Optimization of Depth of Cut in Multi-pass Machining Using Hopfield Type Neural Networks

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Abstract

This paper deals with the problem of optimization of machining parameters for each pass. The developed algorithm is based on minimum unit cost criterion and the objective of the problem is to minimize unit production cost without violating any technological, economic and organizational constraints. A Hopfield-type dynamical network which employs a penalty function approach is proposed for solving the problem formulated by mixed integer linear programming. The results of the proposed approach tested on an illustrative example show that proposed algorithm is both effective and efficient and can be integrated into an intelligent process planning system for solving complex machining optimization problems.

Keywords: Machining parameters, Multi-pass, Depth of cut; Neural networks

1. Introduction

Traditional studies in the field of machining parameter optimization are limited to single-pass operations where total desired depth of cut is removed in just one pass [8]. However one pass is rarely preferred in practice and multi-pass operations where the amount of stock to be removed during machining exceeds the maximum allowable depth of cut are used. The number of passes and subdivision of depth of cut are important parameters in multi-pass machining operations. The followings are the existing approaches for determining and modeling of these parameters; i) consider the depth of cut \(d\) in each pass equal, and determine the number of passes by dividing the total depth of stock to be removed by depth of cut [6], ii) fix the number of passes \(m\) and determine depths of cuts by dividing the stock to be removed by the number of passes \(m\) which are equal again [12]. One of the main drawbacks of these two approaches is to take depth of cuts in each pass equal. In the relevant literature, it was shown that most of the time the optimal solution has unequal depth of cuts. iii) Probably the best-known way of determining the optimal number of passes with unequal depth of cuts is to use a dynamic programming [1, 10]. iv) A relatively new approach, called by optimal sub-division of depth of cut, to determine the optimal number of passes and the corresponding depths of cuts, was proposed by Yellowley and Gunn [15]. In this approach, the optimization is achieved in two stages as described in the next sections. This approach was further developed and matured by Gupta et al. [9]. The authors employed integer programming approach for the second step of the optimization algorithm. Bhaskara et al. [5] and Shunmugan et al. [13] used genetic algorithm approach to optimize the subdivision of depth of cut and number of passes. Al-Ahmari [2] presented a non-linear mathematical model to solve the problem in a single run. Satishkumar et al. [11] investigated the use of non-traditional optimization techniques like genetic algorithms, simulated annealing and ant colony optimization to solve the second step of the optimal sub-division of depth of cut algorithm.

The purpose of this study is to demonstrate the potential of neural networks for machining process optimization. For this purpose, a Hopfield-type dynamical network is employed. The objective of the proposed approach is to minimize total production cost and surface roughness without violating cutting constraints. The production model of Shin and Joo [12] is adopted to illustrate the proposed model and to simplify the comparisons between different optimization methods using illustrative examples.

The rest of paper is organized as follows. In the following section a brief description of machining parameter optimization problem and its mathematical formulation are given. The proposed approach is presented in the next section. The design of the proposed network is described in detail and the proposed algorithm is examined by an illustrative example. Finally, results of the proposed approach are compared to the results of other traditional and non-traditional techniques.
2. Solution Methodology

Yellowley and Gunn [15] proposed a two-stage optimization algorithm, called optimal subdivision of depth of cut, to solve the multi-pass machining problem described above. The authors divided the total production cost minimization problem into two sub-problems. Stage – 1 consist of determining costs for individual finish or rough pass considering various fixed values of depth of cut. A series of depth of cut is defined between minimum and maximum allowable depth of cuts. Minimization of cost for the finish pass can be achieved by using the maximum permissible value of feed under the constraints. In stage 2, number of rough passes \( n_i \), optimal combinations of depths of cut for finish pass \( d_{s_i}^* \) and rough pass \( d_{r_i}^* \) for \( n_i+1 \) (i.e. one finish pass plus \( n \) rough passes) and minimum total production cost are determined.

3. Proposed Approach

In this study, a Hopfield-type dynamical network is proposed for the second stage of the machining parameter optimization problem formulated by Shin and Joo [12]. The proposed network is based on the mixed integer linear programming model of Gupta et al. [9]. To solve the problem, two gradient networks that represent the mixed integer nature of the problem are constructed. To the best of our knowledge this is the first attempt using artificial neural networks for solving second stage of the machining parameter optimization problem.

Hopfield networks are one of the well-known dynamic systems used for optimization problems. The original Hopfield model introduced by Hopfield (1982) consists of a fully interconnected network of neurons capable of performing computational tasks. Each neuron is described by an internal and an external state. The activation function between internal and external states of the neuron can take several forms such as hard limit function, sigmoid function and linear function. In a Hopfield model, the states of the neurons are updated in a random manner. The objective function and the problem constraints are mapped onto a quadratic function that represents the energy of neurons system. The aim is to obtain a configuration minimizing the energy function. Hopfield has proved that with symmetrical weight matrix and non-negative elements on the diagonal of the weight matrix, the energy function, by performing gradient descent, minimizes until convergence to stable states which represent the local minimum values of the energy function. Hopfield and Tank (1985) has showed that if an optimization problem can be represented by an energy function, then a Hopfield network that corresponds to this energy function can be used to minimize this function. However, translation of the optimization problem into an appropriate energy function is, in general, a difficult task. Applying the most common method, penalty function approach, the energy function of the network is set equivalent to the objective function of the problem. Thus, the problem is reduced to an unconstrained form by including the constraints of the problem in the energy function as penalty terms. In penalty function approach, determining the penalty coefficients requires a tedious trial-and-error process.

3.1. Design of The Proposed Network

Gupta et. al [9] formulated the machining parameter optimization problem as a mixed integer linear programming model as follows:

- \( i = 0 \) implies finish pass
- \( i = 1, 2, \ldots, n \) implies \( i^{th} \) rough pass
- \( j = 1, 2, \ldots, m \), implies correspondence to \( j^{th} \) depth of cut

\[
\begin{align*}
1 & \quad h \\
0 & \quad h \\
\end{align*}
\]

\( n \) = The maximum number of rough passes required

Then, the integer programming representation of the problem is as follows:

\[
\sum \sum +
\]

\[
\sum = 1 \quad (15)
\]

\[
\sum \leq 1 = 1, 2, ..., \quad (16)
\]

\[
\sum \sum = \quad (18)
\]
The first constraint (16) implies that there is only one depth of cut selection for the finish pass and the finish pass must always be selected. The second constraint (17) means that there is only one depth of cut selection in case a rough pass is selected. The last constraint (18) indicates that the sum of individual depths of cut is equal to the total depth of stock removal. To construct a dynamical gradient based network representation of the model above, the followings should be carried out: determination of the network architecture, derivation of the energy function representing the proposed network, dynamics and proof of convergence of the proposed network, selection of parameters, and simulation of the proposed network.

3.1.1. Determination of the Network Architecture

The proposed gradient network has two interconnected networks, a maximum network and a continuous network. The maximum network \( X_{0j} \) network is used to assign a depth of cut for the finish pass \( X_{0j} \) and the continuous network \( X_{ij} \) network, \( i>0 \) is used to determine optimal number of rough passes and to assign depth of cut(s) for rough pass(es). \( U_{X_0} \) and \( V_{X_0} \) symbolizes the input and output to the neuron for \( i^{th} \) rough pass and \( j^{th} \) depth of cut, respectively. \( U_{X_0} \) and \( V_{X_0} \) denotes the input and output to the neuron for depth of cut of finish pass, respectively.

The dynamics of the proposed network will be defined in terms of input variables. Outputs of the network will be activated if \( d_j \) value of depth of cut is selected in the \( i^{th} \) pass. Otherwise, the state of the neuron will be set as zero to indicate that the neuron is not activated. Neurons with sigmoidal nonlinearity are used to represent discrete variables, \( X_{ij} \), so the activation function for discrete neurons can take any sigmoidal form with slopes \( \lambda \). In the proposed approach, a tangent-sigmoid function is used to convert discrete neurons to continuous ones. Since all variables of the second network are binary-values, the outputs of the neurons are converged to discrete values by using hard limit transfer function.

3.1.2. Derivation of Energy Function

The energy function for this network is constructed using a penalty function approach. That is, the energy function will consist of the objective function for the static problem plus a penalty function to enforce the constraints. Penalty functions for enforcing inequality and binary constraints are given in Figure 1-a and Figure 1-b. Applying the winner-takes-all (WTA) approach to the \( X_0 \) network, the energy terms for the first constraint can be omitted from the energy function. Also, the WTA guarantees the satisfaction of second constraint in mixed integer linear programming model, that is only one depth of cut is assigned for the finish pass. In addition, it ensures the binary constraint. The energy term for this constraint can also be dropped from the energy function. By applying hard limit transfer function for continuous neurons, the outputs may take values of either zero or one. Thus, the binary constraint \( X_j \in \{0,1\} \) is satisfied for all variables and the energy term for this constraint can be dropped from the energy function. Therefore the energy function takes the following form:

\[
\min \sum \sum a_{ij} + \sum \sum c_{ij} - 1
\]

Once the energy function is determined, it is necessary to consider the equation of motion of the neuron input. The dynamics of the proposed network are obtained by gradient descent on energy function. The computation is performed in all neurons at the same time, referred as synchronous update, so that the network operates in a fully...
parallel mode. The states of neurons at iteration \( k \) are updated as Euler’s numerical integration method. In order to use the proposed Hopfield-type network for solution of the problem, the convergence of the network must be proved. This can be done by showing: energy does not increase along the trajectories, energy is bounded below, solutions are bounded below, and time derivative of the energy is equal to zero only at equilibria. The energy function (19) satisfies the conditions above, hence it can be concluded that the time evolution of the network is a motion in space that tends to the minimum point as \( t \) goes to infinity.

3.1.4. Selection of Parameters
In order to simulate the proposed network, values for the following parameters given in section 4.1.3 must be chosen: (i) The penalty coefficients: \( A, C \) and \( D \), (ii) The scaling factor \( (\eta) \) and activation slope \( (\lambda) \), and (iii) initial conditions (states of neurons, \( UX_{ij} \)).

Because there is no theoretically established method for choosing the values of the penalty coefficients, the appropriate values for these coefficients can be determined empirically. In order to ensure smooth convergence, step size must be selected carefully. The dynamics of the proposed Hopfield-like gradient network will converge to local minima of the energy function \( E \). However, tradeoff problem will exist among the penalty terms to be minimized. In order to satisfy the each penalty term, its associated penalty parameter can be increased. But this causes an increase in other penalty terms and a tradeoff occurs. Thus, determining the appropriate values of the penalty parameters, network parameters and initial states are critical issues associated with gradient type networks.

Here, we proposed a gradient network that all neurons are updated synchronously. The proposed algorithm can be summarized by the following pseudo-code:

**Step 1:** Using a penalty function approach construct an energy function for the machining problem

**Step 2:** Initialize all neuron states to random values,

**Step 3:** Select the step size and activation slope, and determine penalty parameters,

**Step 4:** Iteratively compute the motion equations. Update neuron inputs by the first-order Euler method and then update the neuron output.

**Step 5:** Repeat the iterations \( n \) times and check the cost terms of the energy function penalized. If the required criterion is met proceed to Step 6, otherwise go back to Step 5.

**Step 6:** If the energy is converged to a local minimum, examine the final solution to determine feasibility and optimality.

**Step 7:** Adjust parameters \( A, C \) and \( D \) if necessary to obtain a satisfactory solution, reinitialize neuron states and repeat from Step 3.

4. Simulation Results
In order to evaluate the performance of the proposed gradient network in terms of solution quality, a simulation experiment was conducted.

4.1. Example-1
The example given by Shin and Joo [12] and considered by Gupta et al. [9] is used to validate the performance of the proposed approach. In this problem, for both rough pass and finish pass, minimum and maximum depth of cut are 1 mm and 3 mm, respectively. \( m_i = 20, i=0,1,2,3 \) is taken as 20 for generation of depth of cut series: \( j=1,2,...,20 \). Therefore, \( d_{i0} = 1.0 \) mm, \( d_{i1} = 1.1 \) mm and so on up to \( d_{i20} = 3.0 \) mm. The machining cost of a single rough and finish pass corresponding to each depth of cut are obtained by following the steps in Section 3.2, Stage 1. The results of the proposed algorithm in terms of unit cost for different stocks (6-10 mm) to be removed are given in Table 1.

4.2. Example-2
In the first example, the depth of cuts for both finish and rough passes were in the range of 1.0-3.0 mm. However, from a practical point of view this depth of cut range for finish pass seems to be high and the range is taken as 0.4 mm - 1.2 mm [9]. The depth of cut range for rough passes and all other data are the same as in Example-1. The machining cost of a single finish pass corresponding to each depth of cut is obtained by following the steps in Section 3.2, Stage 1. Table 2 displays the results of the proposed model for different stocks to be removed between 6 mm and 10 mm.

5. Results and Discussion
The problem above is formulated as a Hopfield-like dynamical network and is solved by using MATLAB 6.5. The results of the proposed method and other solutions in the literature are presented in Table 3. The comparison shows...
that artificial neural networks provide more optimal allocation of depth of cut resulting in lower total production cost than computational results of Shin and Joo [12] for both of the examples.

Table 1. Results of the proposed approach for Example-1

<table>
<thead>
<tr>
<th>Total stock to be removed</th>
<th># of rough passes</th>
<th>Finish Pass (mm)</th>
<th>Rough Pass_1 (mm)</th>
<th>Rough Pass_2 (mm)</th>
<th>Rough Pass_3 (mm)</th>
<th>Unit Cost ($/piece)</th>
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<td>-</td>
<td>1.94</td>
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Table 2. Results of proposed approach for Example-2

<table>
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<tr>
<th>Total stock to be removed</th>
<th># of rough passes</th>
<th>Finish Pass (mm)</th>
<th>Rough Pass_1 (mm)</th>
<th>Rough Pass_2 (mm)</th>
<th>Rough Pass_3 (mm)</th>
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<td>3.19</td>
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For the first example, it can be noted that, the proposed model demonstrates more or less, the same production cost that obtained by Gupta et al. [9] and Al-Ahmari [2] for all stock removal sizes. For the second example, for the stock removal size of 8.0, 8.5 and 9.0 mm, the proposed method gives significantly lower costs compared to other studies in the literature. For the other stock removal sizes, the cost remains more or less the same with the ones obtained by Gupta et al. [9] and Al-Ahmari [2]. From the results obtained, it is seen that besides the convergence to feasible and valid solutions, convergence of the proposed network to good quality solutions indicates its general applicability in also other machining parameter optimization problems. The main advantages of the proposed model over other conventional methods such as regression model and mathematical programming can be listed as follows.

- The proposed network has the parallel implementation property and can obtain solutions extremely fast by a dedicated hardware.
- It is possible and simple to implement the existing algorithm and structure without any modification even if new inputs parameters are added.
- It is not problem specific, it can be employed with different objective functions and constraints and can be extended for other machining parameter optimization problems.
- It provides a good representation of non-linear relationship between inputs and outputs.
- It does not employ statistics to find analytical relationships between machining parameters and does not require statistical background.

6. Conclusion

In this study, the problem of optimization of machining parameters was studied in order to minimize unit production cost without violating any machine and part constraints. A Hopfield-type dynamical gradient based network was proposed for the solution of the problem. By using the proposed network approach, the complexity of the considered problem which is expressed by too many constraints was reduced. Some of these constraints were reduced from the energy function using maximum networks while some ones using a logarithmic sigmoid function. The optimization capability of the proposed network was demonstrated by solving the problems considered by Shin and Joo (1992) and Gupta et al. [9], respectively. Compared to conventional methods in the literature, the proposed neural network model provides more or less the same optimal results. Thus, it can be concluded that neural networks provide an analytical alternative to conventional techniques which are often limited by strict assumptions of linearity, variable independence etc. The other advantages of the proposed approach mentioned in the previous chapter, makes it attractive for other machining parameter optimization problems. As a future research, selecting the parameters of the network automatically rather than trial-and-error, testing the performance of the proposed network with different objective functions and constraints will be of our interest.
Table 3. Comparison of results obtained by proposed approach and other approaches in the literature

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References