Stochastic Lot-Sizing: Maximising Probability of Meeting Target Profit

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Abstract

The common objective function in lot sizing models with stochastic demand is maximizing the expected profit or minimizing the expected cost. But there are other important objective functions as well. One of these objective functions is maximizing the probability of achieving a target profit. In this paper, this objective function has been considered in stochastic lot- sizing problem. The optimal solution for single item and single period problems has been obtained. The above mentioned objective function has been converted as a service level constraint. Then the multi period model is developed for which the optimal solution has been gained by static-dynamic uncertainty strategy considering the service level values.

Keywords
Lot sizing, stochastic demand, static-dynamic uncertainty strategy, maximizing the probability of achieving a target profit.

1. Introduction
Maximizing the expected profit or minimizing the expected cost is generally used as the objective function in stochastic production planning problems, but in many cases other objective functions, such as maximizing the probability of achieving a target profit, are adopted by managers (Lau 1980). Sox (1997) described a formulation of the dynamic lot sizing problem when demand is random and the costs are non-stationary. Assuming that the distribution of the cumulative demand is known for each period and that all unsatisfied demand is backordered, the problem can be modeled as a mixed integer nonlinear program. The objective function is minimizing the expected cost. He considers backorder cost in the objective function and an optimal solution algorithm is developed that resembles the Wagner-Whitin algorithm for the deterministic problem but with some additional feasibility constraints. He considers backorder cost in the objective function. Different objective functions are given for newsboy problem in different sources. Schweitzer and Cachon (2000) have discussed descriptive models for newsboy problem. They have surveyed Risk Neutral, Risk-Averse and Risk-Seeking, Prospect Theory, Loss-Averse, Waste-Averse, Stock out-Averse Preferences. The single period problem (SPP)[1] is one of the basic inventory control models. In this model the demand is stochastic. Recognizing of this model is useful for lot-sizing with stochastic demand because in lot sizing problem, sum of the periods are surveyed that each objective function is single period problem. Khouja (1999) has given a literature review about newsboy problem in which he has studied more than ninety cases, each of them about different objective and utility functions. Researchers have observed that maximizing the expected profit may not completely reflect the reality. Actually, Lanzilotti (1958) found that maximizing the probability of achieving a target profit empirically to be more consistent with the attitude of many managers. Subsequently, researchers proposed extensions to the SPP in which the goal is to maximize the probability of achieving a target profit. Kabak and Schiff (1978) solved the SPP under the `satisfying' objective of maximizing the probability of achieving a target profit. They derived the necessary conditions for Q^* and provided a closed-form solution for exponentially distributed demand. One of the analysis in single period problem is cost-volume-profit (C-V-P) analysis. The C-V-P states:

Total profit= Sales Volume × (Unit Selling Price- Unit Variable Cost)- Fixed Cost

[1] - single period problem

Shih (1979) noticed that one of the deficiencies of the stochastic Cost- Volume- Profit (C-V-P) analysis is that although demand is treated as a random variable, the effect of unsold units on the profit is not taken into account. Shih (1979) considered the effects of over production and derived a general probability distribution of π, its expected value, and its variance as a function of Q. Sankarasubramanian and Kumaraswam (1983) maximized the probability of achieving a target profit(P_B) and provided closed-form solutions for Q^* for exponential and uniform demand distributions. Tempelmeier (2007) considers the uncapacitated single-item dynamic lot-sizing problem with stochastic period demands and backordering. He presents a model formulation that minimizes the setup and holding costs with respect to a constraint on the probability that the inventory at the end of any period does not become negative (α service level) and, alternatively, to a fill rate constraint (β service level). Mula (2006) reviews some of the existing literature of production planning under uncertainty. Guan and Liu (2010) studied the stochastic version of lot-sizing problems with inventory bounds and order capacities. They developed two models in stochastic programming: the first one has inventory-bound constraints, and the second one has both inventory-bound and order-capacity constraints.

Şenyigit et al. (2012) have studied an optimum lot-sizing policy based on minimum total relevant cost under price and demand uncertainties by using various artificial neural networks trained with heuristic-based learning approaches; genetic algorithm (GA) and bee algorithm (BA). They have examined combined approaches with three domain-specific costing heuristics comprising revised silver meal (RSM), revised least unit cost (RLUC), cost benefit (CB). It is concluded that the feed-forward neural network (FF-NN) model trained with BA outperforms the other models with better prediction results. In addition, RLUC is found the best operating domain-specific heuristic to calculate the total cost incurring of the lot-sizing problem. Hence, the best paired heuristics to help decision makers are suggested as RLUC and FF-NN trained with BA. Jiang and Guan (2010) develop a dynamic programming algorithm for the scenario-tree-based stochastic uncapacitated lot-sizing problem with random lead times. Their algorithm runs in O(N2) time, where N is the input size of the scenario tree, and improves the recently developed algorithm that runs in O(N3) time. Wong et al.(2011) proposes a stochastic dynamic lot-sizing problem with asymmetric deteriorating commodity, in which the optimal unit cost of material and unit holding cost would be determined. This problem covers a sub-problem of replenishment planning, which is NP-hard in the computational complexity theory. Therefore, they applies a decision system, based on an artificial neural network (ANN) and modified ant colony optimization (ACO) to solve this stochastic dynamic lot-sizing problem. The test results show that the intelligent system is applicable to the proposed problem, and its performance is better than response surface methodology. Tarim and Kingsman (2004) have surveyed the multi-period single-item inventory lot-sizing problem with stochastic demand under the ‘‘static–dynamic uncertainty’’ strategy that was proposed by Bookbinder and Tan (1988). In the static-dynamic uncertainty strategy, the replenishment periods are fixed at the beginning of the planning horizon, but the actual orders are determined only at those replenishment periods when the actual demand for the prior period has been realized. They present a mixed integer programming formulation that determines the optimal order-up-to level values for each period (R_t). Due to the stochastic demand in each period, the actual order quantity is determined after the actual demand in the previous period is specified. They considered a cost function.
The model is (M1 model):

$$\min E\{TC\} = \sum_{t=1}^{N} (a\delta_t + hE\{I_t\} + \upsilon E\{R_t\} - \upsilon E\{I_{t-1}\}) \tag{1}$$

s.t.

$$E\{I_t\} = E\{R_t\} - E\{d_t\} \qquad t = 1, \dots, N \tag{2}$$

$$E\{R_t\} \geq E\{I_{t-1}\} \qquad t = 1, \dots, N \tag{3}$$

$$E\{R_t\} - E\{I_{t-1}\} \leq M\delta_t \qquad t = 1, \dots, N \tag{4}$$

$$E\{I_t\} \geq \sum_{j=1}^{t} \left(G^{-1}_{d_{t-j+1}+d_{t-j+2}+\cdots+d_t}(\propto_t) - \sum_{k=t-j+1}^{t} E\{d_k\} \right) P_{tj} \tag{5}$$

$$\sum_{j=1}^{t} P_{tj} = 1 \qquad t = 1, \dots, N \tag{6}$$

$$P_{tj} \geq \delta_{t-j+1} - \sum_{k=t-j+2}^{t} \delta_k \qquad t = 1, \dots, N, \quad j = 1, \dots, t \tag{7}$$

$$E\{I_t\}, E\{R_t\} \geq 0, \quad \delta_t, P_{tj} \in \{0,1\}$$
$$t = 1, \dots, N, \quad j = 1, \dots, t$$

Notations

a_t: set up cost in period t

h_t: holding cost in period t

υ_t: purchasing cost in period t

δ_t: 1 if a replenishment order is placed in period t and 0 otherwise

I_t: Inventory level at the end of period t

R_t: Order-up-to level variables for each period

d_t: Demand in period t

M: Very large positive number

$G_{d_t}(x)$: Cumulative probability distribution function of demand to period t

P_{tj}: 1 if the most recent order prior to period t was in period t - j+1 and zero elsewhere

N : Number of periods

α_t: Service Level in period t

The objective function (1) minimizes total holding and set up costs. The constraint (2) expresses the inventory balance between the beginning and the end of the periods. Constraints (3) and (4) ensure that the variable R_t must be equal to I_{t-1} if no order is received in period t , and equal to the order-up-to-level if there is a review and the replenishment of an order. The constraint (5) expresses the service level constraint. Constraints (6) and (7) ensure that setup for period t takes place at the same period or the periods prior to it. The initial inventory level is taken as zero. Considering maximizing the probability of achieving a target profit in stochastic lot-sizing problem has not been reported in the literature.

Lau [1] surveyed the objective function that maximizes the probability of achieving a target profit in the newsboy problem with and without backorder cost. His conclusions are expressed in the following section.

Notations

D : Stochastic demand

B : Target profit

RS : Selling price

υ : Unit cost

Q : Order quantity

$$Q_1^* = \frac{B}{RS - \upsilon} \tag{8}$$

Please note that Q_1^* does not depend on the distribution of D. As you see the optimal order quantity is calculated without considering the setup cost.

2. Model with Setup Cost

2.1 Single period

As mentioned above Lau obtained Eq.(8) . We consider set up cost in the model .When considering the set up cost, it should be added to the expected profit and the optimal order quantity is obtained from Eq. (9):

$$Q_2^* = \frac{B + a}{RS - \upsilon} \tag{9}$$

a: set up cost

2.2 Uncapacitated multi item problem

The mathematical model of this problem is extension of Tarim and Kingsman's model, which is named M1 model in this paper (2004). In their model the multi-period single-item inventory lot-sizing problem with stochastic demand under service level constraint and without backorder cost has been studied and its optimal solution has been determined by static–dynamic uncertainty strategy. The extension of this model is used because static-dynamic approach is more suitable than static approach for solving problems with stochastic demand. There is more resemblance between this approach and the reality; thus the order quantity is determined by specifying the real demand and considering up-to-level inventory at the beginning of each period.

The mathematical model of this problem is expressed below:

$$\max\ Prob(Z_t \geq B_t) \quad t = 1, \dots, N$$

$$\min E\{TC\} = \sum_{t=1}^{N} (a\delta_t + hE\{I_t\} + \upsilon E\{R_t\} - \upsilon E\{I_{t-1}\})$$

$$\text{s.t.}$$

$$E\{I_t\} = E\{R_t\} - E\{d_t\} \qquad t = 1, \dots, N$$

$$E\{R_t\} \geq E\{I_{t-1}\} \qquad t = 1, \dots, N$$

$$E\{R_t\} - E\{I_{t-1}\} \leq M\delta_t \qquad t = 1, \dots, N$$

$$E\{I_t\} \geq \sum_{j=1}^{t} \left(G^{-1}_{d_{t-j+1}+d_{t-j+2}+\dots+d_t}(\alpha_t) - \sum_{k=t-j+1}^{t} E\{d_k\} \right) P_{tj}$$

$$\sum_{j=1}^{t} P_{tj} = 1 \qquad t = 1, \dots, N$$

$$P_{tj} \geq \delta_{t-j+1} - \sum_{k=t-j+2}^{t} \delta_k \qquad t = 1, \dots, N, \quad j = 1, \dots, t$$

$$E\{I_t\}, E\{R_t\} \geq 0, \quad \delta_t, P_{tj} \in \{0,1\} \qquad t = 1, \dots, N, \quad j = 1, \dots, t$$

Notations:

Z_t : Stochastic profit in period t

B_t: Budgeted profit level in period t

Other notations are similar to that of M1 model.

Objective function of this problem is consisted of two objective functions: First, Maximizing the probability of achieving a target profit and second, Minimizing total set up, holding and purchasing costs.

In order to maximize the probability of achieving a target profit, this probability should be maximized in each period. As a result, there are N+1 objective functions. Then by utilizing optimal order quantities from single period model with and without set up cost, we can gain service level values required. Lemma (1) is utilized to calculate service level values.

Lemma (1): service level is determined by using these equations:

1- With set up cost
   $\beta_t = F(Q^*_{2t})$
2- Without set up cost
   $\propto_t = F(Q^*_{1t})$

β_t : service level in period t, if set up happens in period t

$\propto_t$: service level in period t, if set up doesn't happen in period t

Q^*_{1t} : Q^*_1 in period t

Q^*_{2t} : Q^*_2 in period t

$F(D_t)$: Cumulative probability distribution function of demand in period t

Proof:

The cumulative probability distribution function is $F(D_t)$. Service level value is the probability that net inventory be equal to or greater than zero at the end of period t, i.e. $Pr(I_t \geq 0)$. $Pr(I_t \geq 0)$ is equivalent to $P(Q_t \geq D_t)$, so $P(I_t \geq 0) = F(Q_t)$.

So by using lemma(1), we can add (10) and (11) constraints and omit the first objective function. Thus, we can write final MIP model as: (M2 model)

$$E\{I_t\} = E\{R_t\} - E\{d_t\} \qquad t = 1, \ldots, N$$

$$E\{R_t\} \geq E\{I_{t-1}\} \qquad t = 1, \ldots, N$$

$$E\{R_t\} - E\{I_{t-1}\} \leq M\delta_t \qquad t = 1, \ldots, N$$

$$E\{I_t\} \geq \sum_{j=2}^{t} \left(G^{-1}_{d_{t-j+1}+d_{t-j+2}+\cdots+d_t}(\propto_t) - \sum_{k=t-j+1}^{t} E\{d_k\} \right) P_{tj} \qquad (10)$$

$$t = 1, \ldots, N$$

$$E\{I_t\} \geq \left(G^{-1}_{d_{t-j+1}+d_{t-j+2}+\cdots+d_t}(\beta_t) - \sum_{k=t-j+1}^{t} E\{d_k\} \right) P_{tj} \qquad (11)$$

$$t = 1, \ldots, N \qquad j = 1$$

$$\sum_{j=1}^{t} P_{tj} = 1 \qquad t = 1, \ldots, N$$

$$P_{tj} \geq \delta_{t-j+1} - \sum_{k=t-j+2}^{t} \delta_k$$

$$t = 1, \dots, N, \quad j = 1, \dots, t$$

$$E\{I_t\}, E\{R_t\} \geq 0, \quad \delta_t, P_{tj} \in \{0,1\}$$
$$t = 1, \dots, N, \quad j = 1, \dots, t$$

Constraint (10) expresses that set up for period t happens in one of the periods prior to it and as we don't have set up in period t, $\propto_t$ is used for the service level value. (11) constraint expresses that set up for period t happens in period t and as we have set up in period t, β_t is used for the service level value.

Solution method

Solution algorithm is described as below:

1- Determination of service level value without set up cost for each period ($\propto_t$).
2- Determination of service level value with set up cost for each period (β_t).
3- Entering service level quantities in M2 model.
4- Solving the model by available software and obtaining optimal solution.

Method for generating problems:

- Random set up cost between 300 to 500
- Random holding cost between 0.2,0.3,0.4 and 0.5
- The demand in each period is normally distributed about the forecasted value with a constant coefficient of variation, $C = \sigma_t / \mu_t = 0.333$. The mean is between 650 and 750.
- Selling price is $4 per unit
- Unit cost is $2
- Profit is between 2300 and 2400
- The initial inventory is zero

Numerical example:

Table (1) presents the information for three problems. Selling price, unit cost and target profit are:

$\mathrm{RS} = 4, \ v = 2, \ B = 2300, \ \mathrm{I}_0 = 0$

Table 1: The information for three problems

		Demand			Set up cost			Holding cost		
		Problem								
		1	2	3	1	2	3	1	2	3
Period	1	740	790	730	500	500	400	0.5	0.3	0.4
	2	730	620	610	700	400	500	0.2	0.5	0.5
	3	700	680	730	400	400	600	0.3	0.4	0.3
	4	780	580	700	230	300	500	0.2	0.2	0.4
	5	650	710	680	300	300	400	0.5	0.3	0.5

Service level value for each period is calculated in one of the following ways:

- **Without set up cost**

  In table (2), Q_{1t}^* values are given.

  Considering Eq. (8) as all of the variables are constant, values of Q_{1t}^* for all periods and problems is:

  $$\mathrm{Q}_{1t}^* = \frac{2300}{4-2} = 1150$$

  And based on values of Q_{1t}^*, service level constraint values, $\propto_t$, are calculated by using normal distribution table for each period and are presented in table(3).

Table 2: Q_{1t}^{*} values

		Period				
		1	2	3	4	5
problem	1	1150	1150	1150	1150	1150
	2	1150	1150	1150	1150	1150
	3	1150	1150	1150	1150	1150

Table 3: service level values ($\propto_t$)

		Period				
		1	2	3	4	5
problem	1	95%	95%	97%	92%	98%
	2	91%	99%	98%	99%	96%
	3	95%	99%	95%	97%	98%

- **With set up cost**

In table (4), Q_{2t}^{*} values are given.

Considering Eq (9), 1500 is calculated as follow:

$$Q_{22}^{*} = \frac{2300 + 700}{4 - 2} = 1500$$

Table 4: Q_{2t}^{*} values

		period				
		1	2	3	4	5
problem	1	1400	1500	1350	1265	1300
	2	1400	1350	1350	1300	1300
	3	1350	1400	1450	1400	1350

And based on values of Q_{1t}^{*}, service level constraint values for each period, β_t, are calculated using normal distribution table. These values are presented in table(5).

Table 5: Service level constraints (β_t)

		Period				
		1	2	3	4	5
problem	1	99.6%	99.9%	99.7%	96.9%	99.9%
	2	98.9%	99.9%	99.8%	99.9%	99.3%
	3	99.4%	100%	99.8%	99.8%	99.8%

By the gained values, M2 model is solved and the results for each problem are given in table (6).

Table 6: Results

			Period				
			1	2	3	4	5
problem	R	1	1150	2369	1639	1150	1150
		2	1150	1150	1150	2051	1472
		3	1150	1150	1150	1150	1150
	I	1	410	1639	939	370	500
		2	360	530	470	1472	762
		3	420	540	420	450	470

The problem has been programmed with LINGO software and the optimal answers have been obtained. The problem has been run on a PC with 2.4GHz CPU. Solution times for five period problems are less than one second.

3. Conclusion and Suggestions

3.1 Conclusion

For managers' satisfaction, using ordinary objective functions like maximizing expected profit or minimizing expected cost isn't enough. Therefore other objective functions like maximizing the probability of achieving a target profit are considered. Considering maximizing the probability of achieving a target profit in stochastic lot-sizing problem has not been reported in the literature. In this article optimal solution in single-item and single-period problems with set up cost has been determined. Then by use the results of this model, multi period model has been expressed and the optimal solution has been determined by static-dynamic uncertainty strategy.

3.2 Suggestions for future research

1- Extension of the model to multi-item with capacity constraints and solving the model with Meta heuristic methods.
2- Comparison of Meta heuristic methods for solving multi item model.
3- Developing and solving single-item and multi-item model with backorder cost.

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