

A Mathematical Model for Train Routing and Scheduling Problem with Fuzzy Approach

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Abstract

Routing and Makeup problems, determine the routing and frequency of trains & the assignment of blocks to trains, but do not take scheduling into consideration. Consequently later on, it may be difficult to find a timetable accommodating all planned trains and satisfying line and station capacity. As an attempt to settle this issue, both the routing & the scheduling of freight transportation have been merged in the present paper. The proposed model embodies three different objectives of: the cost of train formation, the cost of idle time of waiting wagons in stations, & the cost of wagon classifications in shunting stations. It is assumed that, the maximum tonnage and length of trains in each block-section is finite, and is dependent on the critical gradient of the route and also maximum length of internal tracks of stations. In real world application, it is out of reality to assign crisp values to the admissible maximum tonnage of trains in the planning stage. A more applicable method is to consider this parameter as a fuzzy number. The paper intends to present a pure 0-1 model with fuzzy approach, & offers a decomposition-based heuristic algorithm to solve the large-scale problems in a limited amount of time.

Keywords

Train Formation Problem-TFP, Train Routing and scheduling, Pure 0-1 Programming, Fuzzy approach.

1. Introduction

In the most researches, the routing and scheduling problems are investigated individually, and therefore, the final solution may not be optimum. In this paper, both the routing and scheduling of trains are discussed together. Moreover, in real applications, the maximum allowable tonnage of trains in blocks could not be a certain amount and therefore, in this paper it is considered as a fuzzy number.

The Train Formation Problem-TFP and the Train Scheduling Problem- TSP are the major parts of middle term planning. The main goals of TFP are as follows.

- To minimize classifying operations in shunting stations.
- To minimize the train formation costs.
- To minimize the idle time of wagons waiting for trains in shunting stations.
- To maximize the railroad track capacity for train movements.
- To share almost equal wagon classification operations in all shunting stations.
- To yield the optimum scheduling for each wagon

Furthermore the following strategic planning also would be obtained:

- To develop the critical shunting stations with the high wagon classification operations.
- To build new shunting stations if required.
- To procure more locomotives if required.

On the other hand, the general objective of TSP is to obtain the least delays of trains in the mid-stations, and the arrival and departure times of trains to/from stations are determined.

Based on the survey conducted by Cordeau, et al [1], TFP, sometimes called Routing and Makeup Problem¹, is a sub-class of network routing problems. The other sub-classes are Blocking Problem² and Compound Routing and Scheduling Problems³. The followings are the definitions of these three classes presented in the literature:

¹ RMP

1.1 Blocking Problem (BP)

Wagons at station i , which are destined for station j must be added to a block that will next be shipped to station k , possibly transiting by other intermediate stations. Wagons in a block will not be reclassified until the block reaches its final destination. The solution should indicate the routing of freight through the network and the distribution of classification work among stations, but does not specify the trains to be run or the assignment of blocks to trains. It is worth to note that this problem is different from the so-called block-to-train assignment problem. For more detail refer to Krishna, et al [2]. Newton, et al [3] provided a mathematical model for BP which are minimizing the operation costs. Ahuja, et al [4] proposed an algorithm using an emerging technique known as very large-scale neighborhood search that is able to solve the blocking problem to near optimality. Yue, et al [5] introduced a new mathematic model as well as an ant colony algorithm which describe the blocking strategy and various combinations of multi-route O-D pairs in large scale railway network.

1.2 Routing and Makeup Problem (RMP)

In this problem the routing, the frequency of trains and the assignment of blocks to trains should be determined. Furthermore the blocking policy may be either determined endogenously, or be given as an input Verma, et al [6] presented an optimization methodology for the railroad tactical planning problem with risk and cost objectives to determine the routes to be used for each shipment, the yard activities, and the number of trains of different types. Akhavan, and Yaghini [7] proposed a metaheuristic solution method based on elitist ant system to solve Combined Blocking and Train Makeup Problem. Keaton [8] presented a linear 0-1 IP model and a heuristic method based on Lagrangian Relaxation. In addition, Keaton [9], considered a pure strategy constraints for blocking and maximum transit times for each origin-destination pairs and used a dual adjustment method for implementing lagrangian relaxation. Lin [10] introduced two approaches: first a lagrangian one and the second, an implicit enumeration algorithm with ϵ -optimality. This model minimizes the sum of handling and transportation costs. Marin and Salmeron [11]-[12] proposed a local search heuristic for the Tactical Design of rail freight networks, in a series of two papers. Their objectives are minimizing the operating and time costs. Lin [10] presented an implicit enumeration algorithm with ϵ -optimality to solve the TFP model. Shafia et al. [13] introduced a non-linear mathematical model which is abstracted to an integer model. They proposed a robust mathematical model, and a heuristic algorithm to solve the problem.

1.3 Compound Routing and Scheduling Problem (CRSP)

The routing problems, do not take scheduling into consideration, it may be difficult to later find a timetable accommodating all planned trains and satisfying line and station capacity. As a result, in CRSP, both the routing and the scheduling aspects of freight transportation are merged. Zhang, Li [14] developed an integer programming model to determine optimal operations in minimizing the most significant cost figures involved in such operations. Huntley et al. [15] presented a non-linear MIP model and used simulated annealing algorithm. Gorman [16] offered an application of genetic and tabu search algorithms for this problem. Godwin, et al [17], proposed a heuristic algorithm for freight train routing and scheduling in a passenger rail network. Caimi, et al. [18] addressed the problem of routing trains through railway stations for a given timetable and outline two algorithms for this specific problem. Flamini and Pacciarelli [19] addressed a scheduling problem arising in the real time management of a metro rail terminus. It mainly consists in routing incoming trains through the station and scheduling their departures with two objectives, in lexicographical order, the minimization of tardiness/earliness and the headway optimization.

1.4 Outline

The present paper could be classified in CRSP class. The current paper is organized as follows: In section 2 the problem definition is presented along with the proposed mathematical model. In section 3, a fuzzy approach is used, and the resulted mathematical model is proposed. In section 4, a novel heuristic method is proposed to solve the introduced problem. Finally, the concluding remarks are given at the end to summarize the contribution of the paper.

² BP

³ CRSP

2. Problem Definition and Mathematical Model

A railroad network can be considered as a graph consisting of shunting stations as nodes and the potential paths as arcs. Trains move between stations transporting a limited number of wagons. Each train should select a single arc between all arcs originating a particular station. A simple railroad network is shown in Fig. 1. This sample consists of 4 shunting stations. The arc (1,3) shows a path which starts from station no.1 to station no.3 with no stop in station 2. In this example, a train which is planned to move from station 1 to station 4 has four options as follows:

- It can pass arcs numbered (1,2) → (2,3) → (3,4)
- It can pass arcs numbered (1,2) → (2,4)
- It can pass arcs numbered (1,3) → (3,4)
- It can pass arc numbered (1,4)

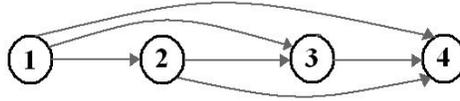


Figure 1: A railroad sample

Passing through each arc, results a particular cost for trains. It is obvious that the cost related to passing arcs (1,2) and (2,3) is equal to passing arc (1,3). The difference between these two cases is the idle time cost of wagons and locomotives in station 2 for the first path which should wait a particular amount of time known as idle time. Trains are two types, direct and indirect. Direct ones travel from their origins to their destinations without stopping in the middle stations. Indirect trains stop in the middle stations waiting for connecting and disconnecting the planned wagons.

In order to model the problem, in each arc, some potential trains are defined to travel. For each train the pre-determined starting time from origin, is known. As the purpose of scheduling is to determine the arrival and departure time of trains and wagons in their passing stations, some of these potential trains are selected according to the arrival time of wagons of each consignment to the stations.

The proposed model is exhibited on the basis of the following assumptions:

- The unit of each consignment is wagon.
- Each train can transport a limited number of wagons according to the topography of each path, and the internal length of stations.
- The arrival time of wagons of each consignment to their origin is predetermined.
- The travel time of trains in block-sections, i.e. distances amongst stations are pre-estimated.

In the proposed model the notations shown in table 1 are utilized:

Table 1: The used notation

Indices	
p	Index for consignments
a	Index for each wagon
k	Index for train
i, j	Indices for stations
Parameters	
θ_{ij}	Required time for passing arc (i, j)
φ_{ij}^k	Departure time of train k in arc (i, j)
$\alpha^{(p,a)}$	Predetermined presence time of wagon a of consignment p in its origin station
Ct_{ij}	Train formation cost for arc (i, j)
Cw^p	The cost of one hour idle or being in service related to each wagon of consignment p
Cc	Shunting operation cost for each wagon
u_{ij}	Maximum allowable wagons hauled by a locomotive in arc (i, j)

s_p	Origin station of consignment p
e_p	Destination station of consignment p
Variables	
$\delta_{ij}^{(p,a),k}$	Equals 1, if wagon a of consignment p is transported by train k in arc (i, j) , and 0, otherwise.
γ_{ij}^k	Equals 1, if train k is formed in arc (i, j) , and 0, otherwise.
$\lambda_i^{(p,a)}$	Equals 1, if wagon a of consignment p is classified in station i , and 0, otherwise.

2.1 The cost of train formation

The objective of this model includes three parts: The model minimizes the cost of each train formation. This cost consists of train personnel wages (including locomotive driver, locomotive driver assistant, repairman, and train chief), consumed oil and gasoline, amortization of locomotive. This cost is equal to $\sum_i \sum_j \sum_k \gamma_{ij}^k \times Ct_{ij}$.

2.2 The cost of idle time of wagons in stations waiting for trains

As the number of trains leaving a particular station for a same destination rises, the waiting time of wagons drops. This cost is equal to: $\sum_p \sum_a \sum_i \sum_k (\delta_{ie_p}^{(a,p),k} \times (\varphi_{ie_p}^k + \theta_{ie_p} - \alpha^{(p,a)}) \times Cw^p)$

2.3 The Cost of wagon classifications in shunting stations

This cost includes wagon separation works from arrived trains and wagon connection works to leaving trains. This cost contains the oil and gasoline of shunting locomotives, plus the wages of locomotive driver, locomotive driver assistant, repairman and shunting operators.

This cost is equal to $\sum_i \sum_p \sum_a \lambda_i^{(a,p)} \times Cc$.

As all the objectives are defined in cost, they are simply added to make a single objective function. This function is

$$Min Z = \sum_i \sum_j \sum_k \gamma_{ij}^k \times Ct_{ij} + \sum_p \sum_a \sum_i \sum_k (\delta_{ij}^{(a,p),k} \times (\varphi_{ie_p}^k + \theta_{ie_p}) \times Cw^p) + \sum_i \sum_p \sum_a \lambda_i^{(a,p)} \times Cc \quad (1)$$

2.4 Constraints

1. In order to ensure that all consignments leave their origins, Eq. 2 is applied to the model.

$$\sum_j \sum_k \delta_{s_p j}^{(a,p),k} = 1 \quad \forall p, a, \quad s_p < j < e_p$$

(2)

2. To ensure that all consignments pass the middle stations successively to arrive their destinations, Eq. 3 is used.

$$\sum_i \sum_k \delta_{ij}^{(a,p),k} - \sum_h \sum_k \delta_{jh}^{(a,p),k} = 0 \quad \forall p, a, \quad s_p < j < e_p, \quad s_p \leq i, h \leq e_p \quad (3)$$

This equation ensures that if wagon a of consignment p , arrives to a middle station, e.g. i , where $s_p < i < e_p$, then it must be departed by a train from the station.

3. To ensure that all consignments reach their destinations. Eq. 4 is applied.

$$\sum_i \sum_k \delta_{ie_p}^{(a,p),k} = 1 \quad \forall p, a \quad \& \quad s_p \leq i < e_p \quad (4)$$

4. Inequality 5 prevents assigning wagons more than the maximum capacity of trains.

$$\sum_p \sum_a \delta_{ij}^{(a,p),k} \leq u_{ij} \cdot \gamma_{ij}^k \quad \forall i, j, k \quad \& \quad s_p \leq i < e_p, \quad s_p < j \leq e_p \quad (5)$$

5. Inequality 6 shows that no wagon leaves its origin earlier than the related "predetermined presence time in origin", i.e. $\alpha^{(a,p)}$.

$$\sum_j \sum_k \phi_{s_p j}^k \cdot \delta_{s_p j}^{(a,p),k} \geq \alpha^{(a,p)} \quad \forall a, p \quad \& \quad s_p < j \leq e_p \quad (6)$$

6. Inequality 7 ensures that all wagons travel from their origins to their destinations successively.

$$\sum_k \phi_{ij}^k \delta_{ij}^{(a,p),k} \geq \sum_k \sum_h \delta_{hi}^{(a,p),k} \times (\phi_{hi}^k + \theta_{hi}) \quad \forall a, p, i, j \quad \& \quad s_p \leq i < e_p \quad \& \quad s_p < j \leq e_p \quad (7)$$

7. Inequality 8 specifies that if a wagon stops in a station, the necessary classification works should be done, and therefore, the relevant costs are added to the objective function.

$$\lambda_j^{(a,p)} \geq \sum_i \sum_k \left(\delta_{ij}^{(a,p),k} - \sum_l \delta_{jl}^{(a,p),k} \right) \quad \forall a, p, j, \quad s_p \leq i < e_p, \quad s_p < j < e_p \quad (8)$$

To clarify the proposed model, the following example is presented:

Example1: Consider the rail network shown in Fig. 1. This network consists of four stations. Assume that all consignments should transport from left to right.

Three consignments are to transport from their origins to their destinations at this network. The relevant data are specified in table 2.

Table 2: The consignments data

	Consignment 1		Consignment 2	Consignment 3
Origin	1		1	2
Destination	4		3	4
The quantity of wagons	18	12	18	21
presence time of wagons at origin (hour)	5	1	4	12

The objective is to determine:

- 1- The number of required trains to transport consignments from their origin to their destinations
- 2- The timetable for each train containing the departure and arrival time in each station
- 3- The timetable for each wagon containing the departure and arrival time in addition to the idle time of wagons in each station.

The related costs for this example are considered as follows:

The wagon classification cost for each wagon: 3 units

The cost for one hour idle or being in service time for each wagon: 2 units

Train formation costs in all arcs are shown in Fig. 2.

$$C_{ij} = \begin{bmatrix} 0 & 2000 & 5000 & 7000 \\ 0 & 0 & 3000 & 5000 \\ 0 & 0 & 0 & 2000 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Figure 2: Train Formation costs for example 1

As an example, in Fig. 2, the train formation cost from station 2 to station 4 is equal to 5000 units.

Moreover, 6 potential trains for each station are defined. The predetermined starting time for each train are indicated in table 3.

Table 3: The departure time of trains

Train number	1	2	3	4	5	6	7
Starting time of trains from station 1 (hour)	1	2	3	4	5	6	7
Starting time of trains from station 2 (hour)	6	8	9	10	11	12	13
Starting time of trains from station 3 (hour)	14	16	17	18	19	20	21

E

The required time for passing each arc is depicted in Fig. 3.

$$\theta_{ij} = \begin{bmatrix} 0 & 5 & 13 & 20 \\ 0 & 0 & 8 & 15 \\ 0 & 0 & 0 & 7 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Figure 3: required time for passing each arc for example 1

As an example the required time for passing arc (1,2) is equal to 5 hours. It should be noted that, in this example it is assumed that, a is the index for each set of three wagons. It means that each set of three wagons connect to each other at their origin and wouldn't disconnect till arriving to their destinations. For all arcs, the parameter u is considered to be equal to 36 wagons. The required time for connecting and disconnecting wagons to/from trains in stations is assumed to be 1 hour. The optimal solution is stated in table 4.

Table 4: The optimum solution of example 1

The final optimum value for objective function: 18615
The formed trains: $\gamma_{12}^4 = \gamma_{13}^5 = \gamma_{23}^6 = \gamma_{34}^5 = \gamma_{34}^7 = 1$
The wagons should stop in stations: $\lambda_2^{(2,1)} = \lambda_2^{(2,2)} = \lambda_2^{(2,3)} = \lambda_2^{(2,4)} = \lambda_2^{(2,5)} = \lambda_2^{(2,6)} = 1$ $\lambda_3^{(3,1)} = \lambda_3^{(3,2)} = \lambda_3^{(3,3)} = \lambda_3^{(3,4)} = \lambda_3^{(3,5)} = \lambda_3^{(3,6)} = \lambda_3^{(3,7)} = 1$
The assigned trains and arcs to wagons: $\delta_{13}^{(1,1),5} = \delta_{13}^{(1,2),5} = \delta_{13}^{(1,3),5} = \delta_{13}^{(1,4),5} = \delta_{13}^{(1,5),5} = \delta_{13}^{(1,6),5} = \delta_{13}^{(1,7),5} = \delta_{13}^{(1,8),5} = \delta_{13}^{(1,9),5} = \delta_{13}^{(1,10),5} = 1$ $\delta_{12}^{(2,3),4} = \delta_{12}^{(2,4),4} = \delta_{12}^{(2,5),4} = \delta_{12}^{(2,6),4} = \delta_{13}^{(2,1),5} = \delta_{13}^{(2,2),5} = \delta_{23}^{(2,3),6} = \delta_{23}^{(2,4),6} = \delta_{23}^{(2,5),6} = \delta_{23}^{(2,6),6} = 1$ $\delta_{23}^{(3,1),6} = \delta_{23}^{(3,2),6} = \delta_{23}^{(3,3),6} = \delta_{23}^{(3,4),6} = \delta_{23}^{(3,5),6} = \delta_{23}^{(3,6),6} = \delta_{23}^{(3,7),6} = 1$ $\delta_{34}^{(1,2),5} = \delta_{34}^{(1,3),5} = \delta_{34}^{(1,4),5} = \delta_{34}^{(1,5),5} = \delta_{34}^{(1,6),5} = \delta_{34}^{(1,7),5} = \delta_{34}^{(1,8),5} = \delta_{34}^{(1,9),5} = \delta_{34}^{(1,10),5} = 1$ $\delta_{34}^{(3,1),7} = \delta_{34}^{(3,2),7} = \delta_{34}^{(3,3),7} = \delta_{34}^{(3,4),7} = \delta_{34}^{(3,5),7} = \delta_{34}^{(3,6),7} = \delta_{34}^{(3,7),5} = 1$

Table 4 can be interpreted as follows:

The first train departs station 1 at time 4:00 and arrives to station 2 at time 9:00. This train contains 12 wagons of consignment 2. At station 2, 21 wagons of consignment 3 connect to the train, then this train leaves station 2 at time 12:00 and arrives to station 3 after 8 hours, i.e. at 20:00. At this station, consignment 2 is separated, and finally the train departs station 3 at 21:00 hauling just remained 21 wagons of consignment 3.

The second train departs station 1 at 5:00 and takes 2 wagons of consignment 2 and also 30 wagons of consignment 1. It arrives to station 3 at 18:00 and the wagons of consignment 2 are disconnected in this station. Finally this train leaves station 3 at 19:00.

3. Applying the Fuzzy Approach to the Model

In classical linear programming, the violation of any constraint renders the solution infeasible, but in real applicable cases, the role of constraints can be different, where the decision maker might accept small violation of constraints but might also attach different degrees of importance to violations of different constraints.

In this paper, the parameter u of the proposed model, is supposed to be imprecise. This parameter affects the right-hand side of inequality 5. As the objective function assumed to be crisp, the Werner's approach is applicable. Applying directly this approach to inequality 5 results a non-linear model, as a result, this inequality is decomposed in to inequalities 9 and 10.

$$\sum_p \sum_a \delta_{ij}^{(a,p),k} \leq u_{ij} \quad \forall i, j, k, \quad s_p \leq i < e_p, \quad s_p < j \leq e_p \quad (9)$$

$$\sum_p \sum_a \delta_{ij}^{(a,p),k} \leq M \cdot \gamma_{ij}^k \quad \forall i, j, k, \quad s_p \leq i < e_p, \quad s_p < j \leq e_p \quad (10)$$

where, M is a big number.

We next present how this approach can be applied to inequality 9. Consider inequality 9, the fuzzy triangular number of parameter u is depicted in Fig. 4.

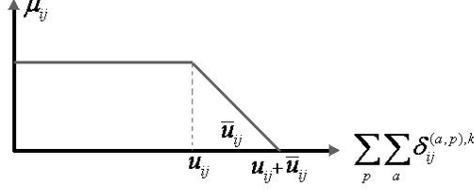


Figure 4: The fuzzy number of parameter u

The membership function is equal to Eq. 11.

$$\mu_{ij} = 1 - \frac{\sum_p \sum_a \delta_{ij}^{(a,p),k} - u_{ij}}{\bar{u}_{ij}} \quad (11)$$

Furthermore, the membership function of the objective function, is equal to Eq. 12.

$$\mu_G = \frac{c^T x - f_1}{f_0 - f_1} \quad (12)$$

Where, $c^T x$ equals the objective function value, and f_1 is the optimum objective value of the proposed model in crisp mode and f_0 is the optimum objective value of the proposed crisp model when the parameter u_{ij} is replaced by $u_{ij} + \bar{u}_{ij}$.

Finally, by introducing one new variable, η , the mathematical model with fuzzy constraint transforms to crisp model (13).

Max η

Subject to :

$$\begin{aligned} \bar{u}_{ij} \eta + \sum_p \sum_a \delta_{ij}^{(a,p),k} &\leq \bar{u}_{ij} + u_{ij} \\ (f_1 - f_0) \eta + c^T x &\leq f_1 \\ \text{Constraints } &2,3,4,6,7,8 \end{aligned} \quad (13)$$

Example 2: Consider example 1 under the introduced fuzzy condition, where, $\bar{u}_{ij} = 3$. At the first step, example 1 is solved with the assumption of $u_{ij} = u_{ij} + \bar{u}_{ij}$ which yields $f_0 = 18435$. In the next step, using model 13, the following solution is achieved.

The optimum objective function is: $\eta = 0.66667$ and the solution is interpreted as follows:

A direct train hauls 30 wagons of consignment 1 from station 1 to 4, starting at 5:00.

The second one departs station 1 at 6:00 and takes 18 wagons of consignment 2. This train arrives to station 2 at 11:00. In this station, 21 wagons of consignment 3 are connected to the train and finally this train leaves station 2 at 12:00, and arrives to station 3 at 20:00. In this station, wagons of consignment 2 are separated from the train. At the end, the train leaves this station at 21:00. It can be concluded that the solution resulted by the applied fuzzy approach is more applicable so that the related cost is substantially reduces in comparison with the crisp mode.

4. Heuristic Method to Solve the Problem

As stated, the specified problem which is a pure binary one, merges the scheduling problem with the routing problem, each of which known to be NP-Hard. Therefore, it could be resulted that the proposed model is computationally very hard to solve by exact software packages, like GAMS, LINGO, CPLEX, etc. Therefore, in order to find a good solution for large-scale problems, a heuristic method is proposed. The proposed heuristic method is as follows:

The problem is divided in two individual problems, one is routing, also known as train formation problem, and the other is train scheduling problem. In other words, the problem is solved in two steps: In the first step, the routing and the frequency of trains are determined and the wagons are assigned to trains. Shafia, et al [13] proposed a linear mathematical model to solve the train formation problem. Moreover, they proposed a heuristic algorithm which is based on decomposition of the main problem, and is solved into two phases. The first phase is to find the routes of compartments and to allocate the necessary trains, such that the train formation cost is minimized. Then, the output of the first phase is considered as the input of the second phase. In phase 2, the exact assignment of the wagons to each train is determined. Note that as the scheduling aspects are not involved with this problem, it is assumed that, the entrance of trains has a Poisson distribution. For more details, refer to [13]. In the remaining parts of the paper, the heuristic algorithm proposed by Shafia, et al [13], is called RT algorithm.

In the second step, the scheduling part of the problem is solved considering all the outputs of the first step as the input data of the second step, i.e. scheduling part of the problem. Shafia, et al [20] proposed a Branch & Bound as well as a Beam Search algorithm to solve a timetabling problem. For the details, refer to [20]. In the remaining parts of the paper, this algorithm is called, SL algorithm. Moreover, Shafia, et al, [21] proposed a Simulated Annealing to solve a fuzzy train scheduling problem.

By these explanations, the proposed algorithm is as follows:

Algorithm 1. The heuristic method to solve the fuzzy train routing and scheduling problem

Step 1. Let $\mu_{ij} = 1, \forall i, j$. Find f_o , and f_1 . For this purpose, at the first step use the RT algorithm to determine the routing and the frequency of trains as well as the assignment of wagons to trains. In the second step run the SL algorithm to find the appropriate time schedule.

Step 2. Considering the amount of $\mu_{ij}, \forall i, j$, Find $\bar{u}_{ij}, \forall i, j$, based on Eq. 11. Then, solve the first step, i.e. the routing problem, using RT algorithm. Considering the outputs of RT, run the SL algorithm.

Step 3. Find the μ_G , based on Eq. 12. If $|\mu_G - \mu_{ij}| < \alpha$, publish the best found solution and terminate the algorithm, otherwise, go to step 4.

Step 4. If $\mu_G < \mu_{ij}$, then go to step 5, otherwise, go to step 6.

Step 5. Consider $\mu_{ij} = \mu_{ij} - \left(\frac{\mu_{ij} - \mu_G}{2}\right)$, and go to Step 2.

Step 6. Consider $\mu_{ij} = \mu_{ij} + \left(\frac{\mu_{ij} - \mu_G}{2}\right)$, and go to Step 2.

Note that the best solution specified in Step 3, is the one that $Min\{\mu_G, \{\mu_{ij}, \forall i, j\}\}$, is more than the other candidates. Moreover, α , is the termination criterion.

Example 3: The Case Study

In this section, a case study which is part of Iranian railway network is considered. The details are specified in [13]. Briefly, the studied network consists of 61 stations in which 11 shunting yards among them can service shunting operations. The topography of the line and maximum length of stations changes in different parts of the network. There are 12 compartments which should be transported by the trains from their origins to their destinations. The distances among all shunting yards, the characteristics of the compartments, the characteristics of studied railway network, and all the related costs are specified in [13], and therefore, not repeated here. By solving the problem in crisp mode, the final solution includes forming 11 trains to transport all the compartments from their origins to their destinations. In this case, the total cost equals 53491 units. The solving iterations are shown in Table 5.

Table 5. The iterations of solving the problem by Algorithm 1.

k	μ_{ij}	μ_G	$ \mu_{ij} - \mu_G $	OFV
1	1	0	1	53491
2	0.5	0.21	0.29	51742
3	0.35	0.41	0.06	48765
4	0.38	0.33	0.07	50013

In the best found solution, in fuzzy case, the required amount of trains reduces one, and the cost reduces by 3781 units in comparison with the crisp mode.

5. Conclusion

The paper is delivered a new linear binary train routing and scheduling model. As it is out of reality to assign a constant amount to the maximum allowable wagons hauled by a locomotive, the fuzzy logic has been applied to the proposed crisp model. Then, to illustrate the model, a simple example was solved in both crisp and fuzzy environments. The results show that, it is impossible to achieve an optimum solution for the large-size problems, during a rational amount of time. Therefore, a heuristic algorithm is proposed based on decomposition of the proposed problem in two individual problems. The results demonstrate that the fuzzy model reduces the cost in comparison with the crisp model.

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