Proceedings of the 2012 International Conference on Industrial Engineering and Operations Management
Istanbul, Turkey, July 3 – 6, 2012

Service Level within an Integrated Model for Warehouse and Inventory Planning

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Abstract

The aim of this paper is to assess the service levels from different point of views by modeling the inventory and warehouse variables. For this reason a main model has been formed and this model has been solved by dividing into inventory sub-model and warehouse sub-model. For the sake of effective implementation of 'order-picking ', the warehouse has been divided into two different areas. These areas are; forward area and reserve area. A predictive approach has been embraced for solvation of the model. A real implementation has been realized in a company acting in the healthcare sector. Because of the nature of the implemented healthcare sector, no limitation has been made for service level. An optimum service level definition approach has been preferred with regard to 'costing'. When it is considered from this point of view, the model is a kind of 'full cost model'. The assessments of numerical results has been made according to different service levels. Not only 'cost effectiveness' but other factors also taken into consideration for the assessment of the results.

Keywords

Service level, stochastic inventory model, warehouse model, Lagrangian relaxation, Forward – Reserve problem.

1. Introduction

Warehouse operations play a critical role in the profitability and success of every business. The way the store and warehouse is managed and run will impact greatly on the company's bottom line. Today's warehouse personnel face major problems in implementing up-to-date methods. They need to be updated on the latest tools and techniques to cost-manage effectively the day-to-day operations. Crucial importance of the matter is the need to adopt and install a systematic and updated approach to store and warehouse operations. In this study we will consider service level on this important factor that warehouses will be defined with separated areas which are forward and reserve areas. In these areas, order-picking activity has to be realized in more effective manner. To sustain consumer satisfaction, the optimal inventory and warehouse management model will be constituted with regard to warehouse systems and inventory cycles. The model includes warehouse and inventory variables and it is solved in two iterations. Although this model includes both the operation cost and penalty cost, it is intended to optimize only operation cost. The reason of this choice is that the optimization of penalty cost in healthcare sector could cause serious drug problems.

For this reason the effects of the service levels on the variables has been examined by selecting high service levels. The researched parts have been presented in details by making comprehensive literature survey in part 2. . Problem description, assumption, variables and model has been explained in details in the part 3. The suggested solution method is given in part 4 and numerical results are presented in part 5.

2. Literature Survey

Scientists have made studies about inventory with effective inventory planning and warehouse management for a long time. Strack and Pochet (2010), Van Den Berg et al. (1998), and Ganeshan (1999) are the most important scientists who interested in inventory management. Studies show that while a real system is modeling, it is required that not only finding economic order quantity, re-order point for each product but also it is required that a new developed model, including warehouse management must be sustained. In literature there are many different studies about warehouse management such as fully cost model, partial cost model, service level, holding costs. Van Den Berg et al. (1998) have formed on separation of first and secondary area and an answer searched for question to which product and how much proportion of it should be stored in first area. In problem of 0-1 first and secondary area which solved by formed difference models, intuitive method stated to give best solution with numerical results. Van Den Berg and Zjim (1999) focus on warehouse systems management problems and their classifications. There are three types of warehouses (for distribution, for production, for common use). Also it has been shown that there are four types of warehouse costs, order, storage, collection and transportation. In order to decide which product in which amount should be assigned to forward area, it is important to look how the collect orders. At the conclusion of the studying on the costs of order –picking times, and their costs have been important factors for warehouse management. Ganeshan (1999) focused the inventory level of warehouses and retailers are analyzed separately and determined that daily demand quantity is appropriate in a probability distribution. This model is solved by SLAM simulation model and results are analyzed for consistency and good results are obtained. Rong et al. (2008) considers inventories, which are in case of partially or fully inadequate condition by indefinite supply lead time, on two different areas with disruption structures and assumes secondary area has only adequate capacity, not unlimited capacity. In conclusion solutions of models are shown as numerical and defined as applicable for vegetable and fruite products in developing countries. Tunç et al. (2008) have focused on improving the process of warehouse order picking systems. As the first study emphasized that order picking time and their costs should be minimized. Besides, the most effective optimal order picking route is Ratliff’s dynamic programming model. At the conclusion, the importance of optimization of the storage systems is emphasized. Lee and Hsu (2009) have examined the two areas structure of warehouse for deteriorated products in assumptions which are determined with time dependent demand structure. In conclusion it specifies costs are highly affected by manufacturing time and this model will be used at demand structures dependent on decreasing or increasing time. Strack and Pochet (2010) has improved a model for inventory management and supports with numerical examples. Generally it elaborates necessity of separation to two different areas and emphasizes necessity of addition capacity in inadequate conditions. In conclusion purpose is explained as to add more decision factors to warehouse and inventory management. Miranda and Garrido (2008) have developed a method for distribution channel and later have utilized this for service level optimization. In their model, `penalty cost` has been assessed in a different way than `operation cost`. There has been two different methods developed in order to define the optimum service levels. Optimum service level has been defined by assessing numerical results on the different service levels. There are several studies that utilize service level approach’ presently. It is recommended that Nahmias (1993), Bookbinder and Tan (1988) , Chen and Krass (2001) could be referred in order to understand clearly the `service level` context that is given in our study and in the other studies.

3. Service Level within an Integrated Model for Warehouse and Inventory Planning

3.1 Problem Description

In implementation part of our study, we define a stochastic model to solve inventory control model for evaluate service level. Also include, modeling a real warehouse system will be analyzed, so that there will be two parts to solve our model. This model consist of two iteration first is based on inventory control sub model, the second is about warehouse sub model. We will solve warehouse sub model by fixing inventory control sub model’s output. If we separate warehouse in two areas, there will be improvements in order picking activity, forward areas will be organized and reserve area will have less inventory. The system’s activities are receiving products, order picking, transmitting to the warehouse, lastly transporting to the customer. Route Y defines the products only assign to the forward area, route Z defines the products only assign to the reserve area, route X defines the products which assigns forward area with advance replenishment and then assign to the reserve area with concurrent replenishment.

3.2 Assumptions for Model

There is safety stock. Concurrent replenishment is only occurred when the warehouse is empty for preventing crushing and accidents. Capacity of two different areas is known. Backorders are satisfied in next period. We know that traditional model results are worse than our new model. Demand is compatible with `normal distribution` and when the demand at supply point exceeds the `re order point` , back-orders emerge. Due to the fact that a real implementation has been made in healthcare sector, `service level` has not been used as a limitation factor.

3.3 Notations

CostRepA: cost of advance replenishment
CostRepC: cost of concurrent replenishment
RN_i**:** the number of renewal i th product
PickCostF: picking cost in the forward area
PickCostR: picking cost in the reserve area
P_i : number of picks
C_{If}: ordering cost for the forward area
C_{Ir}: ordering cost for the reserve area
C_I : average ordering cost
C_{Hi}: inventory holding cost
PC: penalty cost
CapaF: capacity of the forward area
CapaR: capacity of the reserve area
Capa : Total capacity
D_i: demand of i th product in
S_{Ri}: expected cycle backorders for i th product
$L^D{}_i$: lead time demand of the i th product
$\mu_{L^D{}_i}$: average lead time demand of the i th product
$\sigma_{L^D{}_i}$: lead time demand standard deviation of the i th product
h_i : volume of the i th product
α : service level
β : order fill rate
s_i : safety stock of the i th product

3.4 Decision Variables

Qi: order quantity of product i
ri: reorder point of product i
i:1,…,I defines products
xi
1 ; if the product i is supplied to the reserve area; picked from the forward area
and if j locations at least are allocated to product i in the forward area
0 ; otherwise
yi
1 ; if the product i is supplied directly to the forward area from the suppliers
and picked from the forward area only
0 ; otherwise
zi
1 ; if the product i is assigned to the reserve area and picked
from the reserve area only
0 ; otherwise,

3.5 Model

Objective function and limitations are expressed in the following formulatons:

Min:

$$\sum_{i=1}^{I} \mathrm{CostRepA} * (x_i) \tag{1}$$

$$+\sum_{i=1}^{I} \mathrm{CostRepC} * (x_i) * RN_i \tag{2}$$

$$+\sum_{i=1}^{I} C_{Ir} * z_i * (D_i/Q_i) + \sum_{i=1}^{I} C_{Ir} * x_i * (D_i/Q_i) + \sum_{i=1}^{I} C_{If} * y_i * (D_i/Q_i) \tag{3}$$

$$+\sum_{i=1}^{I} \mathrm{PickCostF} * P_i * (x_i + y_i) \tag{4}$$

$$+\sum_{i=1}^{I} \mathrm{PickCostR} * P_i * z_i \tag{5}$$

$$+ \sum_{\boldsymbol{i=1}}^{\boldsymbol{I}} C_{\mathbf{H}i} * (Q_i/2 + R_i - {\mu_L}^D{}_i) \tag{6}$$

$$+\sum_{i=1}^{I} \mathrm{PC} * (D_i/Q_i) * \left(\int_{Ri}^{\infty} ({L^D}_i - R_i)\, f({L^D}_i)\, d\, {L^D}_i\right) \tag{7}$$

Subject to:

$$\sum_{i=1}^{I} [x_i * (Q_i + R_i - {\mu_L}^D{}_i) * h_i + (Q_i + R_i - {\mu_L}^D{}_i) * h_i * y_i] <= \mathrm{CapaF} \tag{8}$$

$$\sum_{i=1}^{I} [(Q_i + R_i - {\mu_L}^D{}_i) * z_i * h_i + (Q_i + R_i - {\mu_L}^D{}_i) * h_i * x_i)] <= \mathrm{CapaR} \tag{9}$$

$$\sum_{\mathrm{i}=1}^{\mathrm{I}} (\mathrm{xi} + \mathrm{zi} + \mathrm{yi}) = 1 \tag{10}$$

$$0 < \alpha < 1 \tag{11}$$

$$\mathrm{xi}, \mathrm{zi}, \mathrm{yi}, \mathrm{Ri}, \mathrm{Qi} >= 0 \quad \forall i \tag{12}$$

Equation 1, defines the cost replenishment advance at highest level that is realized between forward and reserve area. This replenishment type is realized once at the beginning of the period for each product that follows the X-route. Equation 2, defines the concurrent replenishment cost that is realized simultaneously between forward and reserve area. Equation 3, defines ordering cost for reserve and forward area respectively. Equation 4 and 5 define order picking costs for forward and reserve area respectively. Equation 6, defines inventory holding cost. Average inventory level has been calculated as per Hadley and Whitin (1963). Equation 7, defines penalty cost. Equation 8, defines that the products that follow the X and Y routes should not exceed the forward area capacity. Equation 9, defines that the products that follow the X and Z routes should not exceed the reserve area capacity. Equation 11, defines the variation range of the service level.

4. Suggested Solution Method

The model is a kind of non linear mixed integer model due to its nature of including both inventory and warehouse variables. Since the solution of these models somewhat difficult, heuristic sequential approach method has been utilized, (Strack and Pochet, 2010). In this heuristic sequential model, main model has been divided into two sub models. These sub-models are: inventory sub-model and warehouse sub-model. At the first instance, in our main model we have kept inventory relevant parameters and inventory sub-model has been solved with the aid of Lagrangian relaxation, in the later stage the results of inventory sub-model has been kept constant and applied to warehouse sub-model and final solution was obtained.

4.1 Inventory Sub Model

Min $\sum_{i=1}^{I} C_{Hi} * (Q_i/2 + R_i - \mu_L{}^D{}_i)$
$+\sum_{i=1}^{I} C_{Il} * z_i * (D_i/Q_i) + \sum_{i=1}^{I} C_{Il} * x_i * (D_i/Q_i) + \sum_{i=1}^{I} C_{Ib} * y_i * (D_i/Q_i)$
$+\sum_{i=1}^{I} PC * (D_i/Q_i) * (\int_{Ri}^{\infty} (L^D{}_i - R_i)\, f\,(L^D{}_i\,)d\, L^D{}_i)$
Subject to:
$\sum_{i=1}^{I} [\quad x_i * (Q_i + R_i - \mu_L{}^D{}_i) * h_i + (Q_i + R_i - \mu_L{}^D{}_i) * y_i * h_i] <= \text{CapaF}$
$\sum_{i=1}^{I} [(Q_i + R_i - \mu_L{}^D{}_i) * z_i * h_i + (Q_i + R_i - \mu_L{}^D{}_i) * x_i * h_i)] <= \text{CapaR}$
$0<\alpha<1$
x_i, zi, yi, Qi , $\mu_L{}^D{}_i$, R_i,>= 0 ∀i.

To extract the model from warehouse variables (x_i,y_i,z_i) , ordering cost has been kept independent from the followed routes and Equation 14 has been derived from Equation 13.

$$\sum_{i=1}^{I} C_I * (D_i/Q_i) \tag{13}$$

$$C_I = (\text{Ort}C_{Il} + \text{Ort}C_{Ib})/2 \tag{14}$$

Equation 14 is the average of the ordering costs that were resulted from the earlier orders. And it will be used in the solution of inventory sub-model in Equation 13.

In the same way, the limitations have been extracted from warehouse variables, capacity limitations has been defined in Equation 15 as a single limitation.

$$\sum_{i=1}^{I} [(Q_i + R_i - \mu_L{}^D{}_i) * h_i \ <= \text{Capa} \tag{15}$$

Capa notation in Equation 15 has been derived by using Equation 16. This is obtained by summing the Capa, CapaF and CapaR in Equation 16.

$$\text{Capa} = \text{CapaF} + \text{CapaR} \tag{16}$$

Inventory sub-model is in an extracted state from warehouse variables with the above equations. Inventory sub-model will be solved with Lagrangian relaxation method.

The aim of this method, to find the order quantity by minimizing the annual Total Cost (TC).

$$L(\lambda, Q_i) = \sum_{i=1}^{I} C_{Hi} * (Q_i/2 + R_i - \mu_L{}^D{}_i) + \sum_{i=1}^{I} PC * (D_i/Q_i) * \left(\int_{R_i}^{\infty} (L^D{}_i - R_i)\, f(L^D{}_i)\, dL^D{}_i\right) + \sum_{i=1}^{I} C_I * (D_i/Q_i) + \lambda * \left(\left(\sum_{i=1}^{I} [(Q_i + R_i - \mu_L{}^D{}_i)] * h_i\right) - (Capa)\right) \quad (17)$$

$$Q_i = \sqrt{(2D_i(PC(\int_{R_i}^{\infty} (L^D{}_i - R_i)\, f(L^D{}_i)\, d\,L^D{}_i) + C_I)/(C_{Hi} + 2\lambda)} \quad (18)$$

After defining the order quantity equation with Equation 18, it will be necessary to define the re order point (R). Re order point (R) is calculated with Equation 19.

$$R_i = \mu_L{}^D{}_i + s_i \quad (19)$$

Safety stock is calculated with Equation 20.

$$s_i = z * \sigma_L{}^D{}_i \quad (20)$$

If the standard deviation of the lead time demand is known, table value of cycle success probability against constant multiplier could be found by referring the z normal distribution table. In this case $\sigma_L{}^D{}_i$ defines the standard deviation in the lead time demand. Lau ve Lau (2002) suggest in their paper the Equation 21 to find the expected backorders in every cycle which will be resulted in the supply lead time that is complied with normal distribution. In the equation while ϕ defines (standard normal distribution density function) , ω defines (cumulative density function).

$$S_{Ri} = \int_{R_i}^{\infty} (L^D{}_i - R_i)\, f(L^D{}_i)\, d\,L^D{}_i = \mu_L{}^D{}_i\, \phi(z) - \sigma_L{}^D{}_i\, \omega(z) \quad (21a)$$

$$S_{Ri} = (\mu_L{}^D{}_i - R_i)[1 - \phi(z)] + \sigma_L{}^D{}_i\, \omega(z) \quad (21b)$$

$$S_{Ri} = (-s) * [1 - \phi(z)] + \sigma_L{}^D{}_i\, \omega(z) \quad (21c)$$

4.2 Warehouse Sub Model

Min $\sum_{i=1}^{I} CostRepA * (x_i)$
$+\sum_{i=1}^{I} CostRepC * (x_i) * RN_i$
$+\sum_{i=1}^{I} C_{Il} * z_i * (D_i/Q_i) + \sum_{i=1}^{I} C_{Il} * x_i * (D_i/Q_i) + \sum_{i=1}^{I} C_{Ib} * y_i * (D_i/Q_i)$
$+\sum_{i=1}^{I} PickCostF * P_i * (x_i + y_i)$
$+\sum_{i=1}^{I} PickCostR * P_i * z_i$
Subject to:
$\sum_{i=1}^{I} [\, x_i * (Q_i + R_i - \mu_L{}^D{}_i) * h_i + (Q_i + R_i - \mu_L{}^D{}_i) * y_i * h_i] <= CapaF$
$\sum_{i=1}^{I} [(Q_i + R_i - \mu_L{}^D{}_i) * z_i * h_i + (Q_i + R_i - \mu_L{}^D{}_i) * x_i * h_i)] <= CapaR$
$\sum_{i=1}^{I} (x_i + z_i + y_i) = 1$
x_i, zi, yi, Qi , $\mu_L{}^D{}_i$, $R_i >= 0\ \forall i$.

In the warehouse sub model, the objected point is to define which product will be allocated in the forward area and /or reserve area and after the allocation which routes will be followed by the product(s). To make decisions on these issues, and to attain the product numbers in the best or required level for forward area it is necessary to know , how the forward and reserve areas will be shared from the total area, and a decision will have to be made together on how many products will be allocated into the forward area.

5. Conclusions

In this study it has been approached into forward-reserve problem with a different point of view. For improving the order picking activity, the warehouse has been divided into two different area (forward and reserve) and the variables in the model has been formed with regard to this structure. Model results has been assessed on the service level. Because of the nature of the implemented sector (drug distribution company), service levels has been kept reasonably high and the comments have been made from this levels.

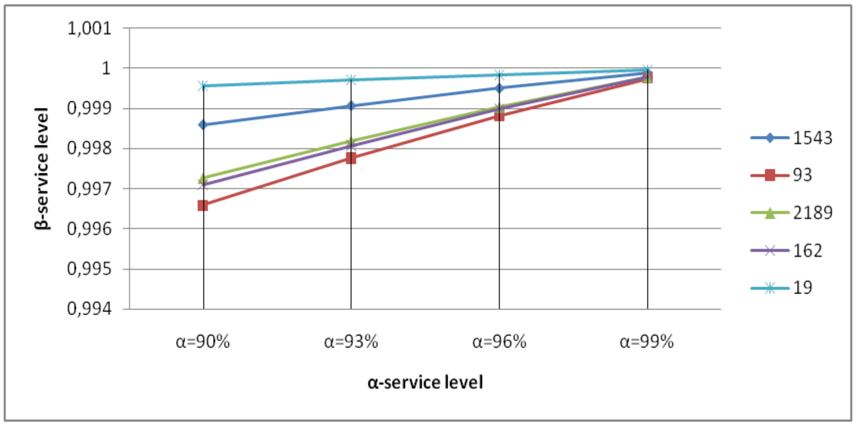

Figure 1: α service level and β service level

Figure 1 shows the comparison of α and β service level for five different products. When the α value increases, β values will be very close to each other. Specially, at the level for α = %99 ‘ the β’ values follows a very close trend. This result shows that α and β service level variations are not at the same rate.

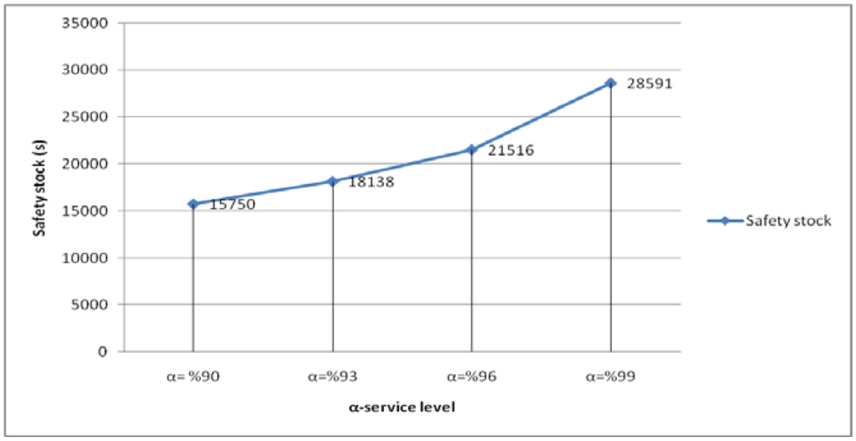

Figure 2: α service level with safety stock

As it is understood from Figure 2’, when the α values is increased, the safety stock is also increased. Since the increase in the safety stock, is dependable on the normal distribution complied supply lead time standard deviation, the equivalent increase in the α’ value, will not be reflected on the safety stock in the equal rate.

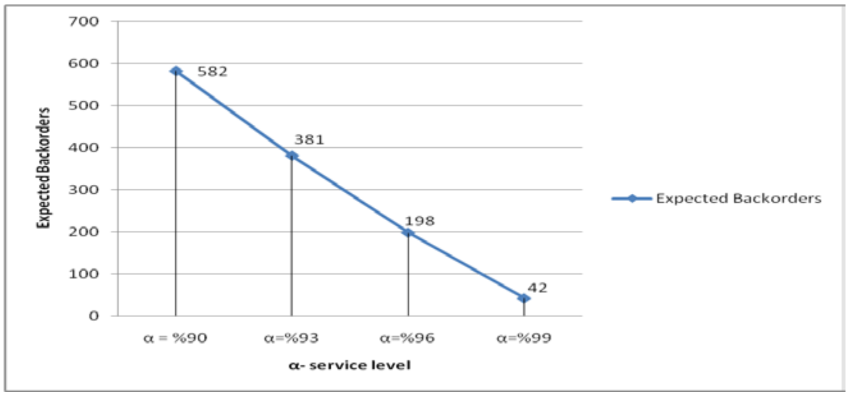

Figure 3: α service level with expected backorders

Figure 3 shows the effect of increase in the α value, on the total backorders in cycle. With the increase in the α' value between (%90-93), the magnitude of the decrease of the expected backorders, is more than that of the magnitude of the decrease in (%93-96) range. The reason of that, when the , α value is increased , with the increase in the re-order point (R) and safety stock level, the improvement in the amount of expected backorders will slow down. As it is understood from Figure 4', when the α value is increased, total order quantity in a cycle will be reduced. Although the decrease will not be realized in an equal manner in the α ranges, biggest decrease will be realized in the (%96-99 α) range.

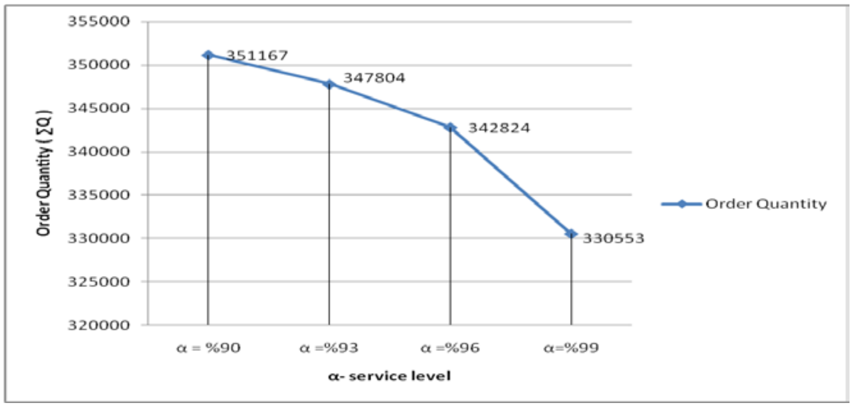

Figure 4: α service level with total order quantity

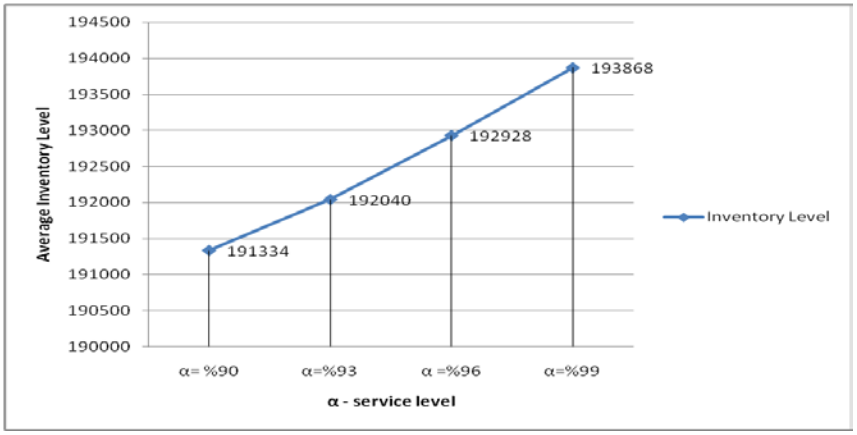

Figure 5: α service level with average inventory level

Figure 5 shows the effect of increase in α value on the average inventory level as well as intended inventory investment. When it is assessed from this point of view, it is seen that increase in the α-service value, also increases the average inventory level. A heuristic algorithm has been suggested in this study to define the effect of stochastic demand together with the variation in the service level on the other variables. There has been a lot of novelty features in the presented study. The most important peculiarity presented in to the relevant literature in this study is `integrated model is assessed together with the service levels`. Among the results, two different service levels assessment unit (α and β) are compared and effect of α service level on the average inventory level, order quantity, expected backorders and safety stock has been investigated. While the model is formed, operation and penalty cost have been assessed together. Since the researched company remains in the drug distribution sector, the α service level that is to be defined has to be assessed before the associated costs. For the companies acting in this sector it is a pre-requisite to reach %96 α service level as breaking point for maintaining both service level and cost effectiveness success. However, when we give priority to service effectiveness, %99 α service level has to be selected. If the future studies, to be considered as relevant to implementation and different sector companies are studied, then service level has to be taken as a limitation and penalty cost has to be excluded from the model. In the future academic studies, both penalty and operation cost have to be considered together, and a formulation has to be developed for maintaining the best (optimum) service level that will minimize the total cost.

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