

An Integer Programming Approach to Air Traffic Management Problem

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Abstract

The main objective of this paper is to introduce a unique formulation for aircraft routing in a 3-D mesh that provides the exact time of arrival to any node that an aircraft visits and the average speed between within two consecutive nodes. While a number of different objectives could have been evaluated, we focused on the early arrivals and delays. Since the model provides exact arrival time to any given node, mid-air collision problem is successfully handled. In traditional aircraft routing problems, mathematical models are build using air-segments and discrete time intervals. Since the aircraft movement is handled using discretized time, mid-air collision problem cannot be handled precisely. Furthermore, the traditional models cannot guarantee the optimal usage of the air-space due to large safety distances requirements between any given airplane-pairs in order to avoid the mid-air collision. Hence the proposed mathematical model brings a new perspective to the aircraft routing problem.

Keywords

Aircraft routing, 3-D mesh network, exact arrival times, exact speed

1. Introduction

With tremendous increase in the demand for air travelling, this industry has been encountering with serious challenges for decades. While the demands for the airport and air-space usages are increasing, both academic and industry efforts are not able to address these challenges adequately. A number of stakeholders with different objectives influence the usage of the airspace. Airline companies, airport authorities, air-traffic controllers and the government are the main stakeholders and all have different objectives in the usage of air-space. Physical capacities of airports, available aircraft-fleets, crew-requirements, fuel cost and finally the safety considerations are some of the main factors that should be carefully incorporate into aircraft routing problem.

Based on International Air Transport Association (IATA) Economics 2012, the revenue passenger kilometers (RPKs) for international scheduled passenger traffic has risen from 150 billion monthly in 2002 to 275 billion in 2012. Likewise, the available seat kilometers (AKSs) calculated for international scheduled passenger traffic has been raised from 210 billion monthly in January 2002 to 360 billion in January 2012 (IATA, 2012). Increasing traffic in most part of the world without adequately updating the infrastructure further increase the challenges for air-traffic controllers. While the air-transportation industry looks like a lucrative business, airline companies have been struggling to survive during past 2 decades despite increasing passenger demand, more fuel-economic aircraft designs and advanced technological solutions for such as booking, optimization etc. Therefore, finding alternative methods to improve air-traffic operations such as reducing cost, improving traffic, increasing efficiency etc. are essential for the industry to deliver the expected services to the society.

Air-traffic congestion is experienced under two circumstances: First around airports when traffic overpasses the airport's arrival and departure capacity; Second, around an air sector when the number of airplanes in the sector exceeds the defined capacity. Different geographies are facing different problems. While the congestion problem

occurs around the airports in USA due to runway and gate capacities, Europe faces difficulties mostly in their airspace (between airports). Due to the geographical size of USA, the air-traffic between airports is not a significant challenge for the time being. In their works, Bertsimas et al. (2011) summarize that according to the US Department of Transportation and Air Transport Association, due to limited capacities in airports, over 26% of all flights were delayed on arrival by more than 15 minutes; more than 3% of flights were cancelled in 2007 costing approximately \$12 billion for the economy. Similarly, European airlines reported over \$5 billion expenses for the similar problem.

Increasing worldwide demand for the air travelling necessitates the development of advanced decision making support systems to help Air Traffic Controllers (ATC). Such support systems should be capable of defining alternative routes for incoming/outgoing traffic within a short time-frame. Furthermore, the proposed support system should incorporate all the constraints (safety and economical) into decision making process. Currently, the ATC personal deals with flight routing both on the ground and in the air by instructing pilots concerning their next move (it may be speed change, altitude change etc.). Since most planning is done by personnel without significant decision support system, air traffic controllers face extreme pressures in their works. Both academicians and industry (including NASA) tackle this challenge and produces abundance of scientific solutions to reduce the workload of ATC personnel. The mathematical model proposed in this paper further improves the existing efforts by incorporating the aircrafts' exact speeds and exact location at certain location. Consequently, the mathematical model presented in this paper precisely handles the calculation of fuel usage cost and ensures the avoidance of mid-air collision.

The remainder of the paper is organized as follows. In section 2, a brief literature review pertinent to our work is discussed. In section 3, model is introduced. Finally in section 4, the conclusions and future works are summarized.

2. Literature Review

Air Traffic Flow Management (ATFM) problem has been studied in many different areas during past decades. One of the first ATFM optimization model is introduced by Odoni (1998). He formulated the problem as a mathematical mode and provided an algorithm to determine the optimal ground holding patterns. Vossen et al. (2012) provided an in-depth summary of the ATFM in a book-chapter. In their works, Gupta and Bertsimas (2011) and Bertismas et al. (2011) present mathematical models to solve Large-scale traffic management problems. Integer programming model presented in Bertismas et al. (2011) aims at tackling the planning of the ATFM problem in a large geography such as US airspace. Similar to many other ATFM models, their work is also using discrete time-increments. The work of D'Ariano et al. (2011) introduces a rerouting algorithm around airports. Lindsay et al. (1993) proposed a disaggregated deterministic binary programming model by considering the airport and airspace constraints for assigning airborne and ground holding Bertsimas and Patterson (1998 and 2000) presented a binary programming model for the deterministic, multi airport TFMP that address capacity restrictions on the en route airspace. Another example for the time-space network based models was proposed in Corolli et al. (2010). They focused on identifying most number of alternatives for the airlines to perform their flights.

In recent years a new ATFM concept has been proposed namely Free Flight Concept (FFC). The idea is to give pilots a flexibility to determine the best routes under the given conditions. While initial ideas for the FFC go to the work done by Dutch National Aerospace Laboratory in 1997 (Noekstra, 2002), recent studies provide stronger foundation for the concept. Nowadays, the airline industry has consensus on the realization of FFC in near future. Dell'Olmo and Lulli (2002) proposed a free flight path scenario that considers a network with no fix routes. Ma et al. (2004) present a model on multi commodity dynamic network flow for short term air traffic flow management. Krozel et al. (2006) presented a routing and scheduling algorithm for ATFMP including ground delays, route selection and airborne holding, aligning with Collaborative Decision Making philosophy. Geng and Cheng (2007) presented an integer programming approach to determine the routes open to certain users during given time periods. Lulli and Odoni (2007) focused on European airspace in his deterministic and stochastic integer programming models. Bertsimas et al. (2001) introduced a comprehensive model that covers all the phases of flights such as ground and air delay, cancellation, rerouting and continued flights. They have considered the fairness of priorities between ground holding and airborne delays and also have solved a small the model in US market. Churchil et al. (2009) introduced an airspace volume instead of air sector so that the aircraft connectivity can be enforced. Alonso-Ayuso (2011) focused on the collision avoidance problem in ATM. They developed a mixed-integer linear optimization model to tackle collision avoidance.

In this paper, the aircraft routing problem is modeled in a 3-D mesh network. Unlike the most solutions discussed above, the proposed model use nodes and arcs instead of air-segments or air sectors. Furthermore, in the proposed model, the time is considered as a decision variable. On the other hand, most other solutions use time as an input as the decisions are only made at the discrete time intervals. Hence, the goal of the proposed mathematical model is to determine which arcs the airplanes will follow, what would be their speed at the beginning and end of the arc, and finally what are their departure and arrival times to the arcs they follow.

In sum, the contributions of this work is standing on continuous time bases, so that it make it possible to track the flight in each node and arc, and give the possibility of rerouting precisely while respecting the collision avoidance. Moreover, by considering the average flight speeds at each arc, estimating the real fuel consumption cost is possible. Most other previous works modeled the fuel consumption cost as a function of delay times. The variation of the speed which inherently affects the fuel usage has not been taken in to account. The proposed model enables us to optimize the aircraft routing by minimizing traveling times, operating cost, fuel cost, air and sound pollution, while safety and technical constraints are respected. Moreover the elegant formulation enables the optimal usage of the airspace by reducing the safety distances between aircrafts.

3. Formulation of Aircraft Routing Problem

The model presented in this section determines the best routes for all the flights that are under consideration during a time-interval in defined air-segment. The airplanes are assumed to be traveling on a bi-directional network. The network is defined by its nodes (*Node i*) and the bi-directional arcs connecting the nodes (*arc(i,j)*) The solution to the problem is obtained through deciding the arrival and departure times at each node and speed between two nodes. Safety distances between flights and mid-air collision avoidance are explicitly modeled. Hence the model ensures that no two flights from opposite directions can be present on the same arc at the same time. Furthermore, two flights on the same direction are separated from each other with a minimum safety distance. While large body of the literature focus on modeling the similar problem using discrete time periods, the presented method provides arrival and departure times to a node in continuous time so that the results guarantees that no two flights can present at the same place at the same time. While the model is discussed as a planning tool for multi-aircraft routing, it can be used as a route planner for a single aircraft where the routes of all the other aircrafts in the air-space are used as input parameters.

The mathematical model is built on a 3-D mesh network where an aircraft route follows a number of connected arcs. In order to navigate in the airspace, aircrafts strictly follow neighbor nodes using the connecting arcs. There are three types of nodes in the system. First, the arriving nodes which are nodes located outside of the mesh. An aircraft arriving to the airspace has to enter to the network through one of these outside nodes. Similarly, an aircraft, intended to leave the airspace has to reach one of these nodes. The second type of nodes is the ground nodes where an aircraft uses the node for landing and departure. Finally the internal nodes which are the connecting nodes enable an aircraft to establish the route between the ground nodes and the external nodes.

Input Parameters

Below, the definitions of the parameters, used in the model are given.

$ORG(f)$: Entry point of flight f

$DST(f)$: Exit point of flight f

e^f : Earliest possible departure time of flight f at its destination point

l^f : Latest possible arrival time of flight f at its destination point

P_e^f : Penalty cost of flight f for arriving or departing early

P_l^f : Penalty cost of flight f for arriving or departing late

- $ST^{ff'}$: Required safety distance time for flight f' following departure of flight f within the same nodes
- $D_{(i,k)}$: Distance of arc (i,k)
- ρ^f : Fuel coefficient for the flight f
- $Schedule(t)$: $[(a_{org(t)}^f, d_{org(t)}^f), \dots, (a_{ik}^f, d_{ik}^f), \dots, (a_{dst(t)}^f, d_{dst(t)}^f)]$ The flight schedule indicates the flight arrival and departure times at every node from origin to destination (take off to landing).
- $v_{(i,k)}^f$: Speed of flight f at arc (i,k)
- $t_{(i,k),m}^f$: Required time for flight f to travel arc (i,k) with speed m

Variables

In this section, the decision variables used in mathematical model are described.

- $a_{(i,k)}^f \in \mathcal{R}^+$ Arrival time of flight f at node k , travelling from node i
- $d_{(i,k)}^f \in \mathcal{R}^+$ Departure time of flight f from node i , travelling to node k
- $X_{(i,k)}^f = \begin{cases} 1 & \text{if flight } f \text{ travels arc } (i, k) \\ 0 & \text{otherwise} \end{cases}$
- $Z_{(i,k)}^f = \begin{cases} 1 & \text{if flight } f \text{ travels arc } (i, k) \\ 0 & \text{otherwise} \end{cases}$
- $L_{S'S}^f = \begin{cases} 1 & \text{if flight } f \text{ can change its speed from } f \text{ to } f' \\ 0 & \text{otherwise} \end{cases}$
- $\alpha_{ik}^{ff'} = \begin{cases} 1 & \text{if flight } f \text{ is followed by flight } f', \text{ travelling arc } (i, k) \\ 0 & \text{otherwise} \end{cases}$
- $\theta_i^{ff'} = \begin{cases} 1 & \text{if flight } f \text{ leaves node } i \text{ before arrival of flight } f' \\ 0 & \text{otherwise} \end{cases}$
- $\beta_{(i,k)}^{ff'} = \begin{cases} 1 & \text{if } f \text{ uses arc } (i, k) \text{ prior to } f' \text{ uses arc } (k, i) \\ 0 & \text{otherwise} \end{cases}$

Indexing

- $\omega(i)$: The list of nodes connected to the *node* i :
 $\omega^-(i)$ directed to *Node* i , $\omega^+(i)$ directed from *Node* i
- $\delta(i)$: The list of arks connected to *node* i

3.1 Objectives

The main objective of the proposed mathematical model is to determine the route for a set of flights on a network that consists of N nodes and M arcs with a minimum cost. While the cost of navigating F flights in the given airspace may vary, in the proposed model, the sum of the early arrival and late arrival penalties, and fuel consumption cost is used to determine the cost function. Hence the objective function of the mathematical model is as follows:

$$\text{Minimize} \quad \sum_{f \in F} \left(P_e^f T_e^f + P_l^f T_l^f + \frac{\sum_{(i,k) \in \delta(i)} (a_{(i,k)}^f - d_{(i,k)}^f) v_{(i,k),m}^f}{\rho^f} \right) \quad (1)$$

In the Equation 1, the fuel cost is calculated on each arc (i,k) as a product of fuel consumption rate $\pi_{ik}^f = V_{(i,k)}^f / \rho^f$ and the flight time $t_{(i,k)}^f = a_{(i,k)}^f - d_{(i,k)}^f$ which leads to a nonlinear equation. Since the distance between two nodes is known, then the Equation 1 can be linearized by replacing the current term $\frac{\sum_{(i,k) \in \delta(i)} f(a_{(i,k)}^f - d_{(i,k)}^f) v_{(i,k),m}^f}{\rho^f}$ by $\sum_{m \in M} \left(\frac{D_{(i,k)} X_{(i,k)}^f}{\rho^f} \right)$ where $X_{(i,k)}^f = 1$ if flight f follows arc (i,k) , 0 otherwise.

On the other hand, penalty cost of early or late arrival of flight has been stated as: $P_e^f T_e^f + P_l^f T_l^f$ where:

$$T_l^f = a_{(i,d)}^f - Sch^f \quad \forall if, \quad \forall d \in DST(f) \quad (2)$$

$$T_e^f = Sch^f - a_{(i,d)}^f \quad \forall if, \quad \forall d \in DST(f) \quad (3)$$

Equations (2 and 3) identify the lateness or earliness where the index d refers to a dummy node. A dummy node is used as a sink node.

3.2 Constraints

In addition to the constraints given in Equations (2 and 3) following constraints are introduced.

$$\sum_{d \in ORG(f)} X_{(d,i)}^f = 1 \quad \forall i \in \omega^+(d), \forall f \in F \quad (4)$$

$$X_{(i,k)}^f \leq 1 \quad \forall i, \forall k \in \omega^+(i), \forall f \in F \quad (5)$$

$$\sum_{j \in \omega^-(i)} X_{(j,i)}^f = \sum_{k \in \omega^+(i)} X_{(i,k)}^f \quad \forall i, \forall f \in F \quad (6)$$

$$\sum_{d \in DST(f)}^n X_{(i,d)}^f = 1 \quad \forall i \in \omega^-(d), \forall f \in F \quad (7)$$

Equation (4) ensures all the flights enter to the airspace through one of the dummy nodes in Equation (5), it is ensured that each flight travel a certain arc at most once. And after, in Equation (6), traveling between every adjunct node is considered. In other word, once an aircraft enters to airspace, it continues its journey by selecting one of the neighboring nodes. Due to technical limitations of an aircraft, an aircraft approaching to *node i* from *node k* can only continue its journey on one of the technical feasible arcs. Equation (7) forces flights to exit the airspace through exit dummy nodes.

Following set of equations deal with aircraft's arrival and departure time based on the routes chosen before and the speed of flight at every traveling arc.

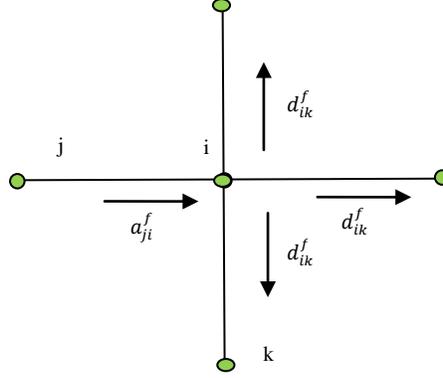


Figure 1: Selection of the next arc

$$a_{(i,d)}^f \geq e^f \quad \forall d \in ORG(f), \forall (i,d) \in \delta^-(d), \forall f \in F \quad (9)$$

$$a_{(i,d)}^f \leq l^f \quad \forall d \in DST(f), \forall (i,d) \in \delta^-(d), \forall f \in F \quad (10)$$

$$\sum_{j \in \omega^-(i)} a_{(j,i)}^f = \sum_{k \in \omega^+(i)} d_{(i,k)}^f \quad \forall i, \forall f \quad (11)$$

$$d_{(i,k)}^f \leq X_{(i,k)}^f M \quad \forall i, \forall (i,k) \in \delta^+(i), \forall f \in F \quad (12)$$

Equations (9-10) constraint the earliest and latest arrival/departure times. Equations (11-12) ensure that aircrafts travelling on $arc(j,i)$ select one of the alternative arcs after node i while there is no stopping on an intermediate node. X_{ik}^f is the decision variable.

$$\sum_{m \in Spd} I_{(i,k),m}^f = X_{(i,k)}^f \quad \forall i, \forall (i,k) \in \delta^+(i), \forall f \in F \quad (13)$$

$$v_{(i,k)}^f = v_{(j,i)}^f + \sum_{m \in Spd} \Delta_m I_{(i,k),m}^f \quad \forall i, \forall (i,k) \in \delta^+(i), \forall (j,i) \in \delta^-(i), \forall f \in F \quad (14)$$

$$\sum_{m \in Spd} v_m Z_{(i,k),m}^f = v_{(i,k)}^f \quad \forall i, \forall (i,k) \in \delta^+(i), \forall f \in F \quad (15)$$

$$\sum_{m \in Spd} Z_{(i,k),m}^f = X_{(i,k)}^f \quad \forall i, \forall (i,k) \in \delta^+(i), \forall f \in F \quad (16)$$

$$t_{(i,k)}^f = \sum_{m \in Spd} Z_{(i,k),m}^f \times T_{(i,k),m}^f \quad \forall i, \forall (i,k) \in \delta^+(i), \forall f \in F \quad (17)$$

$$a_{(i,k)}^f \geq d_{(i,k)}^f + t_{(i,k)}^f \quad \forall i, \forall (i,k) \in \delta^-(i), \forall f \in F \quad (18)$$

In here the speed is discretized. We assume that an aircraft can change its speed from one arc to another by a factor of Δ_m . In Equation (13), it is ensured that if the $arc(i,j)$ is part of $flight f$'s route, then there exists a speed for $flight f$ on that $arc(i,j)$. The speed increment by $\mp \Delta$ from one arc to the next arc is handled in Equation (14). Equations (15-17) converts the speed increment to flight time. Finally in Equation (18), arrival time to a node is calculated from the

departure time from the previous node and the travelling time between two nodes. The travelling time between two nodes $t_{(i,k)}^f$ is retrieved from the look-up table (Table 2).

Table 1: Speed

S_{ik}^f	1	2	...	w
Speed	V_1	V_2	...	V_w

Table 2: Speed and distance

Speed	Arc Length		$D_{(1,2)}$	$D_{(2,3)}$...	$D_{(n-1,n)}$
	Speed Index					
V_1	1		$t_{S_1,12}$	$t_{S_1,23}$...	$t_{S_1,n-1 n}$
V_2	2		$t_{S_2,12}$	$t_{S_2,23}$...	$t_{S_2,n-1 n}$
\vdots	\vdots		\vdots			
V_w	w		$t_{S_w,12}$	$t_{S_w,23}$...	$t_{S_w,n-1 n}$

Figures 2, 3 and 4 illustrate the flight conditions that are formulized by Equations (19-27). Figure 2 describe the necessity of safety distance between two flights on the same arc. In Figure 3 and 4, the limitation for any two aircraft to be present in the same arc/node at the same time.

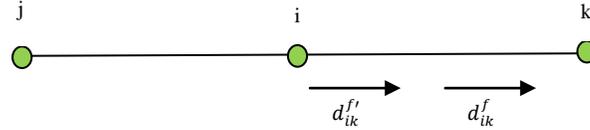


Figure 2: Safety distance between two consecutive flights

$$d_{(i,k)}^{f'} - d_{(i,k)}^f \geq ST^{ff'} - M(1 - \alpha_{(i,k)}^{ff'}) \quad \forall (i, k) \in \delta^+(i), \forall ff' \in F \quad (19)$$

$$d_{(i,k)}^f - d_{(i,k)}^{f'} \geq ST^{f'f} - M(1 - \alpha_{(i,k)}^{f'f}) \quad \forall (i, k) \in \delta^+(i), \forall ff' \in F \quad (20)$$

$$\alpha_{(i,k)}^{ff'} + \alpha_{(i,k)}^{f'f} \leq 1 \quad \forall (i, k) \in \delta^+(i), \forall ff' \in F \quad (21)$$

Equations (19-21) ensure that any two aircraft travelling in the same direction at the same arc are separated from each other by a safety distance of $ST_{(i,k)}^{ff'}$

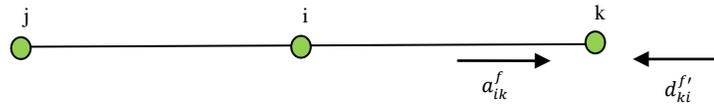


Figure 3: Flight from opposite directions

$$d_{(k,i)}^{f'} - a_{(i,k)}^f \geq ST^{ff'} - M(1 - \beta_{(i,k)}^{ff'}) \quad \forall (i, k) \in \delta^+(i), \forall (k, i) \in \delta^-(i), \forall ff' \in F \quad (22)$$

$$a_{(i,k)}^f - d_{(k,i)}^{f'} \geq ST^{f'f} - M(1 - \beta_{(i,k)}^{f'f}) \quad \forall (i, k) \in \delta^+(i), \forall (k, i) \in \delta^-(i), \forall ff' \in F \quad (23)$$

$$\beta_{(i,k)}^{ff'} + \beta_{(i,k)}^{f'f} \leq 1 \quad \forall (i, k) \in \delta^+(i), \forall ff' \in F \quad (24)$$

Equations (22-24) guarantee that no two flights travelling to opposite directions can travel on the same arc at the same time. In other word, after leaving of the first flight the arc, the second flight can enter that arc with the minimum safety distance of $ST_{(i,k)}^{ff'}$. For instance, if flight f is first travelling on arc (i,k) , then $\delta_{ik}^{ff'}$ will be 1 and Equation (22) will set the departure time of flight f' to $d_{ki}^{f'}$, after the arrival of flight f at time a_{ik}^f with the minimum difference of $ST_{(i,k)}^{ff'}$.

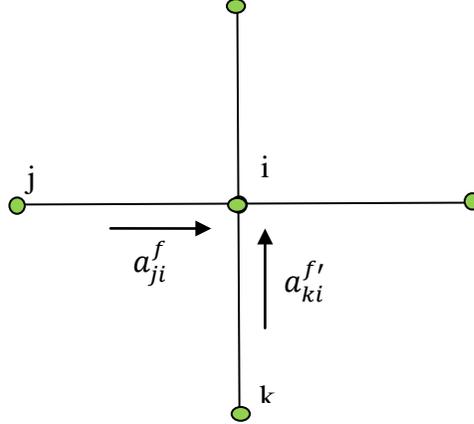


Figure 4: Collision avoidance at node

$$d_{(k,i)}^{f\prime} - a_{(j,i)}^f \geq ST^{ff\prime} - M(1 - \theta_i^{ff\prime}) \quad \forall i \in N, \forall (k,i) \in \delta^-(i), \forall (j,i) \in \delta^-(i), j \neq i, \forall ff\prime \in F \quad (25)$$

$$a_{(j,i)}^f - a_{(k,i)}^{f\prime} \geq ST^{ff\prime} - M(1 - \theta_i^{ff\prime}) \quad \forall i \in N, \forall (k,i) \in \delta^-(i), \forall (j,i) \in \delta^-(i), j \neq i, \forall ff\prime \in F \quad (26)$$

$$\theta_i^{ff\prime} + \theta_i^{f\prime f} \leq 1 \quad \forall i \in N, \forall ff\prime \in F \quad (27)$$

Similar to the Equations (22-24), above Equations (25-27) are used to ensure safety at nodes. These constraints guarantees that any two flights pass through the node i are separated from each other by a minimum safety distance of $ST_{(i,k)}^{ff'}$.

4. Conclusions and Future Works

This paper introduced a unique formulation for aircraft routing problem. The proposed model determines the routes for all the aircrafts in an air-segment. The minimization of fuel consumption cost and the costs associated with the early arrival and delays are considered. The main contribution of the presented mathematical model is its capability of determining the exact times for an aircraft passing through a point in 3-D space. Furthermore, the speeds of aircrafts at these points (nodes) are determined precisely. Keeping in mind that aircrafts are fastest transportation vehicles. Therefore, the discretization of time forces aircrafts to move from one air-segment to another only when the time interval is changed. Clearly, the discretized aircraft motion is not reflecting the reality. Since the whereabouts of the aircraft between two consecutive time intervals cannot be known, large safety distances between aircrafts are imposed in order to assure the safety of aircrafts. Consequently, the discrete-time based models cannot guarantee the optimal usage of the airspace due to large safety distance. Although reducing the time interval would increase the utilization of the airspace, the computational complexity would be significantly increased as a result. Hence, by incorporating exact speed and time in the aircraft routing problem discussed in this paper, following advantages are achieved: i) collision avoidance is ensured; ii) airspace is more effectively used by allowing large number of aircrafts in the region; iii) finally, the fuel consumption cost is formulated more precisely.

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