A Multi-Criterion Decision-Making on Preventive Maintenance

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Abstract

This paper presents a multi-criterion decision-aided maintenance model with three criteria: reliability, maintenance cost and maintenance downtime. The Bayesian approach has been applied to confront maintenance failure data shortage. The model seeks to make the best compromise between these three criteria and establish replacement intervals by using Preference Ranking Organization Method for Enrichment Evaluation (PROMETHEE II) and Analytical Hierarchy Process (AHP), comparing the results obtained from the two decision-making methods, integrating the Bayesian approach with regard to the preference of the decision maker to the problem. Finally, by using a numerical application, the model has been illustrated.

Key words
Preventive Maintenance; Age-Dependent PM policy; PROMETHEE II; AHP; Bayesian Approach;

1. Introduction
Global trade, higher levels of automation and the desire to apply lean production are some factors that increase the demand for effective maintenance [15]. In recent decades industrial and service systems have realized that establishing a proper maintenance policy plays an essential role in achieving their objectives [6, 8]. This paper proposes a multi-criterion decision aided maintenance model with regard to three criteria important to selecting the best maintenance policy. They are maintenance costs, reliability and maintenance downtime criteria. This model not only considers the various aspects of a maintenance problem, but also attends to the preference of a decision maker. Furthermore, Bayesian approach has been applied to overcome failure data shortage. Finally, by using the two familiar decision-making approaches called PROMETHEE II and AHP, this paper tries to choose the best time alternative by comparing the results obtained from the two approaches.

2. Preventive Maintenance
Complex equipment and machinery systems used in the production of goods and delivery of services constitute the vast majority of capital invested in industry [17]. As time passes, the machines age and un-planned failures occur, causing the system performance to drift away from its initial state. Proper maintenance can increase the reliability of a piece of equipment or a system at regular intervals [16]. Preventive maintenance is done periodically before the failure of the system, and hence it is different from corrective or repair maintenance, which is carried out only after the failure of the item or the system [17]. To keep production costs down while maintaining good product quality, preventive maintenance (PM) is often performed on systems subject to deterioration [17]. Making use of CM could be costly for organizations because most of the time, CM takes a long time to have an acceptable effect on a failed system or component [10]. So it would be more rational to study PM models as a basic concept for the purpose of proposing an optimum maintenance model. In addition, PM policies are used for contexts where the component failure rate increases by age and usage [5].

3. PM Models

Although a lot of maintenance models have been created during the past decades, there are few maintenance policies on which all the other maintenance models can be based [20]. There is a categorization proposed by Wang (2002). According to him, there are seven categories of maintenance policies, of which five are preventive. They are age-dependent PM, periodic PM, failure limit, sequential PM and repair limit. He also indicates that the age-dependent policy can be the most common and popular PM. The age-replacement policy and its extensions belong to the age-dependant policy [20]. The age-replacement policy has been chosen as the basis for this research. Also in this paper, it is assumed that the replacement of a piece of equipment or part gives the system a good-as-new performance. In addition, there are two requisites for PM implementation in each system where [5]:

1. The replacement cost of a component (c_p) before failures should be less than the cost of replacement due to failures (c_f)
2. The component failure rate should increase by age and usage.

Maintenance has become one of the most important issues in the manufacturing industry due to high costs involved [17]. According to Maggard and Rhyne (1992) the maintenance can represent between 10 and 40 percent of the production cost in a company. So that maintenance cost cannot be ignored by maintenance managers. But sometimes when the maintenance cost rate is minimized the system reliability measures are also so low that they are not acceptable in practice [19]. It is moreover impossible to capture all of a system's effects in a cost function. Besides, downtime is very important and must not be neglected because the minimum downtime for a piece of equipment could result in undesirable consequences [5]. This paper seeks to determine PM intervals during which the three criteria are in their best compromise with each other.

4. Bayesian Approach

Mathematics has had an important role to extend maintenance models. In order to plan a maintenance program in this research, a failure distribution is needed which has a wear-out characteristic, namely the failure rate should increase with age. The Weibull model is a most prevalent distribution that satisfies this perquisite. It can be shown to be of the form:

$$f(t) = \beta/\eta \ (t/\eta)^\wedge(\beta - 1) \qquad (1)$$

η is called the scale parameter and β is called the shape parameter. In order to estimate the distribution function parameters, historical data are often used; Therefore, a large quantity of data is needed to obtain reliable estimates. But because of the rapid growth of industry, often sufficient historical information about the components or system failures is not available [7]. However, during the process of the system production and its operating time, reliability engineers and specialists find out by intuition about its failure behavior [7]. Bayesian analysis is one way to enter this information into the decision-making process in order to make a more objective decision. Therefore, the Weibull distribution parameters are considered random variables with a priori distributions representing specialist knowledge: $\mu(\eta)$ and $\mu(\beta)$.

5. Mathematical Configurations of the Criteria

In order to evaluate the time alternatives, the mathematical configurations of the criteria are needed. To achieve this goal, the mathematical equations of the three criteria have been illustrated.

The reliability formula which is based on reliability definition is mathematically defined as follows where f(t) is the density function of the component failure behavior.

$$R(t) = \int_t^\infty f(t)\,dt \qquad (2)$$

The second and third equations are the maintenance cost and maintenance downtime formulas.

The details of these formulas are defined as follows:

c_p : Replacement cost before failure

c_f : Replacement cost due to failure

T_p: The time taken to make a preventive replacement

T_f : The time taken to make a replacement due to failure

$$C(t) = \frac{Cp \times R(t) + Cf \times [1 - R(t)]}{(t+Tp) \times R(t) + \left((\int_{-\infty}^{t} x\, f(x)\, dx / [1 - R(t)]) + Tf \right) \times [1 - R(t)]}$$

$$D(t)= \frac{Tp \times R(t)+ Tf \times [1-R(t)]}{(t+Tp) \times R(t) + \left((\int_{-\infty}^{t} xf(x)\, dx / [1-R(t)])+Tf \right) \times [1-R(t)]}$$

By making use of Bayesian approach, the parameters of Weibull distribution are considered random variables and their distributions should be assessed from the specialist information on these variables $\pi(\eta)$ and $\pi(\beta)$. Hence, reliability and cost criteria formulas follow these equations:

$$E(R(t; \eta,\beta)) = \int_{t}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \pi(\eta)\, \pi(\beta)\, f(x; \eta, \beta)\, d\eta\, d\beta\, dx$$

(6)

$$C(t) = \frac{Cp \times R(t)+ Cf \times [1-R(t)]}{(t+Tp) \times R(t) + \left(\int_{-\infty}^{t} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} x\pi(\eta)\pi(\beta) f(x;\eta,\beta)\, d\eta\, d\beta\, dx \right) \times Tf \times [1-R(t)]}$$

6. PROMETHEE One of the Multi-Criterion Decision-Making Methods

By taking a look at decision-making problems in the real world, it can be seen that most of them are multi-criterion. Making decisions in maintenance programs can be a multi-criterion decision problem. A multi-criterion problem is mathematically defined as [1, 2, 3]:

$$\text{Max}\{g_1(a), g_1(a), \ldots, g_i(a), \ldots, g_k(a) \mid a \epsilon A\} \qquad (7)$$

Where A is a finite set of n possible alternatives $\{ a_1, a_2, \ldots, a_n \}$ and $\{ g_1(.), g_2(.), \ldots, g_k(.) \}$ is a set of evaluation criteria.

PROMETHEE methods developed by Brans are one of the best known and most widely used outranking approaches in many applications [9]. In order to apply PROMETHEE, first the performance of the alternatives regarding all criteria needs to be determined. Then, alternatives are compared in pairs for each criterion based on generalized preference functions. Based on the weighted sum of single criterion preferences, positive and negative outranking flows are calculated as a measure of dominance of alternatives. Criteria weights reflect the subjective relative importance of the criteria. The net outranking flow can also be calculated to define a complete preorder on the set of alternatives according to PROMETHEE II [11].

We also need two types of additional information to run PROMETHEE [1]:

1. The information between the criteria that consists of the relative importance of the different criteria and which depends on the preferences of a decision maker. They are shown by w_j , j = 1, 2, … , k .They are considered as norm weights.

2. The information within the criteria is referred to assign a preference function to each criterion. After calculating the differences between each two alternatives for a criterion, $d_i(a, b)$, the decision maker's preferences are needed to identify the indifference threshold, q, that is the largest deviation to ignore, and the preference threshold p, i.e. the smallest deviation considered to be a preference relation between two alternatives. So in this phase, decision maker selects a generalized criterion, $F_i(d_i(a, b))$, to model his preferences for every criterion. After specifying the function parameters by the decision maker, the preference function can be obtained.

   $P_i(a, b) = F_i(d_i(a, b)) \qquad d_i(a, b) > 0 \qquad (8)$

   $P_i(a, b) = 0 \qquad d_i(a, b) < 0 \qquad (9)$

There are six preference functions suggested for decision makers. These functions have satisfied the conditions of many real-world problems.

The PROMETHEE II method has been chosen to rank the alternatives. This ranking is based on net flow φ (a) [5, 1]:

$$\Phi(a) = \frac{1}{n-1} \sum_{x \epsilon A} \sum_{i=1}^{k} [P\,i\,(a, x) - Pi(x, a)]wi. \qquad (10)$$

Therefore, each alternative can get the highest score on the net flow which is the best compromise solution.

7. AHP One of the Multi-Criterion Decision-Making Methods

One of the other Multi-Criterion Decision-Making method to weigh criteria and rank alternatives is the Analytic Hierarchy Process (AHP) which is developed by Saaty [12, 13, 14]. In order to apply AHP, three steps should be passed: Structure of model, Comparative judgment of alternatives and criteria, and Synthesis of the priorities. In the first step, a decision-making problem is arranged in a hierarchy model. Here, the overall objectives of the problem comes at the top level, criteria and sub criteria are arranged in the middle level and the alternatives are at the bottom.

Comparative judgment consists of pair wise comparison of each pairs of criteria or alternatives with respect to each criterion. The results of pair wise comparison are arranged in a pair wise comparison matrix form. The pair wise comparisons are based on a standardized comparison scale, Saaty scale of 1-9, which lies between equal importance's (1) to extreme importance (9) [4].The PROMETHEE II method has been chosen for outranking results in this research. The AHP method also has been chosen for comparing the results of PROMETHEE II and AHP.

8. Numerical Application

In order to evaluate the practical aspects of the model and to see the model's value in practice, a numerical application will be needed. Therefore, this section presents a hypothetical example which is closer to the real situation of a component. The data consist of information about the prior distributions of η and β.Therefore, there are β_1 and η_1 which are the parameters of the Weibull distribution that belongs to β and β_2 and η_2, the parameters of the weibull distribution that belongs to η.They have been obtained from specialist information. Also, the replacement costs before (c_p) and after failure (c_f), the time taken to make a replacement before (Tp) and after failure (T_f) are needed. These values are shown in Table 1. Also, the time alternatives incorporate the interval between 200 and 3000 days with an interval of 200 days between the alternatives.

Table 1. Model's Parameters

c_a (\$)	c_b (\$)	T_f (days)	T_p (days)	β_1	η_1	β_2	η_2
1000	250	3	0.5	3.40	4.15	2.80	2200

The performances of the alternatives are calculated for the three criteria and are shown in Table 2.The calculations in relation to Table 2 have been done by making use of Maple 13 software.

Table 2. Performances of Alternatives ($g_i(t)$)

T (days)	R(t)	C(t)	D(t)
200	0.9904	1.2874	0.0030
400	0.9643	0.6996	0.0014
600	0.9192	0.5315	0.0012
800	0.8566	0.4690	0.0011
1000	0.7802	0.4478	0.0011
1200	0.6947	0.4458	0.0012
1400	0.6051	0.4534	0.0012
1600	0.5162	0.4654	0.0013
1800	0.4314	0.4792	0.0014
2000	0.3538	0.4930	0.0014
2200	0.2849	0.5060	0.0015
2400	0.2258	0.5179	0.0015
2600	0.1762	0.5274	0.0016
2800	0.1358	0.5357	0.0016
3000	0.1034	0.5425	0.0016

In order to see the relationships between these three criteria in the time alternatives whose values have been plugged in Table 2, have been drawn as three curves in Figure 1. The horizontal axes show the time alternatives from 200 to 3000 days and the vertical axes show the values of three criteria in each the time alternative.

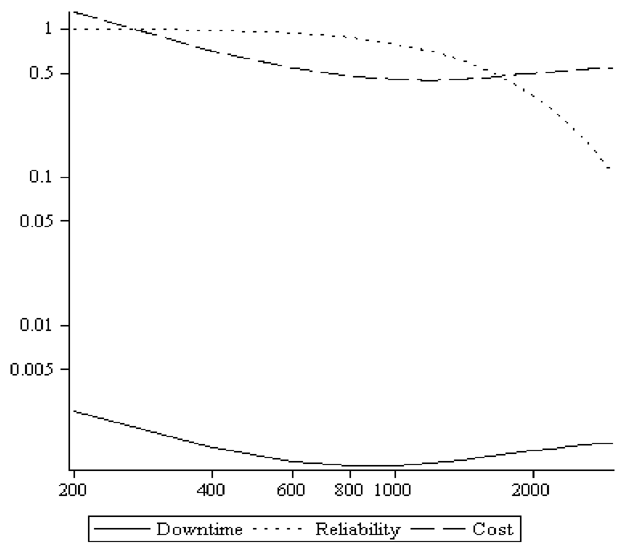

Figure 1: The relationships between these three criteria

It's obvious that the cost criterion should be minimized. As seen in Table 2 and Figure 1, this criterion gets its best value in the time alternative 1200 days. Also, the reliability criterion should be maximized. As seen in Table 2 and figure 1, it is descending during the time alternatives. In fact, it has its best value at point zero. 0 The downtime criterion also needs to be minimized and as can be seen in Table 3 and figure 1, it gets its minimum value in 1000 days. Therefore, for the time alternatives greater than 1200 days, the cost criterion increases during the time and simultaneously the reliability criterion is descending and the downtime criterion is increasing during the time. Therefore, evaluating the alternatives greater than 1200 days is not useful. They cannot result in the best compromise response between these three criteria. Hence, they can be neglected. Finally, there are six alternatives which need to be ranked. There are the time alternatives from 200 to 1200 days.

In order to make use of the PROMETHEE II ranking, the preference function is determined as the linear function, the fifth among the Decision Lab functions and it has been used for all of the three criteria. Their thresholds have been determined according to the preference of the decision maker and they are shown in Table 3.

Table 3. Criteria Threshoulds

	Thresholds	
Criteria	P	q
Reliability	0.0300	0.1000
Cost	0.0150	0.2000
Downtime	0.0011	0.0001

Moreover, the PROMETHEE methods need criteria weights which are chosen by the decision maker. The weights which are assumed for the three criteria in this paper are shown in Table 4.

Table 4. Criteria weights

Criteria	Weights
Reliability	35%
Cost	40%
Downtime	25%

After determining the whole data needed in order to use the Decision Lab software and rank the alternatives, they can be put in the software. Decision Lab 2000 is a multi-criterion analysis and decision-making software. Decision Lab 2000 was designed to be applied to various multi-criterion decision problems and designed for all Windows platforms. After putting the required data in the software, it ranks the time alternatives immediately.The PROMETHEE II ranking is used for the six-time alternatives chosen as shown in figure 2.Figure 2 shows that the best compromise solution is 600 days. It shows that alternative 600 days has made the highest score in net flow; therefore, it is the best compromise solution that PROMETHEE II has determined.

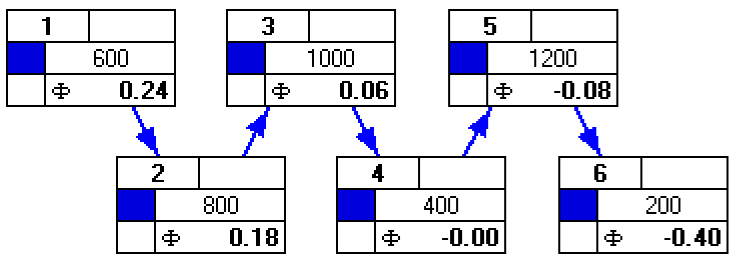

Figure 2: PROMETHEE II ranking 1

In order to make use of AHP ranking, the three steps cited in section 7 have been done. The first step is determining the structure of the model that is depicted in figure 3. The second step consists of comparative judgment that consists of pair wise comparison of each pairs of criteria or alternatives with respect to each criterion. This step has been depicted in figures 4, 5, 6 and 7. It should be noticed that pair wise comparisons have been done by decision makers and by considering the performances of the three alternatives. The third step is synthesizing of the priorities. This step is depicted in figure 8. It can be seen that the alternative 800 days has been chosen by AHP. Moreover, the figure 9 shows the inconsistencies of the model. It should be pointed out that the use of AHP has been made by Expert Choice 11 software.

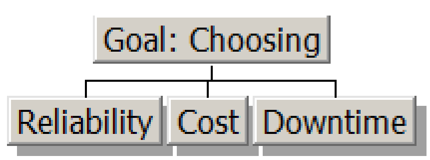

Figure 3: Structure of the model

	200	400	600	800	1000	1200
200		2.0	3.0	5.0	7.0	8.0
400			2.0	4.0	6.0	8.0
600				3.0	5.0	7.0
800					3.0	6.0
1000						4.0
1200	Incon: 0.06					

Figure 4: Pair wise comparisons with respect to Reliability

	200	400	600	800	1000	1200
200		5.0	6.0	7.0	7.0	7.0
400			3.0	4.0	4.0	4.0
600				2.0	2.0	2.0
800					1.0	1.0
1000						1.0
1200	Incon: 0.03					

Figure 5: Pair wise comparisons with respect to Cost

	200	400	600	800	1000	1200
200		6.0	7.0	7.0	7.0	7.0
400			2.0	2.0	2.0	2.0
600				1.0	1.0	1.0
800					1.0	1.0
1000						1.0
1200	Incon: 0.01					

Figure 6: Pair wise comparisons with respect to Downtime

Reliability	Cost	Downtime
	2.0	4.0
		6.0
Incon: 0.01		

Figure 7: Pair wise comparisons with respect to goal

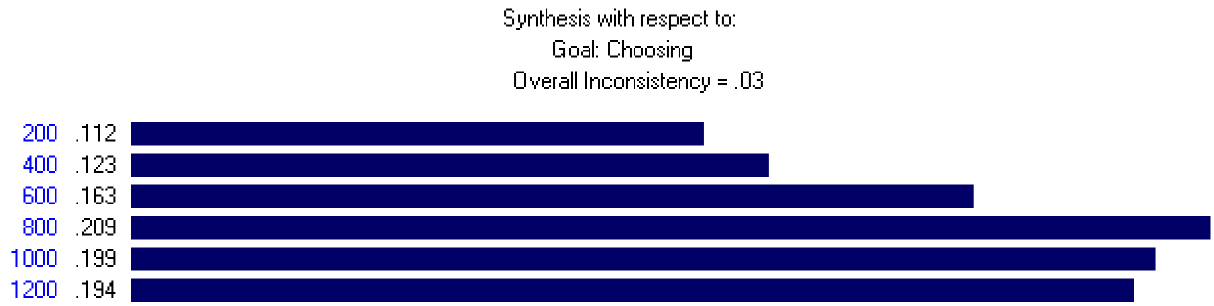

Figure 8: Synthesizing of the priorities

PID	Name	Overall	Goal: Choosing	Reliability (L: .323)	Cost (L: .588)	Downtime (L: .089)
		#Factors	3	6 Alts	6 Alts	6 Alts
0	Facilitator	.0273	.0088	.0610	.0255	.0052

Figure 9: Inconsistencies

10. Conclusion

This paper presents a multi-criterion decision-making model for preventive maintenance planning which determines the best compromise time for replacement of a certain item based on more than one criterion. This model also envisions the difficulty with the shortage of maintenance failure data by making use of Bayesian approach and PROMETHEE II for decision making. One of the most important goals that this paper seeks to reach is to give a broader view of the maintenance managers by considering more than one criterion in making an appropriate decision for replacement of an item in PM problems. Taking these two criteria into consideration in this paper does not imply that they are the most important criteria that need to be considered for replacement of an item in PM planning. It implies that in order to make a complete and timely PM planning which considers many aspects of the problem, decision makers have to study the problem completely and consider the factors which affect a PM planning for replacement of item because ignoring the influential factors in different situations can lead to disastrous results. Comparing the results of PROMETHEE II and AHP can show some differences. It seems that making use of PROMETHEE II can lead to better results. It can be because of mathematical characteristics of PROMETHEE II. Also it seem that the process of determining the preferences in PROMETHEE is more reliable than in AHP.

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