

## **Maintenance optimization models: a review and analysis**

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### **Abstract**

In broad terms, maintenance optimization models includes the mathematical models focused on finding either the optimal balance between costs and benefits of maintenance or the most appropriate time to execute maintenance. Parameters often considered in this optimization are the cost of failure, the cost per time unit of downtime, the cost (per time unit) of corrective and preventive maintenance and the cost of repairable system replacement. The foundation of any maintenance optimization model relies on the underlying deterioration process and failure behavior of the component. Over the last decades, maintenance optimization models have received growing attention, and by now it is a well-established area of research. This paper presents a brief review of existing maintenance optimization models. Several reliable models and methods in this area are discussed and future prospects are investigated.

### **Keywords**

Maintenance optimization model, Preventive maintenance, Corrective maintenance, Risk based optimization, Simulation

### **1. Introduction**

The importance of the maintenance functions and maintenance management has greatly grown in all sectors of manufacturing and service organizations. The principal reason is due to the continuous expansion in the capital inventory, the requirements for the functioning of systems and the outsourcing of maintenance. Maintenance management is gaining importance and support from science is needed to improve it. In theory, maintenance management could have benefited from the advent of a large area in operations research, called maintenance optimization [1, 2].

According to Dekker [1] and Sandve and Aven [3] the interest in development and implementation of maintenance optimization started in the early 1960s by researchers like Barlow, Proschan, Jorgenson, McCall, Radner and Hunter. Well-known models originating from that period are the so-called age and the block replacement models. In the age-type models the timing of the maintenance action depends on the age of the system, however for the block-type models the timing of the maintenance action is known in advance, it depends neither on the age nor on the state of the system [4]. A maintenance optimization model is a mathematical (stochastic) model which aims to quantify costs (in a wide sense) and to find the optimum balance between the cost of maintenance on one side, and the associated cost (benefit) on the other [3]. There has been extensive literature on models for maintenance optimization. Table 1 (Appendix A) represents some basic literature surveys in this area.

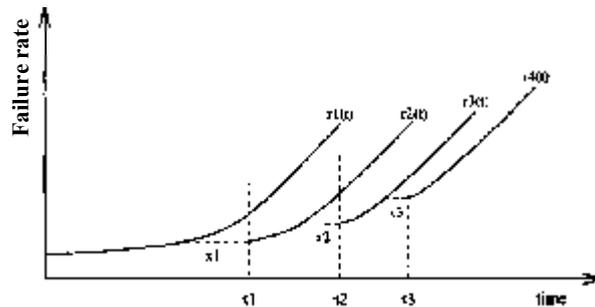
Maintenance optimization is one of the most critical issues in production since the failure of a system during actual operation can be a costly and dangerous event. When a machine fails to operate in a system, it does not only delay the completion time of the operations assigned on it but also affect all the other planned operations in the system. Consequently, the jobs cannot be finished on time and it will induce penalty and bad reputation to the company [5]. This optimization process can utilize different methods. It can be made by adding features and conditions that make the maintenance policy more realistic, for example by taking into account working conditions, the production schedule of the industry, safety issues, perfect and imperfect actions. Generally maintenance optimization models are classified according to the way they describe and represent natural variability and uncertainty in parameter, model and scenario. The use of deterministic methods does not provide information about potential risk which results in non-optimal maintenance planning for process plants. However, Probabilistic models use probability distributions to describe and represent natural variability and uncertainty in different cases [6].

## 2. Models on Optimization of PM Policies

Among the different types of maintenance policy, the preventive maintenance (PM) is widely applied in large systems such as production systems, transport systems, etc. PM consists of a set of management, administrative and technical actions to reduce the components' ages in order to improve the availability and reliability of a system (i.e., reduction of probability failure or the degradation level of a system's component). These actions can be characterized by their effects on the component age: the component becomes "as good as new", the component age is reduced, or the state of the component is lightly affected only to ensure its necessary operating conditions, the component appears to be "as bad as old". The PM corresponds to the maintenance actions that come about when the system is operating. However, the actions that occur after the system breaks down are regrouped under the title of corrective maintenance (CM). Some of major expenses incurred by industry are related to the replacements and repairs of manufacturing machinery in production processes. The PM is a main approach adopted to reduce these costs [7]. Although CM has a direct influence on the components of a system, it was not sufficiently studied. Recently, studies begin to focus on the optimization of PM policies. Traditionally, optimal PM intervention schedules have been obtained using models which involves minimization of the costs incurred in relation to maintenance activities. For considering both PM and CM policies, in the following section several models for optimization of PM policies are reviewed and categorized based on their approach for taking into account CM effect.

### 2.1 Considering CM from the cost point of view

Dedopoulos et al. [8] developed a model which determines the optimal number of PM activities to be scheduled within a time horizon of interest, the extent of the preventive maintenance by means of an age reduction of the unit (Figure 1) and the corresponding optimal value of the expected profit. A single unit working in a continuous mode of operation characterized by an increasing failure rate is considered. For the CM activities, only their corresponding costs are considered.



time after which PM activity  $i$  takes place after the previous one has finished

degree of age reduction when performing PM activity  $i$

( ) failure rate in period  $i$

Figure 1: The age reduction approach for maintenance [8]

Under the title of PM optimization, Tsai et al. [9] presented periodic PM of a system with deteriorated components. Two activities, simple PM and preventive replacement, are simultaneously considered to arrange the PM schedule of a system. The optimal activities-combination standing for determination of the action(s) required for the PM components on each stage by using genetic algorithms and by pursuing system unit-cost life maximization. The CM effect is only taken into account from the cost point of view. The same issue is repeated with Park et al. [10] when the authors tried to minimize the cost of a periodic maintenance policy of a system subject to slow degradation. Each PM relieves stress temporarily and hence slows the rate of system degradation, while the hazard rate of the system remains monotonically increasing. The optimal number and period for the periodic PM that minimize the expected cost rate per unit time over an infinite time span are obtained. The case when the minimal repair cost varies with time is also considered and explicit solutions for the optimal periodic PM are given for the Weibull distribution case.

In order to deal with a maintenance optimization problem for a series system, Duarte et al. [11] developed an algorithm to determine the optimum frequency to perform preventive maintenance in systems exhibiting linear increasing hazard rates and constant repair rate, in order to ensure its availability. Based on this algorithm the authors developed another one to optimize maintenance management of a series system based on preventive maintenance over the different system components. It is assumed that all components of the system still exhibit linearly increasing hazard rate and constant repair rate and that preventive maintenance would bring the system to the as good as new condition. The algorithm calculates the interval of time between preventive maintenance actions for each component, minimizing the costs, and in such a way that the total downtime, in a certain period of time, does not exceed a predetermined value. The maintenance interval of each component depends on factors such as failure rate, repair and maintenance times of each component in the system. It can be observed that the proposed analytical method is a feasible technique to optimize preventive maintenance scheduling of each component in a series system.

### **2.2 Considering CM as A Minimum Failure**

Many studies are proposing the optimized PM policies while they have considered the CM as a minimum failure. Hsu [12] developed an analytical model to perform the joint optimization of preventive maintenance and replacement policies in a queue-like production system with minimal repair at failures. A policy is considered which calls for a preventive maintenance operation whenever  $N$  parts have been processed. If a failure occurs and at least  $K$  preventive maintenance operations have been carried out, the system is replaced by a new one. Otherwise, a failure is handled by minimal repair. An analytical model is developed and the argument of renewal–reward theory is used to provide long-run expected profit per unit time for a given maintenance and replacement policy.

Levitin et al. [13] generalized a preventive maintenance optimization problem to multi-state systems, which have a range of performance levels. Multi-state system reliability is defined as the ability to satisfy given demand. The reliability of system elements is characterized by their hazard functions. The possible preventive maintenance actions are characterized by their ability to affect the effective age of equipment. An algorithm is developed which obtains the sequence of maintenance actions providing system functioning with the desired level of reliability during its lifetime by minimum maintenance cost. To evaluate multi-state system reliability, a universal generating function technique is applied. A genetic algorithm (GA) is used as an optimization technique. Basic GA procedures adapted to the given problem are presented.

Conventional PM policies generally hold same time interval for PM activities and are often applied with known failure modes. The same time interval will give unavoidably decreasing reliabilities at the PM activities for degradation system with imperfect PM effect and the known failure modes may be inaccurate in practice. To avoid this problem, Zhao [14] presented a PM policy with the critical reliability level. Through assuming that system after a PM action starts a new failure process, a parameter so-called degradation ratio is introduced to represent the imperfect effect. The policy holds a law that there is same number of failures in the time intervals of various PM cycles, and same degradation ratio for the system reliability or benefit parameters such as the optimal time intervals and the hazard rates between the neighboring PM cycles. This law is valid to any of the failure modes that could be appropriately referred as a “general isodegrading model”, and the degradation ratio as a “general isodegrading ratio”. In addition, life cycle availability and cost functions are derived for system with the policy. An analysis of the field data of a loading and unloading machine indicates that the reliability, availability and cost in life cycle might be well modeled by the present theory and approach.

Bartholomew-Biggs et al. [15] presented a formulation of the PM scheduling problem which enables the number of PMs to be treated as a continuous optimization variable. This formulation involves the global minimization of a non-smooth performance function. The authors consider models and solution algorithms that can be used to determine PM schedules that optimize a measure of system performance (e.g., minimizing mean cost over a lifetime or maximizing lifetime per unit-cost).

### **2.3 Simultaneous Consideration of CM and PM**

The CM effect on the failure rate of the components and consequently on the global system is often neglected in the PM context. There is a gap in the existing models to evaluate this effect and very little literature offers the possibility to consider it. Samrout et al. [7] proposed a new method that allows taking the CM effect into consideration while planning the PM policy and the CM is no more either minimal repairs or replacements. For this purpose, the proportional hazard function was used as a modeling tool. This method consists of calculating the number of applied

CMs and their efficiency. The age reduction technique was used to determine the “dynamic” number of applied corrective actions. The established comparison shows the importance of the CM effect on the failure rate and consequently on the adopted PM policy.

### **3. Risk Based Optimization Models**

Recently, the risk based approach to maintenance optimization has been addressed by many researchers. Risk based maintenance optimization (RBMO) analyzes the effect on the main objectives of alternative sets of strategies, where each set expresses an alternative maintenance action for each of the relevant components/units under consideration. Thus, it is desirable to establish optimization models for identifying maintenance actions and strategies that are best suitable for achieving the objectives. By evaluating the relationship between costs and benefits associated with each maintenance alternative, the “optimal” strategies can be determined. The models are providing basis for decisions, and are dealing with prediction of future performance of systems under alternative maintenance strategies. Such predictions are normally subject to uncertainties, as the effects of applied maintenance actions on the organizational objectives are hard to determine. This leads managers to a risk based approach [16].

Vatn et al. [17] presented an approach for identifying the optimal maintenance schedule for the components of a production system. Safety, health and environment objectives, maintenance costs and costs of lost production are all taken into consideration, and maintenance is thus optimized with respect to these multiple objectives. This approach to maintenance optimization takes advantage of various fields, e.g., decision theory, risk analysis and reliability and maintenance modelling.

Apeland and Aven [16] presented alternative probabilistic frameworks for risk based maintenance optimization, using a Bayesian approach. The Classical Bayesian and the fully Bayesian approach to risk and risk analysis are investigated. These approaches represent two completely different frameworks for dealing with risk and uncertainty. The authors discuss some key features of the frameworks including uncertainty treatment and type of performance measures to be used and clarified that how risk and uncertainty should be expressed and interpreted in a suitable maintenance optimization. It is concluded that, performing a detailed optimization for all components is not acceptable, that would require too much time and resources. Only in the cases with high risks and large uncertainties, the detailed optimization modeling can be justified.

The risk based maintenance optimization should be closely integrated in the business and maintenance management. In the previous literature only a few aspects of the maintenance management tasks related to the optimization are investigated which implies a need for further studies in this area.

### **4. Optimization Models with Safety Constraint**

Most maintenance optimization models focus on costs of planned and corrective maintenance, but do not explicitly incorporate the safety constraint dimension. There is however an increasing number of models linking maintenance and safety constraints. The study which was done by Vatn et al. [17] and reviewed in previous section, is one of the earliest works on this topic.

Aven and Castro [18] added a new dimension to the minimal repair replacement theory by introducing two types of failures, where one is safety critical. Different levels for the safety constraint are considered. The optimization produces decision support by providing information about the consequences of imposing various safety level requirements.

Vatn and Aven [19] proposed a comprehensive framework for maintenance optimization where decisions with a high safety impact are moved from the maintenance department to the “Safety Board” for a broader discussion. However, since maintenance optimization involves hundreds or thousands of decisions it is not manageable to have these kinds of discussions for all decisions. The Safety Board is involved at several important steps of the maintenance optimization. First in a more general discussion of priority setting and value trade-offs. Then the maintenance department performs required analyses to come up with a draft maintenance program. Together with this proposed program also the main safety issues related to maintenance are highlighted. Then there is a broad discussion of the main safety issues in the Safety Board. In such a discussion, principal guidelines are given to the maintenance department. A revised maintenance plan is then prepared by the maintenance department, and the plan

is finally approved by the Safety Board. The followings are some of the important elements of the proposed framework:

- Identification of members of the safety board;
- The prescriptive normative framework;
- The mathematical framework and preliminary optimization;
- Compilation of the results;
- Critical review of the maintenance plan by the safety board;
- Updating maintenance program and final approval.

As a final remark, the authors listed some pros and cons for their suggested framework. On the positive side, the framework enables the organization to:

- Be more conscious about important maintenance issues that influence safety;
- Ensure that important uncertainties are discussed, and not only hidden behind probability figures.

On the negative side, the following points were listed:

- An extra administrative layer is introduced. This requires resources and time to be allocated, which could have been spent on “real” maintenance;
- There is a risk that too many issues are lifted up to the safety board, and hence it is hard to see the bigger picture. Therefore, it is important to critically assess which topics that should be discussed in the Safety Board;
- If the Safety Board only deals with optimization issues, the focus on the execution of maintenance could be lost. Therefore, it is also recommended that the maintenance department provides reports focusing on the quality of the maintenance execution;
- As for all changes of work processes in an organization it is hard work to really reach the objectives set. So far we have little experience with how a Safety Board will work in practice in this context. However, in the Norwegian railways some of the ideas presented in this paper have been introduced with good results.

The authors mainly focused on the establishment of a preventive maintenance program. However, corrective strategies, and how to follow up the maintenance backlog are also important issues to be discussed by the Safety Board.

## **5. Simulation Optimization Models**

Owing to the complexity of the analytical models, limitations to simplifying them and unrealistic assumptions (such as supposing constant failure rates), many studies have used simulation techniques for modeling and optimization of maintenance policies.

Allaoui and Artiba [20] investigated the hybrid flow shop scheduling problem under maintenance constraints to optimize several objectives based on flow time and due date. In this model, the authors take also on consideration setup, cleaning and transportation times. Since this real life industry problem represents the double complexity (algorithmic and structural–functional), the authors illustrate the approach of integrating the simulation and the optimization to deal with. Using an experimentation study it is shown that the performance of heuristics applied to this problem can be affected by the percentage of the breakdown times.

Rezg et al. [21] presented a joint optimal inventory control and preventive maintenance strategy for a randomly failing production unit which supplies an assembly line operating according to a just-in-time configuration. According to this strategy, the production unit is submitted to an age-based preventive maintenance policy consisting in performing a maintenance action as soon as the unit reaches a certain age  $T$  or at failure whichever occurs first. The building up of a buffer inventory after each maintenance action is jointly considered in order to hedge against future capacity shortage during repair or planned maintenance actions whose respective durations are random. Two approaches have been considered to model the proposed strategy and to determine simultaneously the optimal values of the decision variables which are the age for preventive maintenance  $T$  and the buffer inventory level  $h$ . The first approach is based on a mathematical model expressing the total average cost per time unit over an infinite horizon as a function of the decision variables. Besides its relative complexity, the mathematical model relies on some approximation assumptions. The second approach is based on a combination of simulation,

experimental design and statistical analysis that provided the cost function related to the proposed exploitation strategy in terms of significant main factors and interactions related to the decision variables  $T$  and  $h$ . Besides the fact that the second approach using a simulation model allows to go beyond some restrictive assumptions, it provides a simple estimation of the cost function from which the best values of  $h$  and  $T$  can easily be obtained. For given set of parameters, both approaches were compared, the obtained numerical results were of the same order.

Boschian et al. [4] compared two strategies for operating a production system composed of two machines working in parallel and a downstream inventory supplying an assembly line. The two machines, which are prone to random failures, undergo preventive and corrective maintenance operations. These operations with a random duration make the machines unavailable. Moreover, during regular subcontracting operations, one of these machines becomes unavailable to supply the downstream inventory. In the first strategy it is assumed that the periodicity of preventive maintenance operations and the production rate of each machine are independent. The second strategy suggests an interaction between the periods of unavailability and the production rates of the two machines in order to minimize production losses during these periods. A simulation model for each strategy is developed so as to be able to compare them and to simultaneously determine the timing of preventive maintenance on each machine considering the total average cost per time unit as the performance criterion. The second strategy is then considered, and a multi-criteria analysis is adopted to reach the best cost-availability compromise.

Therefore, it can be concluded that for the situations which are difficult to study analytically, the simulation based approach can be of great help to deal with these problems.

## 6. Conclusion

Based on examination of literature, it is concluded that most of the literature addresses optimization solutions in static environments. However, in today's world of rapidly changing demands, high competition and high maintenance level requirements, it will be increasingly difficult to maintain a good performance when using existing static solution techniques. The research in the field of maintenance optimization should now move towards developing models, algorithms and heuristics that include the dynamic and stochastic aspects of current business. In this context, it is advised to develop approaches which simultaneously optimize the maintenance policies and different aspects of the maintenance management tasks. In this paper some of the existing maintenance optimization models were investigated (Table 2, Appendix B). Among the topics it covers are brief reviews of optimization models for PM policies, risk based optimization models, models under safety risk and simulation models. Based on examination of the literature, further detailed research can also be advised for each of the following issues:

- Optimization models with Simultaneous analysis of CM and PM (rather than assuming CMs as minimal repairs or replacements);
- Optimization models for integrating the risk based maintenance in the business and maintenance management;
- Models and approaches for simultaneously optimize the maintenance policies and different aspects of the maintenance management tasks.

## Appendix A

Table 1: Some basic literature surveys on models for maintenance and maintenance optimization

No.	Author(s)	Title	No. of Ref.	Cited By
1	Pierskalla and Voelker (1976)	A survey of maintenance models: The control and surveillance of deteriorating systems	259	370
2	Sherif and Smith (1981)	Optimal maintenance models for systems subject to failure - A review	524	194
3	Valdez-Flores and Feldman (1989)	A survey of preventive maintenance models for stochastically deteriorating single unit systems	129	349
4	Cho and Parlar (1991)	A survey of maintenance models for multi-unit systems	189	257
5	Dekker (1996)	Applications of maintenance optimization models: A review	132	257

## Appendix B

Table 2: Main contribution of presented papers

No.	Author(s)	Main Contribution
1	Dekker (1996)	To present a literature survey on applications of maintenance optimization models.
2	Dekker and Scarf (1998)	To present a literature survey on applications of maintenance optimization models.
3	Sandve and Aven (1999)	To study the optimal replacement problem of a monotone system comprising $n$ components, where the components are minimally repaired at failures.
4	Boschian et al. (2009)	To investigate the contribution of simulation to the optimization of maintenance strategies for a randomly failing production system.
5	Chung et al. (2009)	To propose a double tier genetic algorithm approach for multi-factory production networks, aiming to keep the system's reliability in a defined acceptable level, and minimize the makespan of the jobs.
6	Ghosh and Roy (2009)	To demonstrate an improved technique involving the maximization of reliability-based benefit-to-cost ratio (BCR).
7	Samrout et al. (2009)	To expound a new method to integrate the effect of CM while planning for the PM policy.
8	Dedopoulos and Smeers (1998)	to investigate the problem of determining how often, when and to what extent PM action should be performed on a deteriorating system
9	Tsai et al. (2001)	To incorporate genetic algorithms in planning periodical PM policy for a system based on maximizing unit-cost life of system.
10	Park et al. (2000)	To identify the optimal number and period for the periodic PM in situation where each PM relieves stress temporarily and hence slows the rate of system degradation, while the hazard rate of the system remains monotonically increasing.
11	Duarte et al. (2006)	To solve the problem of maintenance management of a series system based on preventive maintenance over the different system components.
12	Hsu (1999)	To addresses the joint effects of preventive maintenance and replacement policies on a queue-like production system with minimal repair at failures.
13	Levitin and Lisnianski (1999)	To generalize a preventive maintenance optimization problem to multi-state systems, which have a range of performance levels.
14	Zhao (2003)	To present A PM policy with the critical reliability level is in order to address the preference of field managers
15	B-Biggs et al. (2006)	To introduce a new problem formulation that allows the optimal number of occurrences of PM to be determined along with their optimal timings.
16	Apeland and Aven (2000)	To present alternative probabilistic frameworks for the maintenance optimization, using a Bayesian approach.
17	Vatn et al. (1996)	To present an approach for identifying the optimal maintenance schedule for the components of a production system.
18	Aven and Castro. (2008)	To add a new dimension to the minimal repair replacement theory by introducing two types of failures, where one is safety critical.
19	Vatn and Aven (2010)	To propose a comprehensive framework for maintenance optimization where decisions with a high safety impact are moved from the maintenance department to the "Safety Board" for a broader discussion.
20	Allaoui and Artiba (2004)	To deal with the hybrid flow shop scheduling problem under maintenance constraints to optimize several objectives based on flow time and due date.
21	Rezg et al (2005)	To present a joint optimal inventory control and preventive maintenance strategy for a randomly failing production unit which supplies an assembly line operating according to a just-in-time configuration.

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