A Variable Capacity Parallel Machine Scheduling Problem

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Abstract

Parallel machine scheduling problem is significant because it is a common phenomenon in real life and a sub-problem of multi-stage complex problems. Studies about parallel machine scheduling usually analyzes a single planning period in which all the machines are active in all periods. However, it is not always possible in real life for various reasons. Therefore, this study analyzed the case when there are more than one planning period and different numbers of parallel machines running in each period. The problem was sampled with a real-life application and the success of the resulting solution was discussed.

Keywords
Parallel machine scheduling problem, the assembly-line scheduling, integer programming.

1. Introduction
Production scheduling is practice of determining the processing order and time of jobs that make up a product with a single or multiple machines. Production scheduling may be in many different ways depending on the production type. Considering the processing complexity of scheduling problems, they can be examined under four different headings according to the number of steps that should be considered (French 1982) :

- Single-stage, single-machine problem is the simplest form of problem. Here, all jobs require a single stage of processing to be processed in a single machine.
- Single-stage, parallel machine problem, is similar to the single machine problem. Each job requires a single stage of processing to be processed in one of the parallel machines. However, this problem involves is multiple machines doing the same job.
- In multi-stage problems, each job is involved in a precise priority relationship in the process. Each job is required to be processed in machinery group according to its priority relationship. Multi-stage problems can be examined in two ways as flow shop and job shop. In flow shop problem, all jobs are processed in machine group with the same process sequence. In other words, jobs are subject to the same process sequence (technological constraints) and priority relationship in machines.
- Multi-stage job shop problem is the most common and most complex one in the classification. There are no constraints on the number of process stages of a particular job. In other words, each job in job shop problem has a unique process sequence to be processed in different machines.

The presence of multiple machines that can do the same job is called parallel machines. Scheduling of this type of machines is a more complex problem than single machine scheduling. Parallel machine scheduling problem is significant because it is a common phenomenon in real life and a sub-problem of multi-stage complex problems. These problems can be divided into three groups based on the parts processing time in machines (Pinedo 2002):

- identical, if a part can be produced within the same duration in all of the machines;

- uniform, if a part is not produced within the same duration in all of the machines but time differences can be explained with a parametric relationship;
- unrelated, if production times display an irregular pattern of differences.

This study deal with single-stage parallel machine scheduling problem and all of the machines in the study were assumed to be identical. The criteria used to assess how successful the scheduling is defined as performance criteria (French 1982). It is possible to use a single performance criterion and multiple and conflicting criteria. The performance criteria used most commonly in scheduling literature are listed below (Allahverdi 2008) :

C_{max} :Completion time of the latest job (makespan)

T_{max}: The maximum tardiness

TSC : Total setup cost

TST : Total setup time

$\sum c_j$: Total completion time

$\sum E_j$: Total earliness time

$\sum T_j$: Total tardiness time

$\sum U_j$: Total number of unfinished jobs

$\sum w_j\, c_j$: Total weighted completion time

$\sum w_j\, U_j$: Total weighted number of unfinished jobs

$\sum w_j\, E_j$: Total weighted time of earliness

$\sum w_j\, T_j$: Total weighted time of tardiness

In this study, the objectives of minimization of completion time of the latest job, total early completion and total tardiness were examined together.

The characteristics and objective functions of the process vary in scheduling problems. There are many studies about parallel-machine scheduling problem. For example, Chang (2005) proposed a genetic algorithm (GA) to solve the problem with the objectives of minimization of total tardiness and total completion time. Unlu and Mason (2010) used mixed integer programming (MIP) for parallel machine scheduling problem in their study. Li et al. (2011) proposed a simulated annealing algorithm for the identical parallel machine scheduling problem to minimize the makespan with processing times. Algorithm is tested based on the data with problem size varying from 200 jobs to 1000 jobs. Al-Khamis and M'Hallah (2011) considered the identical parallel machine scheduling problem to minimize the makespan with controllable processing times. The problem's objective is to maximize the expected net profit which corresponds to the sum of the cost of the capacities of the machines and the expected profit of on- time jobs. The stochastic model is solved using an algorithm. Demirel et al. (2011) made use of genetic algorithms in order to minimize the total tardiness. Many studies have usually utilized GA and heuristic methods to solve problems. In most of these studies, process characteristics such as sequence dependent setup times and ready times were considered. Unlike other relevant studies, in this study parallel machine scheduling problem with multiple periods and different numbers of machines in each period was examined.

The problem considered is described in the following section, the experimental results are reported in the third section and the conclusion and recommendations are presented in the last section of the study.

2. The Problem Considered

The problem considered was observed as the scheduling problem of assembly lines of a company manufacturing plastic parts. The production processes of the company consist of five sections: plastic injection, paint shop (wet paint over plastic - metal), screen printing, tooling and assembly. The assembly section contains 12 bench assemblies with one worker each. Semi-finished products come to these benches and they are assembled here. The jobs performed in these benches do not require specialization. In other words, each worker can do any job at bench assembly. In addition, the assembly process is completed in a single stage. So these assembly lines can be scheduled as parallel machines.

The factory has 2 shifts with 7.2 hours of capacity each. Generally, there are 12 operators during shift 2 but 10 operators during shift 1 in the assembly line. The number of the operators and the number of the active bench assemblies are the same in each shift.

The factory uses order-based production. Delivery time is of great importance because the factory serves as a supplier to firms operating on ‘just in time manufacturing” basis. Therefore, the first objective of the problem was determined as minimization of the makespan and the other objectives were minimization of the total tardiness and the total earliness.

The assumptions accepted while establishing the mathematical model of the problem are listed below:

- There are fixed times to complete the jobs.
- Each worker does a particular job within the same time and unfinished jobs are transferred to the next shift.
- There are ready times of jobs.
- The planning period is one day. However, when there are new demands there are changes in customer demands, the scheduling may be revised during the day. Therefore, since each new job is recorded among the pool of jobs to be scheduled, the jobs can be listed in accordance with the time of delivery.

The mathematical model developed based on these assumptions is given below. In the parallel line scheduling problem considered in this study, processing time of the job j did not vary according to the line it was assigned to. The scheduling was performed by taking into account the different capacities in the two shifts. Since the completion times of the procedures before the assembly line were different for each job, ready time of the job j was defined as r_j. The jobs were assumed to be indivisible. The sets, indices, parameters, decision variables and the mathematical model of the problem considered are given below.

Sets:

$N=\{1, 2,\dots, n\}$ job set

$M=\{1, 2,\dots, n\text{-}l+1\}$ order set

$L=\{1, 2,\dots, m\}$ lines set

Indices:

i and $j \in N$ are indices used to show a particular job.

$k \in M$ is the index used to show the job order.

$l \in L$ is the index used to show a particular line.

Parameters:

w_1 , w_2 and w_3 : are the respective weights of z_1 , z_2 and z_3 objectives.

n: number of jobs

m: number of lines

h: number of active lines in the first shifts ($1 \leq h \leq m$, the lines 1 to h are active and lines $h+1$ to m are inactive in the first shift)

a: time of a shift

p_j : processing time of job j

d_j: delivery time of job j

r_j : ready time of job j

M: a large positive number

Decision variables:

C_j: completion time of job j.

C_{max}: completion time of the latest job in m lines is equal to the largest one.

T_j : tardiness in job j $T_j = max\{C_j\text{-}d_j,0\}$

E_j : earliness in job j $E_j = max\{ d_j\text{-}C_j,0\}$

q_j: time remaining till the ready time of job j

y_{jkl}: job j receives 1 if scheduled in k^{th} order in line l but 0 in other cases.

<u>Objective Functions</u>

$$min\ z_1 = \mathrm{C}_{\max} \tag{1}$$

$$min\ z_2 = \sum_{j=1}^{n} T_j \tag{2}$$

$$min\ z_3 = \sum_{j=1}^{n} E_j \tag{3}$$

<u>Scalarized Objective Function</u>

$min\ z = w_1z_1 + w_2z_2 + w_3z_3$ (4)

Constraints

$C_j + M(1 - y_{j1l}) \geq r_j + p_j \qquad \forall j \in N, \forall \in L,\ l \leq h$ (5)

$C_j + M(1 - y_{j1l}) \geq r_j + p_j + a \qquad \forall j \in N, \forall \in L,\ l > h$ (6)

$\sum_j y_{j1l} \geq 1 \qquad \forall l\ \in L$ (7)

$\sum_{j \in N} y_{jkl} \leq 1 \qquad \forall k \in M, \forall l \in \mathrm{L}$ (8)

$\sum_{k \in N} \sum_{l \in M} y_{jkl} = 1 \qquad \forall j \in N$ (9)

$q_j + M(2 - y_{jkl} - y_{ik-1l}) \geq r_j - c_i \qquad \forall j \neq i, i, j \in N,\ \forall k > 1,\ k \in M,\ \forall \in L$ (10)

$C_j + M(2 - y_{jkl} - y_{ik-1l}) \geq c_i + p_j + q_j \qquad \forall j \neq i, i, j \in N,\ \forall k > 1,\ k \in M,\ \forall \in L,\ l \leq h$ (11)

$C_j + M(2 - y_{jkl} - y_{ik-1l}) \geq c_i + p_j + q_j + a \qquad \forall j \neq i, i, j \in N,\ \forall k > 1,\ k \in M,\ \forall \in L, l > h$ (12)

$C_{max} \geq C_j \qquad \forall\ l \in L$ (13)

$T_j \geq C_j - d_j \qquad \forall j \in \mathrm{N}$ (14)

$E_j \geq d_j - C_j \qquad \forall j \in \mathrm{N}$ (15)

$C_j \leq 2a \qquad \forall j \in N$ (16)

$\sum_{j \in N} y_{jkl} - \sum_{i \in N} y_{ik-1l} \leq 0 \qquad \forall\ k, l,\ k > 1$ (17)

$y_{jkl} \in \{0,1\} \qquad \forall\ j \in N, k \in \mathrm{M}, \mathrm{l} \in \mathrm{L}$ (18)

$C_j \geq 0 \qquad \forall j \in N$ (19)

$C_{max} \geq 0$ (20)

$T_j \geq 0 \qquad \forall j \in \mathrm{N}$ (21)

$q_j \geq 0 \qquad \forall j \in \mathrm{N}$ (22)

The model has three objectives. (z_1), objective function (1), represents the minimization of C_{max} which is equal to the largest one among the completion times of all the processes. (z_2), objective function (2), represents the minimization of the total tardiness. Finally, (z_3), objective function (3), represents the minimization of the early completion. The objective function (z) scalarized with the classic weighting method is given in (4). Constraint (5) and (6) indicate that the completion time of the first job assigned to any of the lines should at least be equal to the sum of the processing time, ready time and time of first shift (if line *l* is inactive in first shift) . Constraint (7) was formed in order to ensure that a job is assigned to the first order of each line. Constraints (8) and (9) respectively ensure that a job is assigned to one order in one line at maximum and that each order in each line is assigned a job. Constraint

(10) was used for finding the remaining time for the following job among consecutive ones to be ready. Constraint (11) and (12) were formulated so that the sum of the processing time with this time , the completion time of the preceding job is equal to longer than the completion time of the following job and time of first shift (if line *l* is inactive in first shift). Constraint (13) indicates that C_{max} will be equal to the greatest completion time of jobs. Constraints (14) and (15) determine the tardiness and early completion times of *j* job respectively. Constraint (16) indicates that completion time of any jobs shouldn't exceed the total time of two shifts. Constraint (17) ensures that the jobs are ordered without skipping any order. Finally, constraints (18)-(22) are the pointing constraints of the decision variables.

Table 1: The schedule prepared by the company

Shift	Line	Jobs	Sequence	Number of production	Production time (*hr*)
1	1	1	1	390	3,79
1	2	2	1	6500	11,74
1	3	3	1	2200	5,5
1	4	4	1	724	5,13
1	5	5	1	1300	5,42
1	6	6	1	190	2,9
1	6	7	2	210	3,21
1	7	8	1	710	4,73
1	8	9	1	600	3,75
1	9	10	1	84	0,58
1	10	11	1	440	3,36
1	11	12	1	900	6,88
1	12	13	1	840	6,07
2	1	14	2	500	4,86
2	2	15	2	2300	2,24
2	3	16	2	1725	4,31
2	4	17	2	900	6,38
2	5	18	2	1330	5,54
2	8	19	2	615	3,84
2	9	20	2	170	1,18
2	10	21	2	30	0,2
2	10	22	3	230	1,76
2	11	23	2	730	5,58

3. Experimental Results

The actual data from the supplier of white good manufacturing company were analyzed with GAMS/CPLEX and then the results were compared with the schedule prepared by the company itself. Table 1 shows a schedule prepared by the company for a day with two shifts. On that day, 12 assembly operators worked in the first shift and 9 assembly operators in the second shift. The objective function values for this schedule were calculated as z_1=50328 z_2=18884 and z_3=451196.

Same problem was solved by using the proposed model and the GAMS/CPLEX solver. Time limit is 8000 seconds. Obtained objective function values and their weights are given in Table 2.

Table 2: Solution obtained with different weight values

w_1	w_2	w_3	z_1	z_2	z_3
0.25	0.5	0.25	46512	16047	163474

As can be seen in Table 2, when all objective function values were considered together (w_1=0.25 w_2=0.5 w_3=0.25) and the total tardiness more critical than the others, obtained solution was more successful than existing solution for all objectives. As a result, the proposed model has the capability to solve the real-life problem within reasonable times. Also, it produced highly successful solutions compared with the existing solution.

4. Conclusion and Recommendations

In this study, a mathematical model was developed for parallel machine scheduling problem with multiple planning periods and different numbers of parallel machines in each period. The model had three objectives: to minimize the completion time of the latest job, to minimize the total tardiness and to minimize the total earliness. The model was used for scheduling the lines in the assembly department of a plastic factory and then the success of the obtained solution was discussed. According to the best of our knowledge, no works exists in the scheduling literature on the multi period parallel machine scheduling problem with different capacity in each period. A future study, as a continuation of this one, may develop a heuristic method for solving larger scale problems and then compare the results with those of the mathematical model.

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