

Sub-Contractor Selection in General Contractor Organizations Using Interval-Valued Fuzzy

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Abstract

Selecting sub-contractors based on defining appropriate criteria and applying a practical method is one of the most important concerns in project management applied in general contractor organizations. Because the view points of experts are mainly presented by linguistic variables, fuzzy set theory is appropriate to represent the criteria measures of selecting sub-contractor. In this paper, an interval-valued fuzzy technique has been applied for selecting sub-contractors. Six pre-defined criteria have been defined in an experimental problem using well-known method of brain storming while different strategies including being veto and concerning the majority of experts' view points are used in the decision making process. Results, which are confirmed by local experts show that it is highly noticeable that strategy has influential effect in the process of selecting sub-contractor and it must be set firstly to satisfy the project managers.

Keywords

Project Management, Sub-contractor Selection, General Contractor, Multi Criteria Decision Making, Interval-valued fuzzy set

1. Introduction

A project is a time-tabling work including goals and objectives with some constraints in time, cost and quality. It usually follows a beneficial change or added value in activities, products, and services. Project management consists of some general and technical skills corresponding to the role of management [1]. In the real world, the general contractor mostly Engineering Procurement Construction (EPC) projects hires specialized sub-contractors to perform all or portions of the engineering, procurement and construction work so it is a vital concern to select the most appropriate sub-contractors. In the process of selecting sub-contractors, most agreement criteria are needed to be defined as well as the best method of analyzing is required to be applied [2]. In the construction industry, sub-contractor's performance is a crucial factor in their awards of a new job by a general contractor. Chien-Ho Ko et al., Proposed a Sub-contractor Performance Evaluation Model (SPEM) using Evolutionary Fuzzy Neural Inference Model [3]. The effectiveness of their proposed model is validated by performing case study of a real general contractor. Validation results show that the proposed method accurately measures sub-contractor's performance enhancing the current practice of evaluation. Gokhan Arslan et al, proposed a web-based sub-contractor evaluation system called WEBSSES by which the Sub-Contractors (SCs) can be evaluated based on a combined criterion such as quality of production, efficiency, employment of qualified members, reputation of the company, accessibility to the

company, completion of the work on time, etc [4]. Proposed system enables general contractor to select the most appropriate SCs for their relevant sub-works, speed up the selection process and gain time and cost savings during the bidding process.

A wide range of methodologies are being used for analytical process of criteria, which are defined by numerical measures. But in the practice, there are some criteria, which are not being defined by numeric aspects, so linguistic variables could be implemented in fuzzy set approaches [5]. The well-known approach of Multi Criteria Decision Making (MCDM) provides an effective framework for comparison based on the evaluation of multiple conflict criteria that many concrete problems can be represented by several (conflicting) criteria [6]. Because of the lack of information about the alternatives regarding to all attributes, the classical MCDM methods cannot effectively be applied in solving problems. Hence in the both deterministic and random processes it is a wise decision to tend to be less effective in conveying the imprecision and fuzziness characteristics. This concept has been proposed as fuzzy set theory by Zadeh [7, 8]. Because of much knowledge in the real world is fuzzy rather than precise, fuzzy set theory may be known as a powerful tool to handle imprecise data and fuzzy expressions. In fuzzy sets theory it is often difficult for an expert to quantify his or her opinion as a number in interval [0, 1]. Therefore, it is more suitable to represent this degree of certainty by another method of analyzing. Sambuc [9] and Grattan [10] noted that the presentation of a linguistic expression in the form of fuzzy sets is not enough. Although analytical hierarchical process (AHP) and expert choice (EC) software are well-known techniques for ranking the alternatives to be selected on various area, in this paper a capable method of interval-valued fuzzy [5] has been applied to solve the problem of selecting sub-contractor in an experimental case. Interval-valued fuzzy sets were suggested for the first time by Gorzlczany [11] and Turksen [12]. Also Cornelis et al. [13] and Karnik and Mendel [14] noted that the main reason for proposing this new concept is the fact that in the linguistic modeling of a phenomenon, the presentation of the linguistic expression in the form of ordinary fuzzy sets is not clear enough. Wnag and Li [15] defined interval-valued fuzzy numbers (IVFN) and gave their extended operations. Interval-valued fuzzy sets have been widely used in real-world applications for instance, Kohout and Bandler [16] in a CLINAID system, Sambuc [9] in thyrodian pathology, Gorzlczany [11] and Bustine [17] and in approximate reasoning, Turksen [18, 19] in interval lvalued logic and in preference modeling [12]. Based on definition of interval-valued fuzzy set in [11], an interval-valued fuzzy set a defined on is given by:

$$A = \left\{ \left(x, \left[\mu_A^L(x), \mu_A^U(x) \right] \right) \right\}$$

$$\mu_A^L, \mu_A^U : X \rightarrow [0,1] \quad \forall x \in X, \quad \mu_A^L \leq \mu_A^U \tag{1}$$

$$\bar{\mu}_A(x) = [\mu_A^L(x), \mu_A^U(x)]$$

$$A = \left\{ \left(x, \bar{\mu}_A(x) \right) \right\}, x \in (-\infty, +\infty)$$

Where A is valued fuzzy interval number and also $\mu_A^L(x)$ and $\mu_A^U(x)$ are respectively the lower and upper limits of degree of membership.

After this section, a brief discussion is proposed regarding to the concept behind interval-valued fuzzy. In the third section of paper the characteristics of case study is being presented while applying the method is followed in this section. Results are being discussed in the last section as well as further researches are conducted to do future studies.

2. The Interval-Valued Fuzzy Set and VIKOR

In fuzzy MCDM problems, performance rating values and relative weights are usually characterized by fuzzy numbers. The concept of linguistic variable is very useful in dealing with situations that are too complex or ill-defined to be reasonably described in conventional quantitative expressions [8]. These linguistic variables can be converted to triangular interval-valued fuzzy numbers for ratings and the importance of each criterion as depicted in Table 1 [5].

Table 1: Definitions of linguistic variables

For the ratings		For the importance of each criterion	
Very Poor (VP)	[(0,0);0;(1,1.5)]	Very Low (VL)	[(0,0);0;(0.1,0.15)]
Poor (P)	[(0,0.5);1;(2.5,3.5)]	Low (L)	[(0,0.05);0.1;(0.25,0.35)]
Moderately Poor (MP)	[(0,1.5);3;(4.5,5.5)]	Medium Low (ML)	[(0,0.15);0.3;(0.45,0.55)]
Fair (F)	[(2.5,3.5);5;(6.5,7.5)]	Medium (M)	[(0.25,0.35);0.5;(0.65,0.75)]
Moderately Good (MG)	[(4.5,5.5);7;(8,9.5)]	Medium High (MH)	[(0.45,0.55);0.7;(0.8,0.95)]
Good (G)	[(5.5,7.5);9;(9.5,10)]	High (H)	[(0.55,0.75);0.9;(0.95,1)]
Very Good (VG)	[(8.5,9.5);10;(10,10)]	Very High (VH)	[(0.85,0.95);1;(1,1)]

Let $\tilde{X} = [\tilde{x}_{ij}]_{n \times m}$ be a fuzzy decision matrix for a multi criteria decision making problem in which A_1, A_2, \dots, A_m are n possible alternatives and C_1, C_2, \dots, C_n are m criteria. So the performance of alternative A_j with respect to criterion C_i is denoted as \tilde{x}_{ij} . As illustrated in Fig. 1, \tilde{x}_{ij} and \tilde{w}_i are expressed in triangular interval-valued fuzzy numbers [5].

$$\tilde{x} = \begin{cases} (x_1, x_2, x_3) \\ (x'_1, x_2, x'_3) \end{cases}$$

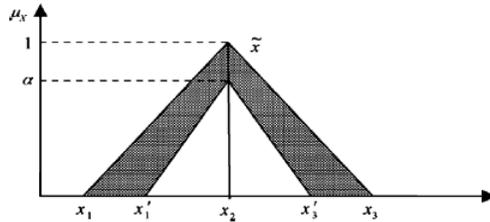


Figure 1: Interval-valued triangular fuzzy number.

The \tilde{x} can be also demonstrated as $\tilde{x} = [(x_1, x'_1); x_2; (x'_3, x_3)]$. In a group decision environment with K persons, the importance of the criteria and the rating of alternatives with respect to each criterion can be calculated as [5]:

$$\tilde{x}_{ij} = \frac{1}{K} [\tilde{x}_{ij}^1 + \tilde{x}_{ij}^2 + \dots + \tilde{x}_{ij}^k] \tag{2}$$

$$\tilde{w}_i = \frac{1}{K} [\tilde{w}_i^1 + \tilde{w}_i^2 + \dots + \tilde{w}_i^k] \tag{3}$$

Eqs. (2) and (3) represents the average values of \tilde{x}_{ij} and \tilde{w}_i denoted by experts, where (+) is the sum operator and is applied to the interval-valued fuzzy numbers as defined in Definition 1. So the output is also an interval value fuzzy number. Now the utilized approach to develop the VIKOR for interval-valued fuzzy data (IVF-VIKOR) can be defined as follows [5]:

In the first step the best rating \tilde{f}_i^+ and the worst rating \tilde{f}_i^- for all the criteria are ascertain. For example if the criterion i symbolizes a benefit, then:

$$\tilde{f}_i^+ = \max_j f_{ij}, \tilde{f}_i^- = \min_j f_{ij}. \tag{4}$$

The second step is calculating the \tilde{S}_j and \tilde{R}_j values for $j = 1, \dots, m$, which symbolize the average and the worst group scores for the alternative A_j respectively, with the relations

$$\tilde{S}_j = \sum_{i=1}^n \frac{\tilde{w}_i \times (\tilde{f}_i^+ - \tilde{f}_i^-)}{(\tilde{f}_i^+ - \tilde{f}_i^-)}$$

$$\Rightarrow \tilde{S}_j = [(S_{1j}, S'_{1j}); S_{2j}; (S'_{3j}, S_{3j})] \quad (5)$$

$$\tilde{R}_j = \max_i \left[\frac{[(w_{1i}, w'_{1i}); w_{2i}; (w'_{3i}, w_{3i})] \times [(f_{1i}^+ - f_{1ij}, f'_{1i} - f'_{1ij}); f_{2i}^+ - f_{2ij}; (f'_{3i} - f'_{3ij}, f_{3i}^+ - f_{3ij})]}{[(f_{1i}^+ - f_{1i}^-, f'_{1i} - f'_{1i}^-); f_{2i}^+ - f_{2i}^-; (f'_{3i} - f'_{3i}^-, f_{3i}^+ - f_{3i}^-)]} \right]$$

$$\Rightarrow \tilde{R}_j = [(R_{1j}, R'_{1j}); R_{2j}; (R'_{3j}, R_{3j})] \quad (6)$$

Here, w_i 's ($\sum_{i=1}^m w_i = 1, w_i \in [0,1] \quad i = 1, \dots, n$) are the relative importance weights of the criteria set by the DM.

Third step includes the calculation of the \tilde{Q}_j values for $j = 1, \dots, m$ with the relation

$$\tilde{Q}_j = \frac{\nu \times [(S_{1j} - S_{1j}^+, S'_{1j} - S'_{1j}^+); S_{2j} - S_{2j}^+; (S'_{3j} - S'_{3j}^+, S_{3j} - S_{3j}^+)]}{[(S_{1j}^- - S_{1j}^+, S'_{1j}^- - S'_{1j}^+); S_{2j}^- - S_{2j}^+; (S'_{3j}^- - S_{3j}^+, S_{3j}^- - S_{3j}^+)]}$$

$$+ \frac{(1 - \nu) \times [(R_{1j} - R_{1j}^+, R'_{1j} - R'_{1j}^+); R_{2j} - R_{2j}^+; (R'_{3j} - R'_{3j}^+, R_{3j} - R_{3j}^+)]}{[(R_{1j}^- - R_{1j}^+, R'_{1j}^- - R'_{1j}^+); R_{2j}^- - R_{2j}^+; (R'_{3j}^- - R_{3j}^+, R_{3j}^- - R_{3j}^+)]}$$

$$\Rightarrow \tilde{Q}_j = [(Q_{1j}, Q'_{1j}); Q_{2j}; (Q'_{3j}, Q_{3j})] \quad (7)$$

Where

$$\tilde{S}^+ = \min_j \tilde{S}_j, \quad \tilde{S}^- = \max_j \tilde{S}_j, \quad (8)$$

$$\tilde{R}^+ = \min_j \tilde{R}_j, \quad \tilde{R}^- = \max_j \tilde{R}_j, \quad (9)$$

The final value of \tilde{Q}_j which is shown by Q_j^* can be calculated as follows:

$$Q_j^* = \frac{Q_{1j} + Q'_{1j} + 2Q_{2j} + Q'_{3j} + Q_{3j}}{6} \quad (10)$$

The fourth and fifth steps are including the step of ranking the alternatives by sorting each \tilde{S} , \tilde{R} and Q^* values in an increasing order with the result of setting three ranking lists denoted as $\tilde{S}_{[.]}$, $\tilde{R}_{[.]}$ and $Q_{[.]}$.and proposing the alternative j 1equivalent to $Q_{[1]}^*$ (the smallest among Q_j^* values) as a trade-off solution.

3. Application of Method in a Real Problem

In this research work, problem solving has been done in a general contractor organization, which in managers are tending to select the best sub-contractor among three candidates for out-sourcing of the pre-selected parts of project. Three candidates are being considered to be evaluated based on the six experimental criteria. They are selected by a team work using a common technique of brain storming. Four experts, who are able to judge on the criteria corresponding to each candidate, are nominated to fill out the questionnaires. Criteria are exclusively considered in the project are follows as:

- Related Execution History (C_1)
- Available Facilities (C_2)
- Adequacy of Technical Team (C_3)
- Financial Ability (C_4)
- Company's Antiquity (C_5)
- Previous Employers Satisfaction (C_6)

Considering to the method of ranking candidates, different weights are engaged for each criteria number of 1 to 6 by 0.2, 0.3, 0.1, 0.1, 0.2, and 0.1 respectively.

Three candidates Company 1 (A_1), company 2 (A_2) and company 3 (A_3) are selected to be ranked. The selected method is currently applied to solve this problem; the computational procedure is summarized as follows:

In the first step, the number of the committee members is four, labeled as DM1, DM2, DM3 and DM4 respectively. Each decision maker has presented his assessment based on linguistic variable for importance of each criterion and rating performance by a linguistic variable as depicted in tables 2 and 3 respectively. We will proceed to solve the problem using the interval-valued fuzzy. Table 4 shows the final judgment of the decision makers through applying Eqs. (2) and (3).

Step 2: \tilde{f}_i^+ and \tilde{f}_i^- values are computed by Eq (4).

Table 2: The importance of criterion

Criterion	DM1	DM2	DM3	DM4
C_1	MH	MH	H	VH
C_2	M	M	ML	MH
C_3	MH	MH	H	MH
C_4	H	MH	H	VH
C_5	VH	VH	H	MH
C_6	ML	ML	M	ML

Table 3: Decision makers' assessments based on each criterion

Criterion	Alternatives	Decision makers			
		DM1	DM2	DM3	DM4
C_1	(A_1)	MP	MP	M	M
	(A_2)	G	G	MG	G
	(A_3)	G	F	F	F
C_2	(A_1)	MG	G	F	MG
	(A_2)	F	F	F	MG
	(A_3)	MG	MG	F	F
C_3	(A_1)	F	F	F	MG
	(A_2)	F	MG	MG	G
	(A_3)	MG	MG	MG	G

C_4	(A_1)	F	F	F	G
	(A_2)	MG	MG	G	G
	(A_3)	F	F	F	F
C_5	(A_1)	MG	F	F	MP
	(A_2)	G	G	G	MG
	(A_3)	MG	MG	MG	MG
C_6	(A_1)	F	F	MP	F
	(A_2)	MG	G	G	G
	(A_3)	F	F	MP	F

Step 3: \tilde{S}_j and \tilde{R}_j values are computed by Eqs (5) and (6) and are shown in Table 5.

Step 4: \tilde{Q}_j and Q_j^* values are computed by Eqs (7) and (10), selecting $V = 0$, $V = 0.5$ and $V = 1$ and are shown in Table 6. V represents the decision making strategy, considering the majority of criteria or maximum group utility in decision making process with $V > 0.5$ and in the state of being veto in the process with $V < 0.5$.

Step 5: The performance ranking order of the three strategies by interval-valued fuzzy are shown in Table 7.

Table 4: The interval-valued fuzzy decision matrix and weights

	C_1	C_2	C_3
(A_1)	[(1.25,2.5);4;(5.5,6.5)]	[(4.25,5.5);7;(8,9.125)]	[(3,4);5.5;(6.875,8)]
(A_2)	[(5.25,7);8.5;(9.125,9.875)]	[(3,4);5.5;(6.875,8)]	[(4.75,5.5);7;(8,9.125)]
(A_3)	[(3.25,4.5);6;(7.25,8.125)]	[(3.5,4.5);6;(7.25,8.5)]	[(4.75,6);7.5;(8.375,9.625)]
Weight	[(0.575,0.7);0.825;(0.8875,0.95)]	[(0.2375,0.35);0.5;(0.6375,0.75)]	[(0.475,0.6);0.75;(0.8375,0.9625)]

Table 4: The interval-valued fuzzy decision matrix and weights (Continued)

	C_4	C_5	C_6
(A_1)	[(3.25,4.5);6;(7.25,8.125)]	[(2.375,3.5);5;(6.375,7.5)]	[(1.875,3);4.5;(6,7)]
(A_2)	[(5,6.5);8;(8.75,9.75)]	[(5.25,7);8.5;(9.125,9.875)]	[(5,6.5);8;(8.75,9.75)]
(A_3)	[(2.5,3.5);5;(6.5,7.5)]	[(4.5,5.5);7;(8,9.5)]	[(1.875,3);4.5;(6,7)]
Weight	[(0.6,0.75);0.875;(0.925,0.9875)]	[(0.675,0.8);0.9;(0.9375,0.9875)]	[(0.0625,0.2);0.35;(0.5,0.6)]

Table 5: The values \tilde{S}_j, \tilde{R}_j by interval-valued fuzzy

Alternatives	\tilde{S}_j	\tilde{R}_j
(A_1)	[(-25.197,-25.197);3.408;(30.755,34.375)]	[(-2.4,-2.4);0.9;(11.25,11.85)]
(A_2)	[(-27.745,-27.745);0.687;(22.655,25.349)]	[(-1.876,-1.876);0.5;(6.937,7.307)]
(A_3)	[(-25.577,-25.577);2.402;(27.771,31.293)]	[(-2.4,-2.4);0.875;(8.062,9.45)]

Table 6: The values \tilde{Q}_j, Q_j^* with $v = 0, 0.5, 1$

Alternatives	$\tilde{Q}_j[v = 0]$	$\tilde{Q}_j[v = 0.5]$	$\tilde{Q}_j[v = 1]$
(A_1)	$[(-1.41, -1.41); 1; (1.08, 1.08)]$	$[(-1.33, -1.33); 1; (1.07, 1.07)]$	$[(-1.25, -1.25); 1; (1.05, 1.06)]$
(A_2)	$[(-0.99, -0.99); 0; (1.03, 1.03)]$	$[(-1.04, -1.04); 0.03; (1.07, 1.07)]$	$[(-1.08, -1.08); 0.06; (1.1, 1.1)]$
(A_3)	$[(-1.19, -1.19); 0.94; (1.08, 1.08)]$	$[(-1.19, -1.19); 0.8; (1.07, 1.07)]$	$[(-1.19, -1.19); 0.65; (1.06, 1.06)]$

Table 6: The values \tilde{Q}_j, Q_j^* with $v = 0, 0.5, 1$ (Continued)

Alternatives	$Q_j^*[v = 0]$	$Q_j^*[v = 0.5]$	$Q_j^*[v = 1]$
(A_1)	0.224	0.246	0.269
(A_2)	0.013	0.02	0.028
(A_3)	0.276	0.225	0.174

Table 7: Preference order ranking by interval-valued fuzzy

Alternatives	$v = 0$	$v = 0.5$	$v = 1$
(A_1)	2	3	3
(A_2)	1	1	1
(A_3)	3	2	2

4. Summary and Conclusion

In this research work, an interval-valued fuzzy method has been applied to solve the problem of selecting sub-contractors based on the assessment of experts. It is based on selecting the best alternative using pre-defined criteria. An expert team work extracted effective criteria measures in an experimental problem using a common technique of brain storming. Six criteria including related execution history, available Facilities, adequacy of technical team, financial ability, company's antiquity, and previous employers' satisfaction are nominated exclusively related to this research. Because of the importance of problem in the real word and different opinion of managers in each level of decision, three different strategies have been utilized for ranking the companies and satisfy decision makers. Decision strategies are represented as VIKOR (V) in the decision making process. It is applied in the states of $V=0$ means decision made by veto, $V=0.5$ means there is veto in decision process by 50 percent and $V=1$ means decision is completely made by majority of experts view points. Results showed that different solutions will be outlined with different strategies. If the decision making process respects to the maximum group of experts view points ($V>0.5$) the best alternative is different from the case which there is a veto in the process ($V<0.5$). It confirms that general contractors need to pay more attention in defining decision strategy. For more confidence, results have been evaluated and checked with overall view of some local experts. While results seems to be accepted, means that this method is enough cable to select sub-contractors in engineering procurement construction projects by general contractors who are keen to out-source some parts of their projects.

For researchers interested in this field, it is recommended to focus on develop methodology of application fuzzy sets considering the more important criteria such as environmental concerns, supply management and safety issues in the real area of uncertainty in high level of educational process and applying this method in the other problem mainly in personal selection. It is highly recommended that further studies help decision makers to select the best supplier in supply chain problems.

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