Lifetime Extension of Distributed Power Networks by Evaluating the Joint Gaussian Probabilities of the Power Coverage of Generators to Load Centers Demand

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Abstract

To extend the lifetime of a distributed power network, that consists of several generators supplying several load centers, according to a joint Gaussian probability distribution functions (PDF) of various random variables with estimated probabilities of source-load power coverage. The probability of any generator covering a load center is assumed to be readily known from statistical data. The nth order joint Gaussian PDF of all generators to any load center is calculated according to variations in the value of the mean covered power. This procedure is expanded to all load centers with the assumption that they are independent. The overall joint PDF with probabilities above a certain threshold value, are selected in an algorithm to prolong whole network lifetime by removing redundancies in powering all generators in same time. The contributions of each generator in supplying overall network loads, are then estimated in an intuitive method according to the evaluated probabilities of each generator supplying whole network load. The lifetime extension is demonstrated in a case study example.

Keywords
coverage probability, joint Gaussian, lifetime extension, probability density functions, redundancies removal

1. Introduction

One of the most important economical concern in power generation networks is the lifetime of power generators, as well as associated machines, auxiliary and supplementary equipment. The overall network lifetimes can be extended when generators are loaded adequately, or when there exist a halt in their operation, such as elapsed times for maintenance, overhaul, check out and repair occasions, cases that are difficult to achieve when maintaining power supply to load demand is crucial. Generators are normally synchronized in parallel to increase availability for load demands, which means that all machines are spinning continuously, even for light loads. In low voltage (LV) power distribution networks, such as gas turbines, diesel engines and wind turbines, it is required to activate all generators of such lower ratings to cover distributed load centers continuously. To increase the overall network lifetime, some generators need to be disconnected at certain times, as outlined in many books (Glover 2002), (Freris 2008), (Boyle 2003).

This study investigates probable powering schemes to make the distribution network more resilient to unnecessary high operating costs, and to enhance the extension of the network lifetime, since lifetime and consumed energy are strongly related. With such distributed networks, it is required to arrange for combination groups or subsets of generators, powered at different times during a certain load cycle, and as a result, redundancies of activating all generators are avoided, leading to an increase of the lifetime of the units that are idle or inactive, as well as that their overall operational costs will be reduced. These power networks are analogous to ad hoc sensor networks (Jing 2014) in which subsets of sensors covering target zones, a case widely investigated in the literature for the purpose of increasing ad hoc and wireless sensors network lifetimes, with employing algorithms such as greedy optimum algorithm (Hongwu 2009) distribution coverage algorithm (Dhawan 2009) and energy balance algorithm (Zhang 2009).

A probabilistic study is attempted in this study, in which it is assumed that each power generating unit has an estimated probability of supplying a load center, based on previous statistical data, such as generation availability and continuity, spinning and reserved power, distance from generator to load center, transmission losses, operating costs, ease of accessibility, etc., just to name some. One way to estimate such probability is by studying and analyzing cases of load flows, faults and types of faults, overloaded lines, power directions, power availability, infrastructure issues, that can be established using readily available field databases, and software tools (Power World Corporation), (Easy Power). In this work, we shall consider a special, yet widely anticipated probability distribution function (PDF), which is the Gaussian or Normal function (Miller 2012) with respect to variation in the mean supplied power designated to the load, which is considered to be random in nature within the average default power, that is planned to cover a load center.

Due to the multiple generators supplying one load demand, a multiple-order joint Gaussian probability distribution function (Miller 2012), (Stark 2012) is to be considered in this work. If the individual generator to the load PDF is Gaussian, then the overall joint PDF would also be Gaussian in nature, when it is reasonably assumed that different generators probabilities in supplying a certain load, are independent. The same procedure is conducted for all possible combinations of generators that are operating in parallel. As a result, there is a maximum of 2^k possible combinations for k generators, to supply a certain load location according to aspects such as network connectivity and availability, among other restrictions. Only generators' combinations that have joint probabilities higher than a predefined threshold value, will be selected for investigation, a situation that leads to removing possible redundancies. The method is implemented in several algorithms, such as load perturbation algorithm (Majid 2015), and load switching algorithm (Majid 2016). Figure 1 displays several generation units supplying several load zones with certain generator to load coverage probability (Cp) as discussed above.

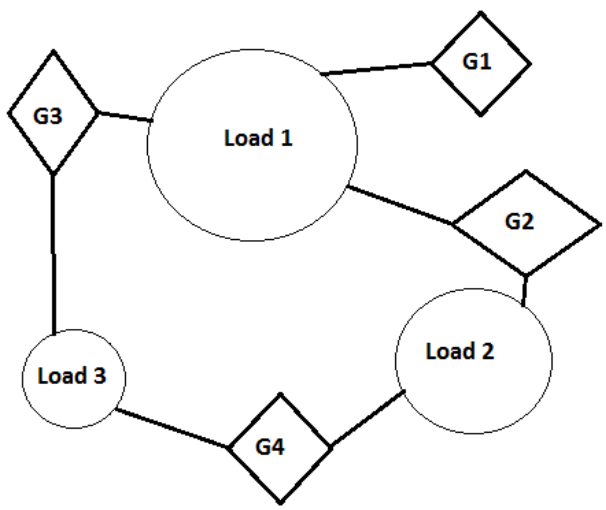

Figure 1. Four generators covering three load targets with different coverage probabilities

It can be noted from Figure 1, that in order to remove redundancies, there exists 4 possible generator subsets or coverage groups which can supply all three target loads in full: {C1=G1,G4}, {C2=G2,G3}, {C3=G2,G4} and {C4=G3,G4}, as depicted in Figure 2. Among these four possibilities, other possible redundancies can also be removed, depending on the values of their probabilities in supplying all load center nodes, as implemented in cost dispatch lifetime extension method (Majid 2020), and cost optimization technique (Majid 2020)

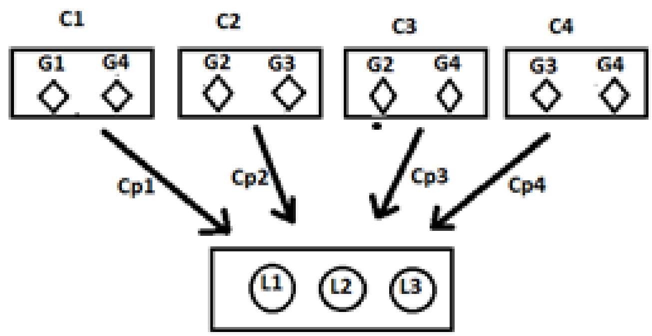

Figure 2. Four generator groups or subsets with full target loads coverage.

Hence, Cp1, Cp2, Cp3 and Cp4 denote the overall or joint probabilities from any number of generators to all loads. To remove redundancies, each group may be energized at a different time within an assumed cyclic period. When several generators are to operate in parallel as a group, within a certain designated time slot, a load dispatch cost analysis may be used to determine the economical sharing of generation among each subset of generators (Hodge 2010). The procedure steps conducted in this study are displayed in Figure 3.

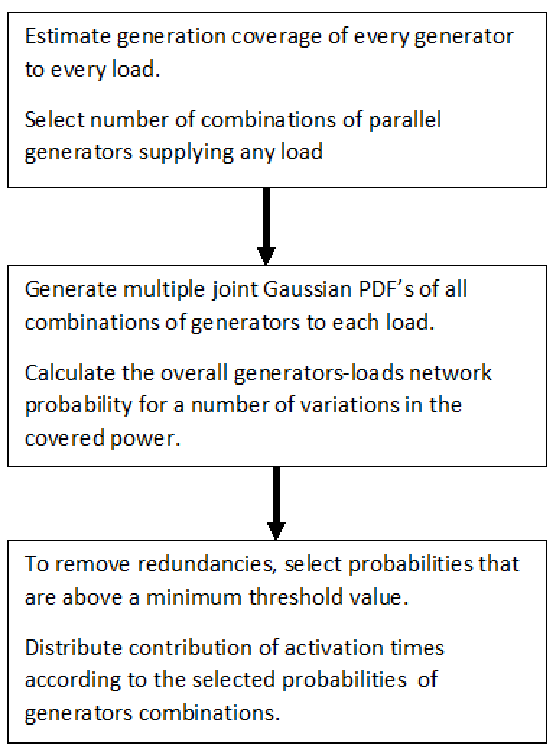

Figure 3. Procedure analysis steps employed in this study.

Redundancy removal is performed intuitively in a two folded attempts, namely; time slotting of each generators' combination, subset or group, as well as adjusting activation time of the individual generator units within each group in a most economic method, according to the calculated joint Gaussian probabilities.

2. Methodology

2.1 Joint Gaussian PDF of a power distribution network

Each generator has a certain probability of supplying power to a load in a sustainable manner, that is governed by network individual element components' probabilities, such as power availability and supply continuity, occurance of different faults, ease of supply and manpower accessibility, etc., which each can be estimated to be having a maximum *speculated* value with a normal PDF with respect of variation in the mean power coverage from a generation node to a load node. We shall assume that the individual element of power coverage probability is not uniform, but follows a Gaussian PDF pattern, with an average value, and a covariance. Consider a generator-load *G-L* network of *N* generators covering *M* target load zones, with *N* random coverage probabilities of joint Gaussian pattern

$$F_X(x) = \frac{1}{\sqrt{(2\pi)^N \ \det(C_{xx})}} \exp{(E)} \qquad (1)$$

where $E = -0.5(x - \mu_x)^T C_{xx}{}^{-1}(x - \mu_x)$, μ_x is an N vector of mean values $=[\ \mu_1\ \mu_2\ ..\ \mu_N]^T$ and C_{xx} is an $N\ x\ N$ covariance matrix

$$\begin{bmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 & \cdots & \rho\sigma_1\sigma_N \\ \rho\sigma_1\sigma_2 & \sigma_2^2 & & \vdots \\ \rho\sigma_1\sigma_N & \cdots & & \sigma_N^2 \end{bmatrix} \qquad (2)$$

It can be deduced that for two-dimensional matrix, as an example, the joint PDF is

$$f_{X,Y}(x, y) = \frac{1}{2\pi\sigma_X\sigma_Y\sqrt{(1-\rho^2\ )}} \exp{(c)} \qquad (3)$$

where

$$c = (\frac{x-\mu_X}{\sigma_X})^2 - 2\rho_{XY}(\frac{x-\mu_X}{\sigma_X})\ + (\frac{x-\mu_Y}{\sigma_Y})^2$$

Figure 4 depicts the joint Gaussian PDF surface plot with $\mu_X = \mu_Y = 1$, $\sigma_X = \sigma_Y = 2$, and $\rho_{XY} = 0.8$. The relation of joint Gaussian probability of Eq. (3) can be represented symbolically by MATLAB as,

$$f\ _{X,Y}\ (x, y) = \ 5/12*exp(-25/36*x\text{^}2 + 5/18*x - 5/18 + 10/9*y*x + 5/18*y - 25/36*y\text{^}2)/pi$$

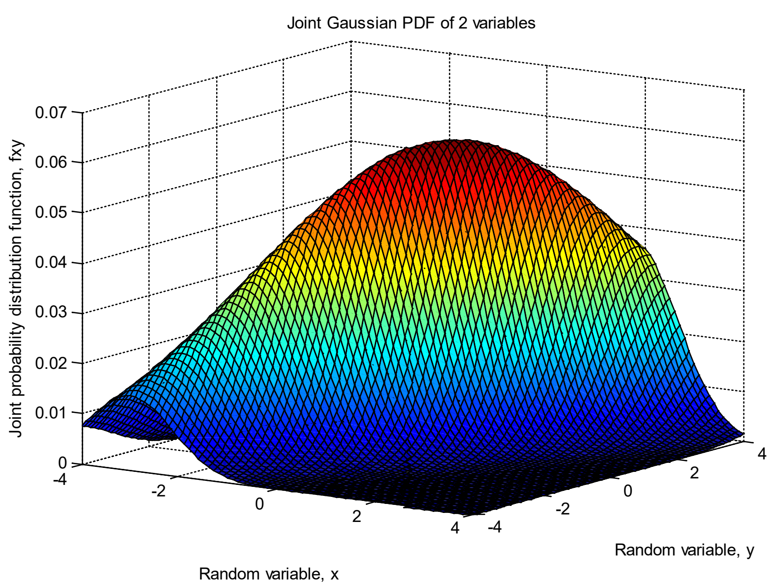

Figure 4. Surface plot of the joint Gaussian probability of two random variables, x and y.

The marginal PDFs of x and y are also Gaussian and evaluated as (Miller 2012)

$$f_X(x) = \frac{1}{\sqrt{2\pi\,\sigma_X^2}} \exp(-\frac{(x-\mu_X)^2}{2\sigma_X^2}) \quad (4)$$

$$f_Y(x) = \frac{1}{\sqrt{2\pi\,\sigma_Y^2}} \exp(-\frac{(x-\mu_Y)^2}{2\sigma_Y^2}) \quad (5)$$

Figure 5 shows that the marginal PDFs of a joint Gaussian function are also Gaussian, and with no correlation, they become independent to each other, that implies that the joint Gaussian PDF is merely the cross multiplication of each marginal PDF.

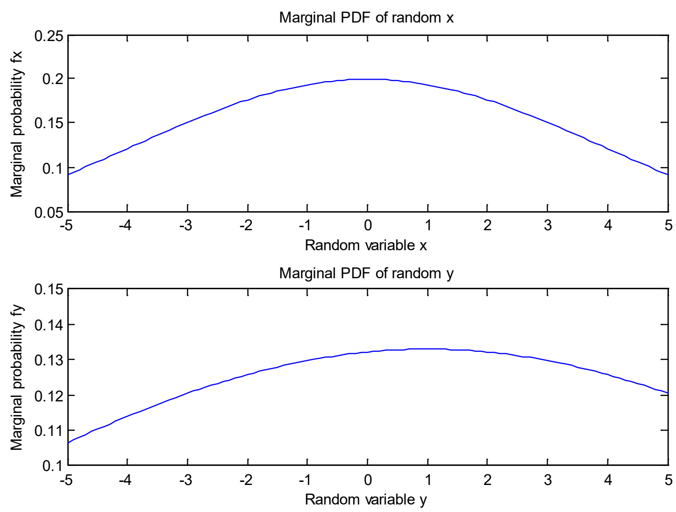

Figure 5. Marginal PDF of two variables joint Gaussian, x and y

We shall extend this procedure for multiple joint Gaussian PDF networks, comprising of several generators, feeding several load centers. Consider a case using Eq. (1) with all coverage probabilities being all mutually uncorrelated. This means that: con(x_i,x_j)=0 for all obj. A direct result of this case, is when the off-diagonal elements of the covariance matrix are zero, That's,

$$C_{XX} = \begin{bmatrix} \sigma_1^2 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \sigma_N^2 \end{bmatrix} \quad (6)$$

the determinant of which can be evaluated as

$$det\ (C_{XX}) = \sigma_1^2 \sigma_2^2 .. \sigma_N^2 \quad (7)$$

and the inverse as,

$$C_{XX}^{-1} = \begin{bmatrix} \sigma_1^{-2} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \sigma_N^{-2} \end{bmatrix} \quad (8)$$

The quadratic form that appears in the exponent of the joint Gaussian PDF, E becomes

$$(x - \mu_x)^T\, C_{xx}^{-1}\, (x - \mu_x) = \sum_{n=1}^{N} \frac{(x_n - \mu_n)^2}{\sigma_n^2} \quad (9)$$

hence, the marginal PDF, $f_X(x)$ can be represented as

$$\frac{1}{\sqrt{2\,\pi^N\,\sigma_1^2\,\sigma_2^2\,..\sigma_N^2}}\exp\left\{-0.5\sum_{n=1}^{N}(\frac{x_n-\mu_n}{\sigma_n})^2\right\} \quad (10)$$

and simplified as,

$$f_X(x)=\prod_{n=1}^{N}\frac{1}{\sqrt{2\pi\sigma_n^2}}\,exp(-\frac{(x_n-\mu_n\,)^2}{2\sigma_n^2}) \quad (11)$$

It can be deduced that uncorrelated Gaussian random variance are independent, and equal to the product of their marginal PDFs. We shall extend this concept to multi-dimensional Gaussian PDF, each with generator-to-load related parameters, in order to find the overall function.

2.2 Quantitative estimation of probability

Network lifetime extension is effected by network coverage probabilities as well as reliability and failure rates, since they will determine the most efficient way of powering the generators and as a result extending the network lifetime. To determine the life extension of a network using a probabilistic method, we need to estimate the initial probabilities from any generator to any load target. These probabilities depend on many quantitative values that can be estimated statistically, or from field operational data. A number of estimation tools, but not limited, are proposed as examples of determining these probabilities. We shall assume that each distibution network adminstrator or controller assign a number of these estimated probabilities, that mostly suit and fit with this work analysis.

2.2.1 Load flow estimate

From operational day-to-day load flow analysis, the flow probability, P_1 from any generation node to a certain load is

$$P_1=\frac{I_i}{\sum_n I_i} \quad (12)$$

where Ii is the statistically average operational current flow from generation node i to the load node, and n is number of generators covering the same load.

2.2.2 Losses estimate

This can typically be the transmission line losses I^2R with line resistance R. Hence line losses probability P_2 is

$$P_2=1-\frac{I_i^2R_i}{\sum_n I_i^2R_i} \quad (13)$$

where R_i is the ith line segment resistance

2.2.3 Line and other faults estimate

This is normally proportional to the line length l, so

$$P_3=1-k\frac{l_i}{\sum_n l_i} \quad (14)$$

where li is the ith line length and k a proportionality constant.

2.2.4 Lifetime estimate

It is assumed that the individual lifetime LT_i is related to the overall network lifetime, and it is dependent on several measurable factors such as line loading effect, line contribution in the network, number of switching over a period of time, maximum line loads, etc., that's

$$P_4 = \frac{LT_i}{LT} \tag{15}$$

2.2.5 Coverage accessibility estimate

In order to supersede all above measurable probabilities, an overall probability estimates, termed P_5 is used as an expected estimate of the power coverage probability, that can be evaluated intuitively from knowledge of field and operational statistical databases.

It must be noted that other estimate categories can be included too. Hence, the probability of power availability from a generating node i to a load node j, can be represented as,

$$P_{ij} = (P_1 \cap P_2 \cap P_3 \cap P_4) \cup P_5 \tag{16}$$

$$P_{ij} = P_{1-4} + P_5 - P_{1-4} P_5 \tag{17}$$

where $P_{1-4} = P_1 \cap P_2 \cap P_3 \cap P_4$.

2.3 Procedure of analysis

Using the general vector Eq. (1), as an example for three generators covering a load center, and assuming their mean values μ=[0 0 0]^T, with

$$C = \begin{bmatrix} 4 & 2 & 1 \\ 2 & 5 & 3 \\ 1 & 3 & 6 \end{bmatrix} \tag{18}$$

The 3-dimensional joint PDF can be found symbolically from Matlab, as

$f_{X,Y,Z}$(x,y,z) = 1/268*exp(-21/134*x^2+9/67*x*y-1/67*x*z-23/134*y^2+10/67*y*z-8/67*z^2)*134^(1/2)/pi^(3/2).

As a result, the overall probability of covering a load center by any number of generators can be evaluated. At this stage, we shall impose different variations in the covered mean power, and evaluate the overall probabilities. It is also noted that the variations of covered power from estimated mean values are proportional to distance locations of their mean values. For any generators' combinations, the probabilities are calculated for certain values of covered power variations, and the minimum possible probabilities are selected as worse case benchmarks, to be considered for evaluation.

As for this example, for 3 generators covering one load, we calculate the probabilities of all combinations of generators, in this case 23=8 different combinations, and we select the minimum probability for all variations, say Δ1, Δ2 and Δ3, which are the covered power variations due to any of the three generators. It is assumed that the overall probability of all generators' combinations to one load center, are independent to each other, that's, $Prob(A \cap B \cap C..) = Prob(A)\, Prob(B)\, Prob(C)..$

Having multiple load centers to be covered by a set of generators, the above process is repeated for each load center, and the overall probabilities are evaluated by cross multiplication of each generator-load probability. With this analysis, we shall select all combinations of generators that cover all load centers, according to their largest probabilities when certain variations of covered power are assumed. In this analysis, probabilities above certain minimum allowed values are selected, whereas other generator contributions are eliminated, and hence the lifetime of the whole network is increased due to the removal of redundancies when all generators operating continuously at the same time.

Intuitively, the contributions of the several generators to all load centers are estimated to be proportional with the calculated probabilities, such that generating units with the highest probabilities, would be activated more in time than others. Other limitations in the overall probabilities can also be imposed to take care of minimum and maximum economic operation of the generators.

3. Results

3.1 Algorithm of used method

A flow chart, that is listed in the Appendix, depicts steps used in the used algorithm, such as evaluating random variables probabilities, joint nth order Gaussian probability, removal of redundancies, and contributions of each generator in supplying the total load demand.

In the first step, probabilities of generators supplying load centers are estimated for N number og generators and M number of load centers. The estimation of these probabilities are based on various factors, such as power supply continuity, units and transmission line faults, etc, as depited in Eqs. (12), (13), (14), (15) and (16), that can be evaluated from statistical field data of the electrical distribution network. The estimation technique is not confined, precise or qualitative, but can be modified or adjusted by network administrators. Further work maybe needed here.

In the second step, the joint Gaussian probability for each combination of generators are calculated, as there are 2^N-1 combinations of generators that can be powered at the same time. This process is repeated for each load center demand, since every combination has to supply total load. In the third step, the joint probabilities are calculated with amount of variations in the mean value of these probabilities, a predefined threshold is introduced, in which only joint probabilities larger than a predefined threshold value are selected. This would control redundancies on the used numebr of generators. Also, the sharing percentage of each generator combination are calculated for the total load demand.

The third step is repeated for a number of variations in the mean values, and hence the average sharing percentage of each generator combination is evaluated, together with contribution of each generator within a combination.The number of variations would control the accuracy and precision of the method used. Finally, all contributions of each generator in the different combinations, are added for all variations that are considered above. Hence, each generator can be powered according to its sharing percentage in all combinations.

3.2 Case study

The above analysis is applied on a case study of a network comprised of 3 generators covering two targeted load centers, denoted as 3G-2L, with values of the mean vector and C matrix for each load as

$$\mu 1 = [0 \quad 0 \quad 0],\ \mu 2 = [1 \quad 0 \quad -1], C1 = \begin{bmatrix} 4 & 1 & 2 \\ 1 & 5 & 3 \\ 2 & 3 & 6 \end{bmatrix}, \text{and } C2 = \begin{bmatrix} 5 & 3 & -1 \\ 3 & 4 & 2 \\ -1 & 2 & 6 \end{bmatrix}$$

where μ_1, C_1 are for load 1 and μ_2, C_2 are for load 2.

Since there are 3 generators X, Y & Z to cover all load demands, there exist 7 different combinations of generators to be investigated with variations in the mean covered power. Table 1 depicts probabilities of all of these combinations for load 1 as *F1* and for load 2 as *F2*, as well as for the total overall load *FT* due to a variation in the mean covered power of a value of *Δ=3*. Tables 2 & 3 are the same but for variation values of *Δ=4* and *Δ=5* respectively. All calculated values are rounded up.

Table 1. Joint probabilities of generator combinations with random variable variation value of 3

	F1	F2	FT
X	0.0028	0.0241	0.6748e-4
Y	0.0060	0.0028	0.1680e-4
Z	0.0096	0.0021	0.2016e-4
XY	0.0070	0.0154	1.0780e-4
YZ	0.0119	0.0062	0.7378e-4
XZ	0.0092	0.0037	0.3404e-4
XYZ	0.0014	0.0014	0.0196e-4

Table 2. Joint probabilities of generator combinations with random variable variation value of 4

	F1	F2	FT
X	0.0012	0.0146	0.1752e-4
Y	0.0030	0.0012	0.0360e-4
Z	0.0053	0.0010	0.0530e-4
XY	0.0019	0.0065	0.1235e-4
YZ	0.0052	0.0020	0.1040e-4
XZ	0.0032	8.0475e-4	0.0257e-4
XYZ	3.8878e-4	0.0014	0.0196e-4

Table 3. Joint probabilities of generator combinations with random variable variation value of 5

	F1	F2	FT
X	3.8507e-4	0.0073	0.0281e-4
Y	0.0012	3.8507e-4	0.0046e-4
Z	0.0025	4.0371e-4	0.0100e-4
XY	3.6509e-4	0.0021	0.0076e-4
YZ	0.0018	4.5933e-4	0.0082e-4
XZ	8.3695e-4	1.1081e-4	0.0009e-4
XYZ	7.3083e-4	0.3943e-4	0.0000e-4

It can be seen from Table 1 that combinations {X, XY, YZ} have total probabilities more than a threshold value of *0.5e-4* when mean power variations of $\Delta=3$, whereas for Table 2, the power variation of $\Delta=4$, shows that the combinations {X, Z, XY, YZ} have total probabilities within the same *0.5e-4* threshold value, and Table 3 indicates that combination {X, Z, XY,YZ} are within this same threshold value.

In order to extend the lifetime of all generators, redundancies of generators' combinations, acting at the same time, are to be eliminated. For example, from Table 1, only generators' combinations {X, XY, YZ}, having total loads probabilities of {0.6748e-4, 1.0780e-4, 0.7378e-4} are to be activated according to these values during a reference time period of 1. Intuitively, the contribution time of any generators combination *CT* can be estimated as

$$CT_i = \frac{Prob_i}{\sum_{i=1}^{k} Prob_i} \tag{20}$$

where k is number of combinations. So for the combinations {X, XY, YZ}, X=27.38%, XY=43.28% and for YZ=29.62%. It can further be deduced that for combination XY, the contribution of X within XY is 80%, hence the total rounded contribution of the three generators are:

$X = 27.38\% + 80\%\, x\, 43.28\% = 62\%$
$Y = 20\%\, x\, 43.28\% + 45.45\%\, x\, 29.6\% = 22\%$
Z=16%.

Table 4 depicts the generators' contributions when the variations of mean covered power equal to Δ=3, Δ=4 and Δ=5 respectively, when the average variations is considered.

Table 4. Generator combinations with random variable variation values 3, 4 and 5

Variations of covered power	GENERATOR		
	X	Y	Z
variation =3	62%	22%	16%
variation =4	61%	15%	24%
variation =5	64%	7%	29%
Average	62%	15%	23%

The values of these variations are comparable with the mean and variances of the assumed PDF's of the three generators. We may consider as many variations as inticipated from realistic situations, and the average of these variations is to be used. As seen from Table 4, generators X, Y and Z share 62.3%, 14.6% and 23% respectively of the total operating time, when average variations are imposed. We can conclude from the above calculations, that 190% of the total generators lifetime is saved.

It must be noted, that whereas a threshold value of *0.5e-4*, has been selected in the overall joint probability, for Table 1, any other appropriate value can be selected, yet the contributions of generators activation, will be changed accordingly. This threshold value depends on many factors, such as limitations on minimum or maximum allowed generated power of generators, accuracy of the method used, estimate of the variation of covered power, etc. Further work may be investigated in selecting this threshold value.

4. Conclusions

The lifetime of a network comprising a number of generators supplying the demand of several load centers, is increased by removing redundancies by activating only some of the generators with individual contributions, using a novel algorithm that calculates each generator probability of supplying the total load. In a case study of 3 generators supplying two load centers, the lifetime is increased by 190%.

In this study, we calculate the joint Gaussian PDF of several assumed random probabilities of network elements, such as maintaining continuous supply probability, different fault probabilities, least cost probabilites, etc. that can readily be estimated from available statistical field data.

It is noted that redundancies removal depends on the amount of variations considered in the mean values of the selected random variables. For every variation, redundancies are removed by selecting the maximum probabilities among the different combinations of generators' combinations. Adequate values of variations are assumed for the same case study example, in which the average is used in the evaluation of the total joint Gaussian PDF.

It is assumed that the joint probabilities of all generators to any load, are independent, and whereas cross multiplication of of all these probabilities is implemented in finding total or resultant probability, other methods can be used, such as calculating the minimum joint probabilities in 2 or 3-dimensional space.

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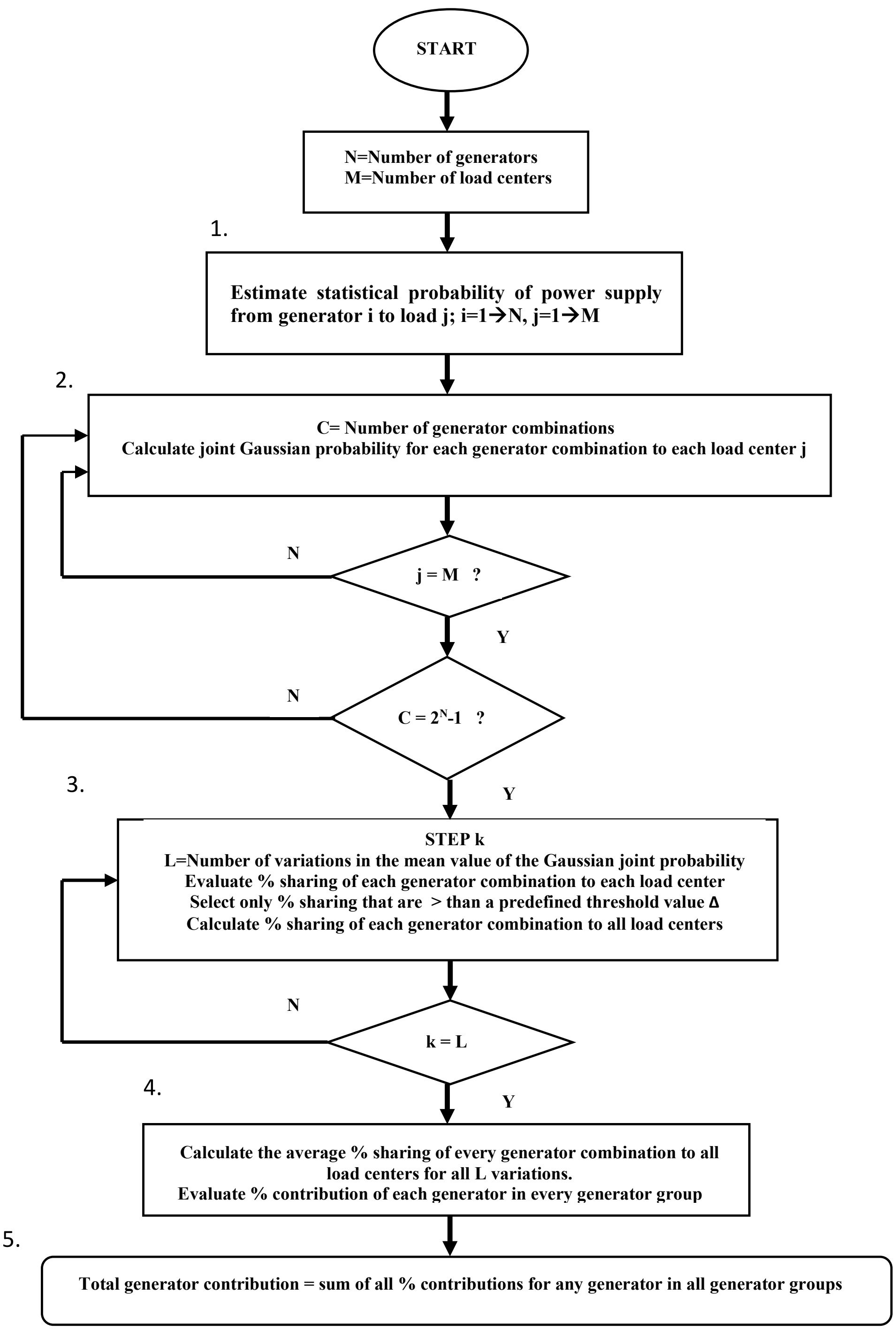
Biography

Amir Majid, Received M.Sc. degree in Electrical Systems Engineering from Surrey University, England, in 1976, and Ph.D. in Electrical Engineering from University of Loughborough, England in 1980. He has an industrial experience of 8 years in power stations, and industrial installations, and an academic experience of over 25 years in multi-national universities, with research in versatile fields of electrical engineering. He has published several books at Amazon arena such as

"Engineering Networks and Circuits with MATLAB Simulations";
https://www.amazon.com/Engineering-Networks-Circuits-Matlab-Simulations-ebook/dp/B07BZJWLP1,
"MATLAB, Based Electric Circuits Analysis";
https://www.goodreads.com/book/show/34837575-matlab-based-electric-circuits-analysis ,
"Computer Engineering Essentials";
https://www.amazon.com/Computer-Engineering-Essentials-Hardware-Programming/dp/179587631X.

APPENDIX


START
N=Number of generators
M=Number of load centers
1.
Estimate statistical probability of power supply from generator i to load j; i=1→N, j=1→M
2.
C= Number of generator combinations
Calculate joint Gaussian probability for each generator combination to each load center j
N
j = M ?
Y
N
C = 2^N-1 ?
3.
Y
STEP k
L=Number of variations in the mean value of the Gaussian joint probability
Evaluate % sharing of each generator combination to each load center
Select only % sharing that are > than a predefined threshold value Δ
Calculate % sharing of each generator combination to all load centers
N
k = L
4.
Y
Calculate the average % sharing of every generator combination to all load centers for all L variations.
Evaluate % contribution of each generator in every generator group
5.
Total generator contribution = sum of all % contributions for any generator in all generator groups