

Developing a hybrid algorithm for distribution network design problem

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Abstract

Distribution network design is one of the important issues in supply chain management. Two important sub-problems of distribution network design are inventory and location decisions. Traditional approaches considered inventory and location decisions separately which might result in suboptimal solutions. This paper considers the design of a distribution network and simultaneously determines the inventory and location decisions. The problem is formulated as a non linear mixed integer mathematical model and a hybrid heuristic algorithm, based on simulated annealing is developed to solve the model. The proposed algorithm finds optimal or near optimal solution in a reasonable computational time.

Keywords

Distribution network design, supply chain management, inventory location problem, location allocation problem

1. Introduction

Distribution network design is one of the main topics in supply chain management [1]. Three planning levels are defined in supply chain management. which are strategic, tactical and operational [2]. Strategic decisions such as determining the number, location and capacity level of facilities have long-lasting effects on the supply chain. On the order hand tactical level such as the inventory control decision are classified as medium term decisions. Due to the high dependency between the supply chain planning levels [3], decision of different levels must be considered jointly to have significant cost saving [4]. Traditionally, in order to model a distribution network, strategic and tactical decisions were considered separately, which might lead to suboptimal results [5]. In this study, we consider a distribution network design problem, and simultaneously determine strategic and tactical decisions. The network which is considered in this study distributes a single product from a single supplier to a set of retailers through a set of distribution centers (DCs). The distribution centers order the products from the supplier and fulfill the demand of retailers. The problem is to determine the optimal number and location of DCs, the allocation of retailers to open DCs and the best inventory level of the DCs in order to minimize the total cost.

The problem is NP-hard [6] and can be formulated as a non linear mixed integer mathematical programming. In order to solve the problem, we have developed a hybrid algorithm; consist of a local search heuristic algorithm and a simulated annealing.

2. Literature review

The existing literature on location problems focuses on finding the optimal number and location of facilities. On the order hand inventory problems determine the optimal inventory level and the best ordering policy of stocking points. Recently joint inventory location problems have been introduced which simultaneously determine the location allocation decisions and the inventory control decisions of the network. Daskin, et al. [7] studied a joint inventory location model while considering safety stock level at DCs. A Lagrangian relaxation was presented to solve the model. Candas et al. [8] discussed the importance of considering inventory as part of the network design models. In this article, the authors presented an integrated approach (simultaneously considering location allocation and inventory decisions) and compared the result with a sequential approach (solving the location allocation first and inventory stocking next) to show the benefits of simultaneous approach. Qi et al. [9] and Snyder et al. [10] respectively proposed a general solution algorithm and a Lagrangian relaxation algorithm to solve the model. Both papers dealt with the uncertain parameters. Gaur et al. [11] developed a bi-criteria model to minimize cost and maximize responsiveness and presented a two stage heuristic to solve the problem. Sourirajan et al. [12] investigated the tradeoff between lead times and inventory risk-pooling benefits and proposed a Lagrangian heuristic to solve the

model. Miranda et al. [1] studied an integrated inventory location problem. The authors proposed an improved Lagrangian heuristic by considering a new constraint to tighten the dual bound of a heuristic.

More recently [13] developed a model for joint inventory location problem in which the number and location of suppliers are also considered as decision variables besides of determining the inventory control and location- allocation decisions of distribution centers. Yao et al. [14] studied a distribution network in which multiple products are shipped from multiple suppliers to the distribution centers. The objective is to find the network configuration decision and inventory control decision in order to minimize the total cost while achieving required customer satisfaction.

3. Model formulation

We consider a distribution network consists of one supplier a set of retailers and a set of distribution centers. The DCs order single-item products from the supplier under a basic EOQ (Q, r) inventory policy and distribute them to the retailers. It is considered that the demand of retailers is independent and follows normal distribution. Moreover, the retailers do not hold any inventory and the inventory of retailers are centralized in a number of DCs in such a way that each retailer is allowed to assign to only one DC. The objective of the problem is to determine the network configuration decisions and also inventory decision, to minimize the total cost of the network, including the fixed cost of establishing DCs, ordering cost, holding inventory (working inventory and safety stock) and the cost of transportation from DCs to retailers.

In order to model the problem, we use the following notation:

- **Sets**

J set of retailers

I set of set of candidate DC locations

- **Indices**

I Index for DCs

j Index for retailers

Input Parameters

F_i : Annual fixed setup cost for DC_i

T: Transportation cost per unit of product per unit of distance

h_i : Inventory holding cost at DC_i per unit of product per year

O_i : Fixed ordering cost per order

C_i : Capacity of DC_i

d_j : Mean monthly demand at Retailer_j

v_j : Variance of monthly demand at Retailer_j

dis_{ij} : Distance between DC_i and Retailer_j

lt: lead time in months from the supplier to each DC

α : level of service that has to be achieved at the retailers

Z α : Standard Normal deviate such that $P(z \leq z\alpha) = \alpha$

- **Decision variables:**

Q_i : Order quantity at DC_i

SS_i : Safety stock level at DC_i

y_{ij} : Binary variable, taking the value 1 if Retailer_j is assigned to DC_i and 0 otherwise

x_i : Binary variable, taking the value 1 if DC_i is open and 0 otherwise.

The components of the objective function are described as follows:

1.1 Working inventory and safety stock cost:

The total inventory of the network consists of working inventory and safety stock. The following formula represents the total inventory holding cost. Where, $Q_i/2$ is the average working inventory.

$$\sum (h_i \cdot Q_i/2 + h_i \cdot SS_i) \quad (1)$$

Replacing the SS_i by its amount in the above formula the total holding inventory cost is calculated as follow:

$$\sum (h_i \cdot Q_i/2 + h_i \cdot Z_{\alpha} \cdot \sqrt{v_j} \cdot \sum (y_{ij})) \quad (2)$$

1.2 Ordering cost:

Ordering cost is calculated by multiplying the number of orders by the ordering cost.

$$\sum \sum O_i \cdot 12 \cdot (Q_i) / \quad (3)$$

1.3 Transportation cost:

Annual transportation cost from DC_i to Retailer_j per unit of product per unit of distance is calculated as follows.

$$12. \sum_{i \in I} (c_{ij} + \dots) \cdot x_{ij} \quad \forall i \in I, \forall j \in J \tag{4}$$

1.4 Fixed setup cost:

Fixed set up cost, which is the cost of establishing DC_i can be obtained as follows:

$$\sum_{i \in I} f_i \cdot y_i \quad \forall i \in I \tag{5}$$

Where y_i is binary variable that is equal to 1 for open DCs and 0 for close DCs.

1.5 Objective function

We formulate the problem as a nonlinear mixed integer mathematical model as follows:

$$\text{Min } \sum_{i \in I} (c_{ij} / + \dots) \cdot \sum_{j \in J} (x_{ij}) + \sum_{i \in I} \sum_{j \in J} c_{ij} \cdot (x_{ij}) + \sum_{i \in I} \sum_{j \in J} (c_{ij}) \cdot x_{ij} + \sum_{i \in I} f_i \cdot y_i \tag{6}$$

Subject to:

$$\sum_{j \in J} x_{ij} = 1 \quad \forall i \in I \tag{7}$$

$$x_{ij} \geq 0 \quad \forall i \in I, \forall j \in J \tag{8}$$

$$x_{ij} \leq C_j \cdot y_i \quad \forall i \in I, \forall j \in J \tag{9}$$

$$y_i \in \{0,1\} \quad \forall i \in I \tag{10}$$

Constraint (7) ensures each retailer can be served by only one DC. Constraint (8) makes sure that retailers are not assigned to closed DCs. Constraint (9) avoid DCs to supply more than their capacity during each order cycle and (10) specifies that y_i, x_{ij} are binary variables.

4. Solution approaches

The problem is NP hard so choosing an exhaustive search algorithm which searches all the solution space is quite time consuming. We propose a hybrid heuristic, consists of a simulated annealing and a hill climbing algorithm to solve the problem. The hill climbing algorithm is a local search algorithm and generates an initial solution during a predetermined number of iterations. This initial solution is applied as an input parameter of the simulated annealing algorithm. The mechanism of the hill climbing algorithm is as follows.

In each iteration of the algorithm, a random feasible configuration for the network is generated and the total annual cost will be computed. At the end, the best found solution will be presented as an input parameter for the next part of the hybrid algorithm.

The next part is a simulated annealing algorithm. The possibility of temporary accepting worse solution avoids the algorithm from being trapped in local optimal. Pseudo code of the algorithm is presented in Figure 1.

We also presented an exhaustive search algorithm to obtain the global optimal solution of the problem and evaluate the results of the proposed heuristic algorithm in terms of solution quality and computation time. Both hybrid and exhaustive search algorithms are coded in C++.

Table 1: Input Parameters used to solve test problems

Per unit per mile transpiration cost from DCs to Retailers	0.5
Ordering cost per order	500
Per unit per year average holding cost	2
Lead time in month	0.1
Level of service	97.5%

5. Computation Results and discussion

The computational experiment involves solving several test problems on a T2350, 1.86 GHZ with 1 GB RAM. The parameters of the problems are constructed as follows. The location of retailers and warehouses are uniformly distributed over the square of (0, 10]. The average monthly demands of retailers are uniformly drawn between [1000, 3000]. The variances of retailers' demands are uniformly drawn between [10,150]. The Capacity of DCs are selected uniformly between [1000, 9000] and the fixed cost of establishing each DC is selected as a proportion of its capacity. The rest of the input parameters are shown in Table 1.

In this research, the stopping condition for hill climbing and simulated annealing algorithms considered to be different depending on the size of the test problems. The associated stopping criteria for each test problem are presented in Table 2. For the simulated annealing algorithm, the number of iterations at a particular temperature or the equilibrium condition is set to be 300 for all the test problems. The initial temperature is considered to be 8000 for all test problems and decrease with a cooling rate of close to one.

We generated 14 test problems and solved them for 10 times. The average total cost and CPU time of the hybrid algorithm is compared with the optimal solution obtained by an exhaustive algorithm, and the gap between total costs is presented. The results are presented in Table 2. According to the Table 2, the presented hybrid climbing algorithm was able to find the optimal solution for 80% of Test problems and near optimal solution with a maximum of 0. 6% for other test problems. The longest time to solve the problems optimally is 270minutes for 10 DCs, 10 retailers, and by using the hybrid algorithm this amount of time reduced to about 50 seconds.

```

Input:
Max1-ite=stopping condition;
Max2-ite=equilibrium condition;
T= Initial temperature
B=Initial configuration
Ψ(B)=cost of B
Let best configuration= B
Let best cost= Ψ(B)
For i=1 to Max1-ite do
    Randomly choose a movement
    For j=1 to Max2-ite
        Generate a new configuration=N
        Compute the total cost=Ψ(N)
        If Ψ(N)< best cost then
            best configuration=N
            Best cost= Ψ(N)
        End if
        Else
            Let Δ= Ψ(N)- Ψ(B)
            Generate 0<K<1
            If K<exp(-Δ/T) then
                best configuration=N
                Best cost= Ψ(N)
            End if
        End for
    Update temperature
End for
Return:
Best configuration
best cost
CPU time
    
```

Figure 1: SA algorithm pseudo code

Table 2: comparison of optimal and. heuristic algorithm's output

NO.	# of DCs	# of Retailers	Optimal algorithm		Hybrid algorithm			Stopping criteria of the hybrid algorithm	
			Total cost	CPU time	Total cost	CPU time (sec)	Gap (%)	N	M
1	4	4	270355	0*	270355	0.018s	0	4000	50
2	3	6	529124	0*	529124	0.018s	0	4000	70
3	4	6	405056	0*	405056	0.110s	0	6000	250
4	4	8	418594	0.031s	553334	0.196s	0	10000	400
5	4	9	420550	0.125s	420550	0.641s	0	4000	500
7	7	7	271439	0.328s	271439	0.626s	0	45000	1000
8	6	8	376825	0.672s	478493	1.674s	0	90000	3000
6	4	10	506152	0.969s	506152	1.916s	0	100000	2000
9	6	9	436369	4.781s	436369	7.389s	0	300000	3000
10	8	8	383104	12.781s	383104	3.783s	0	600000	3000
11	6	10	457757	69.532s	457757	9.839s	0	1000000	3000
12	6	11	509332	222.234s	509332	21.720s	0	2000000	3000
13	9	9	377038	301.985s	377582	9.731s	0.14	2000000	3000
14	8	10	473545	21.77 min	474948	25.659s	0.2	6000000	3000
15	10	10	497032	270.9 min	500102	50.825s	0.6	10000000	3000

* CPU time less than 1 milli sec, N= # of iterations for hill climbing algorithm, M= # of iterations for SA algorithm

Gap=(heuristic cost-optimal cost)/optimal cost*100

6. Conclusion

Distribution network design in which strategic and tactical decisions are simultaneously considered is one of the recent research areas. The problem can be formulated as a non linear mixed integer mathematical programming. This paper has developed an efficient hybrid algorithm in order to solve the problem. The algorithm consists of a hill climbing algorithm in order to generate an initial solution and a simulated annealing to improve the initial solution and protect the algorithm from being trapped in local optimal. The results show that the proposed algorithm is able to find optimal or near optimal solution in a much less CPU time in comparison with an exhaustive approach. This study considers a system that operates under EOQ ordering policy, considering other ordering policy such as quantity discount would be of interest as a future research. It would be also interesting to extend the model for systems with fixed transportation cost from the supplier to distribution centers besides of variable transportation cost.

7. References

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