, the mathematical program is composed of:

- \( y_{ssi} \): Maintainability of the interacting subsystem n°1 in the 1st class of the system;
- \( y_{ss2} \): Maintainability of the interacting subsystem n°2 in the 1st class of the system;
- \( y_{ssN} \): Maintainability of interacting subsystem n°N in the 1st class of the system;
- \( C_{ssi}=\text{constant} \): Minimum maintenance cost with known value (already calculated or planned) for the interacting subsystem n°1 in the 1st class of the system;
- \( C_{ss2}=\text{constant} \): Minimum maintenance cost with known value (already calculated or planned) for the interacting subsystem n°2 in the 1st class of the system;
- \( C_{ssN}=\text{constant} \): Minimum maintenance cost with known value (already calculated or planned) for the interacting subsystem n°N in the 1st class of the system.

The mathematical model of the 2nd problem is the following Linear Program (LMP written in its standard form):

\[
\begin{align*}
\text{Min} & \quad z = \sum_{i=1}^{N} C_{ssi} \cdot y_{ssi} \\
\text{Subject to (under the following constraints):} & \\
\quad y_{ss1} \geq \text{minimum of the objective to be achieved in maintainability for the } S \text{ of n°1 of the 1st class;} \\
\quad y_{ss2} \geq \text{minimum of the objective to be achieved in maintainability for the } S \text{ of n°2 of the 1st class;} \\
\quad y_{ssN} \geq \text{minimum of the objective to be achieved in maintainability for the } S \text{ of n°N of the 1st class;} \\
\quad \text{All } y_{ssi} \geq 0 \text{ et integers (integrity constraint).}
\end{align*}
\]
5.2.3. **Solving both Linear Mathematical Problems (LMP.R and LMP.M) & Interpretations**

The resolution of linear mathematical programs can generally be done by the Simplex method or other methods that require iterations, several lines and even several pages (analytically), which puts the industrial managers under the enormous constraint of time, and in view of the need to make quick tests (sometimes in performance meetings) on the mathematical models of the problems of optimization of the reliability and maintainability of systems attached to a minimization of the maintenance costs of the interacting subsystems of the 1st class for a studied production material system. We aim at the ease of the resolution task, proposing the work with the SOLVER Macro in the Excel environment, which only requires the introduction of the complete and correct mathematical program, to undertake the resolution of our previously formulated canonical form optimization problems.

The SOLVER is a present function in Excel. It is a very powerful tool that allows both optimization and allocation of resources. This tool is often used to solve equations. Indeed, it allows to find the minimum, maximum or closest value to a data while respecting the constraints. The solver therefore has the power to give the best solution, i.e. the optimum.

6. **Conclusion**

The current concept of dependability can be defined as "the totality of an entity's abilities to achieve the specified functional performances, at the right time, for the expected duration, without damage to itself and its environment (CNAM, 2001)". This recent definition of dependability was taken from a technical report carried out in 2001, by the French National Observatory of Arts and Crafts. Thanks to: Our dynamic and global vision of the system allowed by the systemic, in fact, attached to a systemic thinking opposite to the analytical thinking (classical or Cartesian-analytical); An exhaustive definition of the new paradigm of system defined by the systemic, which was given by Rosnay (1975), who wrote that "a system is a set of elements in dynamic interaction, organized according to a goal"; Our general hierarchical diagram of a system showing the classes of its subsystems and the different interactions (Fig. 1), indeed, by considering the holistic character of the company and its interacting production resources, especially its production material systems. & In the framework of the development of approaches to improve the science of breakdowns and failures, in particular its reliability and maintainability components: Our Production System Reliability and Maintainability Approach can respond perfectly to the requirements of industrialists in all sectors in terms of the maximum functional performance of their production material systems crossed with their lowest possible maintenance costs. By allowing the irreproachable mastery of the maintenance costs optimization, the reliability, the maintainability and consequently the availability of the interacting sub-systems of the first class of a studied system.

By consciousness and in view of the complexity of solving linear mathematical programs in addition to the considerable amount of time required to do so, we prefer working with the data analysis cell thanks to the SOLVER complement of Excel, which is used often and in a diverse field of applications, such as the maximization and the minimization of objective functions in the field of economy, the industry in general, the construction industry and military in particular.

Our research perspectives were conceived only during the development of the sections of this paper, in particular, the section developing the deployment of our production system reliability and maintainability approach, as well as the section describing the pillars of system maintainability, where we thought deeply to integrate the commitment and tenacity of the company (from top management to line operators) in a Total Productive Maintenance project, as a system approach oriented towards improving the efficiency of the production system and making one of the foundations of the company's management philosophy (management culture). In this context, we proclaim that we will work on two next research projects that will be the object of two primary works entitled: The Total Productive Maintenance, from a System Approach Applied to the Company and making a Principle of Quality Management to the Company Culture Regeneration; Optimization of the System Maintainability in a Distributed Context on the Company.

**References**


Biographies

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