

Three-Level Service Contract between Manufacturer, Agent and Customer (Game Theory Approach)

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Abstract

Warranty as a kind of service contract today plays a role key in business and legal transactions. In this paper, we present a manufacturer, an agent and a customer's model under different service contracts suggestions. The manufacturer's profit is maximized by determining sale price, warranty period and warranty price. In addition we obtain optimal maintenance cost or repair cost for the agent's model to maximize the agent's profit. Moreover, the customer maximizes his/her profit by choosing one of the options that are suggested by the manufacturer and the agent. We also consider the interaction between the manufacturer, the agent and the customer by using game theory approach. Nash equilibrium is obtained under non-cooperative game. In the non-cooperative game the manufacturer, the agent and the customer choose their best strategy separately and simultaneously. In the semi-cooperative game the manufacturer and the agent make their best strategy together and then they dominate the customer in a conventional way.

Keywords

Expected profit, Free replacement, Game theory, Repair, Service contract

1. Introduction

Nowadays, warranty has an important effect on purchasing products. Each customer prefers to buy a product which has a reliable warranty during its lifetime which can make the customers sure about their purchase. In other words, the customers are interested in buying the products with free or low repair charge whenever their purchased product needs repairs. This subject can be formed as service contracts which are attached to the products. Therefore, the service-providers such as agent and manufacturer recently have offered some service contracts to increase their profit. In addition, the customer is interested in using the suggested service contracts. Different service contracts may be suggested by service providers. In the literature, there are different types of service. We categorize them in four groups as follow:

Some researchers study about free replacement, in which the manufacturer replaces the failed item by new one free in charge for the first time and repairs it in low cost during warranty period. (Rinaska and Sandoh [4], Zhou, et.al.[1] and Wu, et.al.[2]). There is a situation in which the manufacturer has to compensate the consumers by refunding the price paid (money back warranty) in addition to free replacement (Boom [11]).

Moreover, in the second group outsourced services are considered. Based on this assumption, when a failure occurs, an external agent rectifies all failure under different options. (Asgharizadeh and Murthy [3], Murthy and

Asgharizadeh [8], Murthy and Yeung [7], Jackson and Pascual [6]). Game theory approach is used to determine an optimal structure for the agent's price.

Maintenance options are considered as a type of service contract in the third group. For instance some researchers present models that the customers can negotiate about the parameters of maintenance such as availability, reliability, time intervals between controls and etc. (Wang [5], Maronick [10]). Lifetime warranty policies and models for predicting failures and estimating costs for lifetime warranty policies are presented in some researches.(Chattopadhyay and Rahman [12]).

In addition to above papers, Hartman and Laksana [9] discussed some warranty contracts which include restrictions on repairs and renewals. A significant shortcoming of all these papers is that they model service contracts only with regard to the manufacturer or the agent. In other words, they consider the interaction of the customer with the manufacturer or the agent. But recently the interaction of the customers with manufacturers and agents has been considered simultaneously. Therefore each customer could arbitrarily choose one their suggested options. For instance, most of high-tech products are supported by the third part (agent) warranty in addition to the manufacturer warranty. iPad 2, one of the products of Apple factory, is a known example which is supported by both Applecare (as the manufacturer warranty) and Squaretrade (as the agent warranty).

In this paper, we propose the manufacturer, the agent and the customer's models under different warranty options. The manufacturer suggests two options to the customer: (1) free replacement per failure during warranty period, (2) no warranty. The manufacturer profit is maximized by determining optimal sale price, warranty price and warranty period under his/her options. On the other hand, the agent offers three options: (1) to repair the failed products after warranty expiration for a fixed cost per failure, (2) to repair all failures occurred during lifetime of product for a fixed price, (3) to repair the failed products for a fixed cost per failure during lifetime. In the agent's model under his/her warranty options, we determine optimal maintenance cost or repair cost by maximizing the agent's profit. Moreover, the customer maximizes his/her profit by choosing one of the options that are suggested by the manufacturer and the agent. We consider the interaction among the manufacturer, the agent and the customer by using game theory approach. In addition, the model is considered under the non-cooperative and semi-cooperative games. In the non-cooperative game we consider static and dynamic games. In the static game the manufacturer, the agent and the customer choose their best strategy separately and simultaneously. On the other hand, in the dynamic game we assume the manufacturer has more power than the agent and the agent has more power than the customer. Therefore, the agent makes the best strategy by dominating customer in a conventional way. In addition, the manufacturer also makes the best strategy by dominating the agent in a conventional way. In the semi-cooperative game we consider the manufacturer and the agent cooperating together as one service-provider. In such situation they have more power than the customer. Therefore we obtain the optimal solution under the dynamic game. For each scenario, numerical examples are presented to illustrate our models.

The remainder of this paper is organized as follows. We give the notation and assumptions underlying our model in Section 2. In Section 3, we formulate the problem, including models of the manufacturer, the agent and the customer, and discuss about their optimal solution. Section 4 presents a game theory approach including non-cooperative static and dynamic game. Semi-cooperative game theory approach is discussed in Section 5. Finally, the paper concludes in Section 6 with some suggestions for further research in this area.

2. Notation and Assumption

This section introduces the notation and formulation of our model. Here, we state decision variables and input parameters and assumptions underlying our models.

2.1 Decision Variables

P_{ip}	Sale price of the manufacturer under option i	P_a	Maintenance cost received by the agent
T_w	Warranty period for the manufacturer	C_{ir}	Repair cost of the agent per unit under option i
P_w	Warranty price of the manufacturer	R_i	Revenue per time unit under option i (customer)

2.2 Parameters

L	Lifetime of product	X	Working time of product after last repair
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C_p	Production cost per unit	λ	Failure rate
P_s	Salvage value of failed product	r	Aging rate of product
C_r	Repair cost per unit for agent itself	Π_{Mi}	Profit of manufacturer under option i
N_1	Number of failed items during product lifetime	Π_{Ai}	Profit of agent under option i
N_2	Number of failed items during warranty period	Π_{Ci}	Profit of customer under option i
N_3	Number of failed items after warranty expired	Π_{SP}	Profit of service provider
X_i	Operational time of product after ith repair and before (i+1)th failure		

2.3 Assumption

The proposed models in this paper are based on the following assumptions:

1. There is just one customer, one agent and one manufacturer.
2. The intensity of failure is increasing function of time based on Jackson and Pascual's model [6]. This kind of aging behavior by a failure hazard, $\lambda(t)$, given by $\lambda(t)=\lambda_0+rt$, where λ_0 is the initial failure rate when the product is new, at $t=0$, and t is the age of product.
3. The number of product failures in the specified time interval follows Poisson distribution and the failure rate is $\lambda(t)$.
4. Warranty's price, P_w , is a linear function of warranty period, T_w , $P_w=\beta T_w$, where $\beta>0$ depends on the number of failures which is estimated by the manufacturer.

3. The Proposed Model

In this section we propose models of the manufacturer, the agent and the customer. Regarding decision variables of manufacturer and agent, the customer will find the best option to maximize the profit.

3.1 The manufacturer's model

The manufacturer offers the following two options to the customer:

M1: The manufacturer has to replace the failed item free in charge during the warranty period. M2: No warranty.

3.1.1 The best strategy of the manufacturer's model under option M1

The manufacturer's profit= Sale price+ Warranty price- Production cost- (Production cost- Salvage value)*Number of failures during warranty period

$$(1) \quad \Pi_{M1} = P_{1p} + P_w - C_m - (C_m - P_s)E(N_2)$$

According to (1) the problem is to determine sale price, P_{1p} , warranty period, T_w , and warranty price, P_w . Since N_2 is a random variable by exponential distribution ($N_2 \sim \exp(\lambda_2)$), we use its expected value. Therefore,

$$(2) \quad E(N_2) = \sum_{N_2=0}^{\infty} N_2 \frac{e^{-\lambda_2} \lambda_2^{N_2}}{N_2!} = \lambda_2 = \int_0^{T_w} (\lambda_0 t + \frac{1}{2} r t^2) dt = \lambda_0 T_w + \frac{1}{2} r T_w^2$$

We put the first derivation of the manufacturer's model equal to zero, to obtain T_w . Since the manufacturer's model is concave, we have:

$$(3) \quad T_w^* = \frac{\partial P_w / \partial T_w - (C_p - P_s) \lambda_0}{r(C_p - P_s)} = \frac{\beta - (C_p - P_s) \lambda_0}{r(C_p - P_s)}$$

Please note that according to assumption (4), P_w is a linear function of T_w , $P_w=\beta T_w$. Solving $\pi_{M1}=0$ for zero profit regard to P_{1p} gives:

$$(4) \quad P_{1p} = C_p - (\beta T_w) + (C_p - P_s)(\lambda_0 T_w + \frac{1}{2} r T_w^2)$$

Since (4) is an increasing linear function of P_{1p} , the optimal P_{1p}^* occurs at the highest price that is possible for the manufacturer to charge the customer. Therefore,

$$(5) \quad P_{1p}^* = k_1 [C_p - (\beta T_w) + (C_p - P_s)(\lambda_0 T_w + \frac{1}{2} r T_w^2)]$$

for some $k_1 > 1$.

3.1.2 The best strategy of the manufacturer's model under option M2

The manufacturer's model under option M2 would be:

The manufacturer's profit= Sale price- Production cost

$$(6) \quad \Pi_{M2}(P_{2p}) = P_{2p} - C_p$$

Solving $\pi_{M2}=0$, for zero profit regard to P_{2p} gives:

$$(7) \quad P_{2p} = C_p$$

Since (7) is an increasing linear function of P_{2p} , the optimal P_{2p}^* occurs at the highest price that is possible for the manufacturer to charge the customer. Therefore,

$$(8) \quad P_{2p}^* = k_2 C_p$$

where $k_2 > 1$.

3.2 The agent's model

The agent has three options to offer to the customer that they are:

A1: To repair the product when a failure occurs for fixed cost per failure (after expiration of warranty). A2: To repair all failures occurred during lifetime of product for a fixed price. A3: To repair the product when a failure occurs for fixed cost per failure (during lifetime).

3.2.1 The best strategy of the agent's model under option A1

The agent's profit= (Revenue of repairing failed product after expiration of warranty- Repair cost) *Number of failed product after T_w .

$$(9) \quad \Pi_{A1}(C_{1r}) = (C_{1r} - C'_r)E(N_3)$$

The number of failed product after expiration of warranty, N_3 , is a random variable with exponential distribution ($N_3 \sim \exp(\lambda_3)$). Therefore, the expected value of N_3 is:

$$(10) \quad E(N_3) = \lambda_3 = \int_0^{L-T_w} \lambda(t) dt = \lambda_0(L - T_w) + \frac{1}{2} r(L - T_w)^2$$

Solving $\pi_{A1}=0$ for zero profit regard to C_r gives:

$$(11) \quad C_{1r} = C'_r$$

Since (9) is an increasing linear function of C_{1r} , the optimal C_{1r}^* occurs at the highest price that is possible for the agent to charge the customer. Therefore,

$$(12) \quad C_{1r}^* = k_3 C'_r$$

for some $k_3 > 1$.

3.2.2 The best strategy of the agent's model under option A2

The agent's model under this option would be:

The agent's profit= Maintenance price- Repair cost for agent*Number of failures during lifetime of product- Penalty cost due to delay in repairing queue.

$$(13) \quad \Pi_{A2}(P_a) = P_a - N_1 C'_r - \alpha \left(\sum_{i=1}^{N_1} \max\{0, Y_i - \tau\} \right)$$

According to assumption one, there is not any queue and the penalty cost is ignored. Then (13) would be:

$$(14) \quad \Pi_{A2}(P_a) = P_a - N_1 C'_r$$

The number of failures in the lifetime of product, N_1 , is a random variable with exponential distribution ($N_1 \sim \exp(\lambda_1)$). We use the expected value of that.

$$(15) \quad E(N_1) = \lambda_1 = \int_0^L \lambda(t) dt = \lambda_0 L + \frac{1}{2} r L^2$$

Solving $\pi_{A2}=0$ for zero profit regard to P_a gives:

$$(16) \quad P_a = C'_r \left(\lambda_0 L + \frac{1}{2} r L^2 \right)$$

Since (16) is an increasing linear function of P_a , the optimal P_a^* occurs at the highest price that is possible for the agent to charge the customer. Therefore,

$$(17) \quad P_a^* = k_4 C_r' (\lambda_0 L + \frac{1}{2} r L^2)$$

for some $k_4 > 1$.

3.2.3 The best strategy of the agent's model under option A3

The agent's model under this option is as follow:

The agent's profit = (Revenue of repairing failed products - Cost of repair for the agent) * Number of failed products during lifetime.

$$(18) \quad \Pi_{A3}(C_{3r}) = (C_{3r} - C_r') E(N_1)$$

Solving $\pi_{A3} = 0$ for zero profit regard to C_{3r} gives:

$$(19) \quad C_{3r} = C_r'$$

Since (24) is an increasing linear function of P_a , the optimal P_a^* occurs at the highest price that is possible for the agent to charge the customer. Therefore

$$(20) \quad C_{3r}^* = k_5 C_r'$$

for some $k_5 > 1$.

3.3 The Customer's Model

The customer would choose one of the agent or manufacturer's option to maximize his/her profit. Therefore the customer has three options:

C1: (M1 and A1), Paying P_w to the manufacturer for free replacement per failure during T_w , and after that, paying C_{1r} cost per failure to the agent.

C2: (M2 and A2), Paying P_a to the agent, therefore each failure would be repaired free in charge.

C3: (M2 and A3), Paying C_{3r} cost per failure to the agent during lifetime.

3.3.1 The best strategy of the customer's model under option C1

Under option C1, the customer's model would be as follow:

The customer's profit = Revenue obtained by the product - Sale price - Warranty price - Repair cost after T_w .

$$(21) \quad \Pi_{C1}(R_1) = R_1(T_w + \sum_{i=1}^{N_1} X_i + X) - P_{1p} - P_w - C_{1r} N_3$$

We note that the time of new product replacement is ignorable. Also the total downtime of product for the customer is negligible, because the mean total (waiting + repair) time is very small in relation to mean time to failure [3]. Therefore:

$$(22) \quad R(T_w + \sum_{i=1}^N X_i + X) \approx RL$$

where N represents the number of failures. According to (11) and (22), (21) would be:

$$(23) \quad \Pi_{C1}(R_1) = R_1 L - P_{1p} - P_w - C_{1r} (\lambda_0 (L - T_w) + \frac{1}{2} r (L - T_w)^2)$$

Solving $\pi_{C1} = 0$ for zero profit regard to R_1 gives:

$$(24) \quad R_1 = \frac{1}{L} (P_{1p} + P_w + C_{1r} (\lambda_0 (L - T_w) + \frac{1}{2} r (L - T_w)^2))$$

Since (24) is an increasing linear function of R_1 , the optimal R_1^* occurs at the highest price that is possible for the customer. Therefore,

$$(25) \quad R_1^* = \frac{k_6}{L} (P_{1p} + P_w + C_{1r} (\lambda_0 (L - T_w) + \frac{1}{2} r (L - T_w)^2))$$

for some $k_6 > 1$.

3.3.2 The best strategy of the customer's model under option C2

When the customer chooses option C2, his/her profit would be as follow:

The customer's profit = Revenue obtained by product - Sale price - Cost of agent's service contract.

$$(26) \quad \Pi_{C2}(R_2) = R_2 L - P_{2p} - P_a$$

Solving $\pi_{C2} = 0$ for zero profit regard to R_2 gives:

$$(27) \quad R_2 = \frac{1}{L}(P_{2p} + P_a)$$

Since (27) is an increasing linear function of R2, the optimal R2 occurs at the highest price that is possible for the customer. Therefore,

$$(28) \quad R_2^* = \frac{k_7}{L}(P_{2p} + P_a)$$

for some $k_7 > 1$.

3.3.3 The best strategy of the customer's model under option C3

The customer's model under option C3 is:

The customer's profit= Revenue obtained by product- Sale price- Repair cost during the product's lifetime.

$$(29) \quad \Pi_{C3}(R_3) = R_3L - P_{2p} - C_{3r}E(N_1)$$

Solving $\pi_{C3}=0$ for zero profit regard to R_3 gives:

$$(30) \quad R_3 = \frac{1}{L}(P_{2p} + C_{3r}E(N_1))$$

Since (30) is an increasing linear function of R_3 , the optimal R_3^* occurs at the highest price that is possible for the customer. Therefore,

$$(31) \quad R_3^* = \frac{k_8}{L}(P_{2p} + C_{3r}(\lambda_0L + \frac{1}{2}rL^2))$$

for some $k_8 > 1$.

4. The Non-Cooperative Static Game

We have a set of players (N=3), and for each player, we also specify the set of strategies and corresponding profit available to that player [13]. The components of the three-person game are described as follow:

- N= {M, A, C} where the manufacturer, the agent and the customer are represented by M, A and C respectively.
- The set of strategies available to the player i, S_i , $i= M, A, C$, are
 $S_M = \{M1, M2\}$
 $S_A = \{A1, A2, A3\}$
 $S_C = \{C1, C2, C3\}$
- The payoff of each player i, $u(i)$, $i=M,A,C$, is
 $u(M) = \{ \pi_{M1}(P_{1p}, P_w, T_w), \pi_{M2}(P_{2p}) \}$
 $u(A) = \{ \pi_{A1}(C_{1r}), \pi_{A2}(P_a), \pi_{A3}(C_{3r}) \}$
 $u(C) = \{ \pi_{C1}(R_1), \pi_{C2}(R_2), \pi_{C3}(R_3) \}$

In this section we present the static and dynamic scenarios as a non-cooperative static game. In the static game the players obtain their best strategy simultaneously and separately. Therefore, Nash equilibrium is obtained according to Table1. Please note that in Table1 all strategies of players are presented. The numerical examples are also presented to illustrate our model.

5. The Semi-cooperative Game

In this section we assume that the manufacturer and the agent cooperate together and act as a unit, service-provider. Therefore the customer faces the following options:

(SP1) During the warranty period, when a failure occurred, the service-provider tries to repair the fault free of charge, in the specified time (defined in the contract), otherwise the service-provider replaces it without charge. Please note, after expiration of the warranty, repair per each fault will appear on cost of C_{1r}

(SP2) The customer prefers not to buy the service contract.

The customer's profit under option 1:

$$(32) \quad \Pi_C = RL - P_p - P_w - C_r(\lambda_0(L - T_w) + \frac{1}{2}r(L - T_w)^2)$$

$$(33) \quad R^* = \frac{k_9}{L} (P_p + P_w + C_r (\lambda_0 (L - T_w) + \frac{1}{2} r (L - T_w)^2))$$

Since the manufacturer and the agent cooperate together, the service-provider has more power than the customer. We will regard the interaction between the service provider and the customer as a Stackelberg game. Based on the customer's decision variable, the profit of the service-provider is maximized. Therefore the model would be:

$$(34) \quad \begin{aligned} \text{Max } \Pi_{sp} &= P_p + P_w - (C_p - P_s) \sum_{i=1}^{N_2} \max\{0, \text{sgn}(Y_i - \tau + \varepsilon)\} \\ &\quad - C_p - C'_r \sum_{i=1}^{N_2} \max\{0, \text{sgn}(\tau - Y_i)\} + (C_r - C'_r) E(N_3) \\ \text{s.t.} \\ R^* &= \frac{k_9}{L} (P_p + P_w + C_r (\lambda_0 (L - T_w) + \frac{1}{2} r (L - T_w)^2)) \\ k_9 &> 1. \end{aligned}$$

We would illustrate our model by presenting numerical examples.

6. Numerical Examples

Example 1: We set the parameters of model as below:

$\lambda_0=0.6$ (peryear), $r=0.3$ (per square year), $L=6$ (years), $C'_r=100(10^3)\$$, $C_p=800(10^3)\$$, $\beta=450(10^3)\$$ and $P_s=500(10^3)\$$.

We also suppose, $k_1=1.2$, $k_2=1.1$, $k_3=1.1$, $k_4=1.2$, $k_5=1.1$, $k_6=1.1$, $k_7=1.1$ and $k_8=1.1$.

We obtain the optimal solution of the manufacturer, the agent and the customer's model under their strategies (Table-1).

Table 1: Profits of manufacturer, agent and customer, $(\Pi_M, \Pi_A, \Pi_C)(10^3)\$$ to Obtain Nash equilibrium-Example.1

		A1	A2	A3
C1	M1	(145,67.5,34.6)	(145,180,34.6)	(145,90,34.6)
	M2	(80,67.5,34.6)	(80,180,34.6)	(80,90,34.6)
C2	M1	(145,67.5,32.6)	(145,180,32.6)	(145,90,32.6)
	M2	(80,67.5,32.6)	(80,180,32.6)	(80,90,32.6)
C3	M1	(145,67.5,34.2)	(145,180,34.2)	(145,90,34.2)
	M2	(80,67.5,34.2)	(80,180,34.2)	(80,90,34.2)

According to Table 1, Nash equilibrium would be (M1, A2, C1). It means that the manufacturer prefers to suggest his/her first option. The agent's preference is to offer his/her second option. And the customer has the most profit by choosing his/her first option. The first option which is selected by the customer is also suggested by the manufacturer. Therefore, the agent wouldn't obtain his/her maximum profit. Therefore:

$$\begin{aligned} \Pi_{M1}^* &= 145(10^3) \$, T_w^* = 1 \text{ year}, P_w^* = 450(10^3) \$, P_{1p}^* = 870(10^3) \$ \\ \Pi_{A2}^* &= 67.5(10^3) \$, P_a^* = 1080(10^3) \$ \text{ and} \\ \Pi_{C1}^* &= 34.6(10^3) \$, R_1^* = 346(10^3) \$. \end{aligned}$$

Now we illustrate the model (34) by presenting the following example.

Example 2: Consider the parameters of the example 1.

We have:

$$\begin{aligned} \text{Max } \Pi_{sp} &= P_p + P_w - (500) \sum_{i=1}^{N_2} \max\{0, \text{sgn}(Y_i - 0.007 + \varepsilon)\} - 800 - 100 \sum_{i=1}^{N_2} \max\{0, \text{sgn}(0.007 - Y_i)\} + (C_r - 100) E(N_3) \\ C_r &= \frac{1}{(0.6(6 - T_w) + 0.3/2(6 - T_w)^2)} \left(\frac{6}{1.1} R^* - (P_p + P_w) \right) \end{aligned}$$

We substituting the Cr in the above function and we reach the following results:
 $T_w^* = 1.5$ year, $P_w^* = 675(10^3)$, $P_p = 697(10^3)\$$, $C_r = 105(10^3)\$$, $R^* = 329(10^3)\$$

7. Conclusion

In this paper we have presented a three level warranty between the manufacturer, the agent and the customer by using game theory approach. Under different service contracts suggestions, the manufacturer's profit is maximized by determining sale price, warranty period and warranty price. In addition, we obtain optimal maintenance cost or repair cost for the agent's model to maximize the agent's profit. Moreover, the customer maximizes his/her profit by choosing one of the options that are suggested by the manufacturer and the agent. We also consider the interaction among the manufacturer, the agent and the customer under non-cooperative and semi cooperative games. In the non-cooperative game, Nash equilibrium is obtained in the static scenario. We also consider the manufacturer and the agent cooperate together and act as one service-provider which has more power than the customer under the semi cooperative game. By presenting some numerical examples, we illustrate the obtained results of each scenario.

There is much scope in extending the present work. For example, each customer can buy more than one product, therefore he/she can select different portfolio of service contracts for each product. In such case, queue may be created, therefore the agent has to pay penalty if the repair of failed product is delayed. Also, risk parameter can be considered for the manufacturer, the agent and the customer that have a considerable effect on their decisions.

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