Inventory Model Involving Controllable Lead Time through New Approach

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Abstract

Lead time, the time between ordering date and actual date goods received, is one of the most important categories in studies of inventory management. In most Inventory models, lead time is considered as a pre-specified parameter, whether it is probable or certain, but has not been considered as a controllable and variable one. Although In many instances, lead time can be controlled and reduced by paying additional cost, on the other hand, reducing the lead time improves service level and reduces safety stock. As a result, it reduces the required investment in stock system and reduces errors resulted from forecasting demand in the lead time period. One of the goals of inventory management systems is to balance between these two types of costs. In this paper, we try to present an inventory management system with controllable lead time. To solve this problem, we utilize an exact approach which is easy in application. Moreover, results obtained from the proposed algorithm are exact. The results from computational experiments demonstrate the success of our approach in most of the instances.

Keywords
Lead time, inventory management, controllable and variable lead time, exact approach.

1. Introduction

Controllable lead time

Additional costs to reduce lead time, usually composed of three components are as follows:

1.1 Administrative costs

Administrative costs include reduction costs that are applied to the system at the time of preparation and ordering the goods. For instances, we ask whether employees do overtime or use part-time staff to reduce administrative costs. Hence, overtime costs and hiring part-time employees cost for reducing of lead time is classified in administrative costs.

1.2 Transportation costs

Time of transporting goods from supplier to customer, is one of the main components of lead time. Often, the speed of transportation and the cost ratio is reversed. For instances, transportation from supplier to the customer can be through sea, railway, road, air or a combination of them; that is, each method has a specific speed and costs.

1.3 Accelerating cost of Supplier operations

Suppliers' production time, is a small part of suppliers lead time. In fact, non-productive time includes an important component of lead time of the suppliers. There is no doubt that the proposed amount to supplier, the impact of priority is getting a lot. This alone affects lead time reducing. Also, if a buyer willing to supply some of the cost of preparation, the lead time will be reduced. Overall, the supplier point of view, to reduce lead time, it must act to bring a heavy investment in raw materials, components and finished products inventory. These costs, supposedly is transmitted to the customer, if it calls for shorter lead time. It should be noted that although cost reduction can be divided into three mentioned groups, but there are cases that should any of these groups are divided into sub-groups, each of them has its own reduced costs. As stated above, lead time can change with allocation of predetermined costs. However, in negotiations to determine lead time, lead time shown as a competitive variable because both the supplier and the buyer must have benefit in lead time reduction. That is, if buyers offer better suggestion, get high priority and the lead time is sooner considered. So often in the real world, lead time of the inventory system is the decision variable.

Considering lead time as a decision variable has a brief history. In inventory control books, this issue has not yet been considered. Although some authors have recognized this problem and the need to study lead time as a decision variable have pointed, but have not paid a serious discussion in this area. Start new job in this area goes to 1990, an article is published with the vision of controllable lead time, and they developed the model. In this article, only for current lead time with minimizing the average total cost, a model was presented. He assumed that the demand follows the Poisson distribution. In 1991 they published a new article in which the lead time is determined by considering specific economic quantity. Ben-Daya [3], proposed a model in which the two parameters (economic order quantity and lead time), was considered as a decision variable. Latest research that has been done in this field is done by I. iang-YuhOuhang, Neng-Che Yen and Kun-Shan Wu in 1996. In this paper, reduced cost has a linear relationship with the lead time. In this model, by assuming a specific level of service to determine the allowable slack, economic reorder quantity and optimal lead time is presented.

Later, Ouyang, Chuang (2000) considered a mixture periodic review inventory model in which both the lead time and the review period were considered as decision variables. Instead of having a stock-out term in the objective function, a service level constraint was added to the model. Pan and Yang (2002) presented an integrated inventory model with controllable lead time. The model was shown to provide a lower total cost and shorter lead time compared with prior researches. Chuan Lee (2005) first assumed that the lead time demand follows a mixture of normal distribution, and then relaxed the assumption about the form of the mixtures of distribution functions of the lead time demand and applied the mini-max distribution free procedure to solve the problem.

In this paper, we present a new approach for obtaining reorder point and lead time that can be used for every distribution function such as normal, poison, uniform distribution. By using this approach we can obtain the exact amount of reorder and lead time. This paper is organized as follows: In section 2, we present former model development. In Section 3, we define the parameters. Then, the problem modeling is presented in Section 4. The proposed approach for new problem is described in Section 5. In Section 6, we present a numerical example. Finally, the last section deals with conclusion.

2. Former Model Development

The former model is as follows: determining the lead time and the economic allowed slack, with respect to cost reduction linear function and normal distribution function for demand during the lead time. This model is a model developed by Iang model [4] considering the slack as a decision variable. In this model assumes that the reduced cost function linearly depends on the lead time and demands distribution follows a normal distribution. This model assumes that some slack in one period is considered as back order to be fulfilled in the next period and also some of them are considered as the lost sales. This percentage is determined by market conditions. Also the inventory system is permanently reviewing the system. Means, by reaching the level of predetermined stock, the new order is ordered.

3. Parameters

D: average demand per year (unit)
Q: Ordering Quantity
A: Fixed ordering cost
h: Inventory cost per year for single product
π: Constant costs per unit for slack
π_0: profits from the sale of each unit

Also the below assumptions are considered in the model:

1- lead time (L) is considered as certain and assume that demand in lead time (x), follows the normal distribution with mean μL and SD = $\sigma\sqrt{L}$.
2- Re-order point r = average demand during the Lead time + safety stock (SS)

In fact, we divide the L into n part. Each split point represents that one part of lead time is reached to minimum. R (L) indicates the cost function between these points. Thus, the total cost function of the system is calculated separately in the each interval of n part that reduced into their minimum.

4. Problem modeling:

The previous definition is assumed that the slack is allowed and demand for Lead time of (x) follows the normal probability distribution. The distribution has mean $L\mu$and $SD = \sigma\sqrt{L}$. With these assumptions, the average slack at the end of the period is calculated by following equation:

$$B(r) = \int_r^{\infty} (x - r) f(x)\, dx$$

3- Inventory is permanently under review. Re-order is done, when Stock levels reach below the re-ordering point.
4- Lead time (L), is composed of n independent part. Each part i, has a minimum a_i and a normal value, b_i and the reduced cost per times unit, c_i. Also for easily modeling, will assume that:
$C_1 \leq C_2 \leq \ldots \leq C_n$
To achieve this assumption, we should sort ascending the cost coefficients (c_i) in the notations. In other words, the earlier you take action to reduce lead time, the less cost you will pay.
5- If we assume $L_0 = \sum_{j=1}^{n} b_j$ and L_i is length of lead time where, i ($i=1, 2 \ldots$) part of Total parts, have reached its minimum; L_i will define as follows:

$$L_i = \sum_{j=1}^{n} b_j - \sum_{j=1}^{i} (b_j - a_j) \quad ; \quad i = 1,2,\ldots,n$$

L_0: The current delivery time
L_i: lead time after reducing i part

For example:

$$L_3 = \sum_{j=1}^{n} b_j - (b_1 - a_1) - (b_2 - a_2) - (b_3 - a_3)$$
$$= L_0 - (b_1 - a_1) - (b_2 - a_2) - (b_3 - a_3) =$$
$$(b_1 + b_2 + b_3 + b_4) - b_1 - b_2 - b_3 + a_1 + a_2 + a_3 \rightarrow L_3 = b_4 + a_1 + a_2 + a_3$$

For example, suppose that n=4, means that lead time is divided into four parts, so we will have:
$L_0 = b_1 + b_2 + b_3 + b_4$
$L_1 = b_2 + b_3 + b_4 + a_1$
$L_2 = b_3 + b_4 + a_1 + a_2$
$L_3 = b_4 + a_1 + a_2 + a_3$
$L_4 = a_1 + a_2 + a_3 + a_4$

$i=1$	$R(L) = c_1 (L_0 - L)$	$L \in (L_1, L_0)$
$i=2$	$R(L) = c_2 (L_1 - L) + c_1 (b_1 - a_1)$	$L \in (L_2, L_1)$
$i=3$	$R(L) = c_3 (L_2 - L) + c_2 (b_2 - a_2) + c_1 (b_1 - a_1)$	$L \in (L_3, L_2)$
$i=4$	$R(L) = c_4 (L_3 - L) + c_3 (b_3 - a_3) + c_2 (b_2 - a_2) + c_1 (b_1 - a_1)$	$L \in (L_4, L_3)$

With respect to above definition, we can define $R(L)$ (reducing costs in each period) for the $L \in (L_i, L_{i-1})$ as follows:

$$R(L) = C_i (L_{i-1} - L) + \sum_{j=1}^{i-1} C_j (b_j - a_j)$$

4.1 How to calculate the cost of slack:

Average number of back-ordered in each period is equal to $\beta\, B(r)$ and average of lost sales will be equal to $(1-\beta) B(r)$. β ($0 \leq \beta \leq 1$) indicates a percentage of demand that is faced with a slack that should be compensated as back order. This percentage is determined by market conditions.

So the annual slack costs would be:

$$\frac{D}{Q}[\pi + \pi_0(1-\beta)]\mathrm{B(r)}$$

On the other hand, to calculate the average net inventory, it should be noted that the stock in end of period will be $r-x$. If $x \leq r$; that is, demand during the lead time is smaller than the stock order points, and will be equal to $\beta\,(r-x)$. In this case, if $x > r$; that is, demand is more than order point stock, stock is zero. B is percent of the slack that should be compensated. So, the average stock in the end of period or order immediately before the new order is:

$$\int_0^r (r-x)f(x)dx + \int_r^\infty \beta(r-x)\, f(x)\, dx$$
$$= r\int_0^\infty f(x)dx - \int_0^\infty xf(x)dx + \int_r^\infty (x-r)\, f(x)\, dx + \beta\int_r^\infty (r-x)\, f(x)\, dx$$
$$= r - \mu L + B(r) - \beta B(r) = r - \mu L + (1-\beta)B(r)$$

And the average inventory immediately after reaching the order is:

$$Q + r - \mu L + (1-\beta)B(r)$$

So average storage cost is:

$$h[\frac{Q}{2} + r - \mu L + (1-\beta)B(r)]$$

Average annual cost:

average annual cost = constant ordering cost + storage cost + slack cost + costs for reducing lead time

Or:

$$EAC(Q,L,k) = A\frac{D}{Q} + h\left[\frac{Q}{2} + r - \mu L + (1-\beta)B(r)\right] + \frac{D}{Q}[\pi + \pi_0(1-\beta)]B(r) + \frac{D}{Q}R(L) \rightarrow$$

$$EAC(Q,L,k) = A\frac{D}{Q} + h\left[\frac{Q}{2} + k\sigma\sqrt{L}\right] + \left[h(1-\beta) + \frac{D}{Q}[\pi + \pi_0(1-\beta)]\right]\sigma\sqrt{L}\,\xi(k)$$
$$+ \frac{D}{Q}\left[C_i(L_{i-1} - L) + \sum_{j=1}^{i-1} C_j\left(b_j - a_j\right)\right], L \in (L_i, L_{i-1}), equation\ (1)$$

5. Algorithm for Solving Problem

Equation (1), consider, in this equation for each interval t, the three variables Q, L and k decisions are considered. Assuming service levels being found in the inventory system studied, the order point value can be obtained and also can be shown:

$\mu L_0 \leq r \leq \alpha\, r$ $\qquad$ $\alpha \geq 1$

L0: The current lead time

5.1 Algorithm description

Step 1) Determine the desired number for α.

Step 2) In Equation (1) for maximum r (means $\alpha \times r$) and for $L_{i-1} \leq L \leq L_i$, the optimal value of Q can be calculated:

$$Q = \sqrt{\frac{2D[A + (\pi_0(1-\beta) + \pi)B(r) + R(L)]}{h}}\ ,\quad \text{equation (2)}$$

And for values of rand L found in step two, calculate the cost function. Then the optimal value *L* is obtained for each specific *r*.
Step 3) Reduce the amount of rand at the same level of *r*, for various *L* in the interval $[L_i, L_{i-1}]$, the optimal *Q* value obtained again and calculate the cost function value for different values of *r*, *Q*, *L*. calculate optimal value of *r*, *Q*, *L* and the EAC.
Step 4) Continue this algorithm to reach the minimum *r* (μL_0). In each level of r, calculate optimum *L* and *Q*.
Step 5) Calculate the cost function value for *r*, *Q*, *L*, at different levels. Select the minimum amount as the average annual minimum costs. *r*, *Q* and *L* Values of This level is selected as optimal *r*, *Q*, *L*.
If the value of *r* is large, the implementation of this algorithm by using hand calculations is very difficult. Therefore, to run the algorithm, we will use of computer programs. Note that with any form of demand function in lead time, the algorithm can be used. Just replace the value *B(r)* in equation (2) and run the algorithm.
A) Value of slack function, in case demand has normal distribution in lead time:
The definition of slack function has been defined by Hadly (1963).

$$B(r) = \int_r^\infty (X-r)f(x)\,dx = \int_r^\infty (X-r)\frac{1}{\sigma\sqrt{L}\sqrt{2\pi}}EXP^{\frac{-1}{2}\left(\frac{x-\mu L}{\sigma\sqrt{L}}\right)^2}dx =$$

$$\int_r^\infty \frac{X}{\sigma\sqrt{L}\sqrt{2\pi}}EXP^{\frac{-1}{2}\left(\frac{x-\mu L}{\sigma\sqrt{L}}\right)^2}dx - r\int_r^\infty \frac{1}{\sigma\sqrt{L}\sqrt{2\pi}}EXP^{\frac{-1}{2}\left(\frac{x-\mu L}{\sigma\sqrt{L}}\right)^2}dx$$

Change following variables:

$$\frac{x-\mu L}{\sigma\sqrt{L}} = U \quad \rightarrow \quad \frac{1}{\sigma\sqrt{L}}dx = dU \quad , \quad X = U\sigma\sqrt{L} + \mu L$$

We have:

$$\int_{\frac{r-\mu L}{\sigma\sqrt{L}}}^\infty \frac{U\sigma\sqrt{L} + \mu L}{\sigma\sqrt{L}\sqrt{2\pi}}EXP^{\frac{-1}{2}U^2}\sigma\sqrt{L}\,dU - r\,P(X > r) =$$

$$\sigma\sqrt{L}\int_{\frac{r-\mu L}{\sigma\sqrt{L}}}^\infty \frac{U}{\sqrt{2\pi}}EXP^{\frac{-1}{2}U^2}dU + \mu L\int_{\frac{r-\mu L}{\sigma\sqrt{L}}}^\infty \frac{1}{\sqrt{2\pi}}EXP^{\frac{-1}{2}U^2}dU =$$

$$\sigma\sqrt{L}\frac{1}{\sqrt{2\pi}}EXP^{\frac{-1}{2}\left(\frac{r-\mu L}{\sigma\sqrt{L}}\right)^2} + (\mu L - r)\,P(X > r)$$

B) Value of slack function, in case demand has exponential distribution in lead time:

$$B(r) = \int_r^\infty (x-r)L\lambda EXP^{-L\lambda x}dx = L\lambda\int_r^\infty xEXP^{-L\lambda x}dx - rL\lambda\int_r^\infty EXP^{-L\lambda x}dx$$

$$= \left(-\frac{x}{L\lambda}EXP^{-L\lambda x}\Big|_r^\infty + \int_r^\infty \frac{1}{L\lambda}EXP^{-L\lambda x}dx\right)L\lambda - Lr\lambda\int_r^\infty EXP^{-L\lambda x}dx = Lre^{-L\lambda r}$$

$$+(1-L\lambda r)\int_r^\infty EXP^{-L\lambda x}dx = LrEXP^{-L\lambda x} + (1-L\lambda r)\left(\frac{1}{L\lambda}EXP^{-\lambda Lr}\right)$$

$$= \frac{1}{L\lambda}EXP^{-L\lambda r}$$

C) Value of slack function, in case demand has uniform distribution in lead time:

$$B(r) = \int_r^b (x-r)\frac{1}{L(b-a)}dx = \frac{1}{L(b-a)}\int_r^b (x-r)dx$$

$$= \frac{1}{L(b-a)}\left(\frac{x^2}{2} - rx\,\Big|_r^b\right) = \frac{1}{L(b-a)}\left(\left(\frac{b^2}{2} - rb\right) - \left(\frac{r^2}{2} - r^2\right)\right)$$

$$= \frac{1}{L(b-a)}\left(\frac{b^2 - r^2}{2} + r^2 - rb\right) = \frac{1}{L(b-a)}\left[\frac{(b-r)(b+r)}{2} - \frac{2r(b-r)}{2}\right]$$

$$\frac{1}{L(b-a)}\left[\frac{(b-r)[(b+r)-2r]}{2}\right] = \frac{1}{L(b-a)}\left[\frac{(b-r)(b-r)}{2}\right] = \frac{(b-r)^2}{2L(b-a)}$$

D) Value of slack function, in case demand has Poisson distribution in lead time:

$$B(r) = \sum_r^\infty (x-r)\frac{EXP^{-L\lambda}L\lambda^X}{X!} = \sum_r^\infty X\frac{EXP^{-L\lambda}L\lambda^X}{X!} - \sum_r^\infty r\frac{EXP^{-L\lambda}L\lambda^X}{X!} =$$

$$L\lambda \sum_{r}^{\infty} \frac{EXP^{-L\lambda} L\lambda^{X-1}}{(X-1)!} - r \sum_{r}^{\infty} \frac{EXP^{-L\lambda} L\lambda^{X}}{X!} = L\lambda\, [\bar{F}(r-2) - r\, \bar{F}(r-1)]$$

Where F(x) is the cumulative distribution function and $\bar{F}(x)$ is its complementary function.

6. Numerical Example

In order to explain how to solve the mentioned problem, suppose that an inventory system is available with the following specifications:

D=520 units per year, A=200 USD on the unit over the year, π=30 USD on the unit, π_0=130 USD on the unit, μ=10 units per week,σ=3 units per week, h=50 USD on the unit over the week, Service level=95%.

Lead time consists of three components with the information as Table 1. Problem is solved by presented algorithm for cases β=0, β=0.25, β=0.5 (Table 2). Table 2 expresses the optimum amount of lead time, economic ordering quantity, and total cost for any case of *B*. In this instance, we assume that lead time has normal distribution.

Table 1: Three components with the information

Reduce costs per week (Ci)	Minimum time (a_i) per week	Normal lead (b_i) per week	Components of lead time period
20	3	10	1
75	4	9	2
110	4	11	3

Table 2: Numerical solution

i	B=0			B=0.25			B=0.5		
	Li	Qi	EACi	Li	Qi	EACi	Li	Qi	EACi
Normal	18	211	26036	23	193	25123	23	172	23918

7. Conclusion

In this paper, we present a new model in which lead time is considered as a variable and controllable parameter. Costs are an issue that engages many managers; one of these costs is inventory cost; and one of the important factors for reducing the inventory cost is to use the convenient lead time. If we order goods with high lead time, our cost increases because the holding cost increases. On the other hand, if we order goods in low lead time, we must pay additional cost because of reduced lead time. Therefore, we must use a convenient lead time to balance these kinds of cost. In this paper, first of all, we formulate mentioned problem and after that, to solve the modeled problem, we present an exact method. This method can be used for any kind of probability distribution function in lead time period including normal, exponential, uniform, and Poisson distribution.

References

Ben-Daya. M., Abdul Raouf,. Inventory Models Involving Lead Time As Decision. *J. Opl. Soc*. vol. 45, pp. 579-582, 1994.

Chuan Lee. W., Inventory model involving controllable backorder rate and variable lead time demand with the mixtures of distribution. *Applied Mathematics and Computation*, vol. 160, no. 3, pp. 701-712, 27 January 2005.

Hadley, G., and Whitin, T.M., "Analysis of Inventory Control ", Prentice-Hall, Inc, Englewood Cliffs, N. J., 1963.

Iao. C. J. I., and Shyu. G., An Analytical Determination of Lead Time with Normal Demand. *Int. J. Opns. Prod. Mgmt*. vol. 11, pp. 72-78, 1991.

Iang. I., Ouhang. You, Neng-Che. Yen and Kun- Shan Wu., Mixture Inventory Model with Backorders and Lost Sales for Variable Lead Time. *J. Opl. Res. Soc*. vol. 47, pp. 829-832, 1996.

Ouyang. Liang Yuh., Bor Ren Chuang., A periodic review inventory model involving variable lead time with a service level constraint, *International Journal of Systems Scienc*e, vol. 31, no. 10, pp. 1209-1215, 2000.

Pan janson. Chao and Yang, Jin-Shan., A study of an integrated inventory with controllable lead time. *International Journal of Production Research*, vol. 40, no. 5, pp. 1263-1273, 2002.

Silver. F. A., and Peterson. R., Decision Systems for Inventory Management and Production Planning. John Wiley, New York, 1985.