

Multi-Period Cell Loading in Cellular Manufacturing Systems

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Abstract

This paper focuses on a multi-period cell loading problem on a thermo siphon manufacturing station of a company. The aim of study is firstly to minimize the number of tardy jobs (n_T) in a multi-period planning horizon and then deal with the scheduling of tardy jobs. Even though minimizing n_T is studied extensively in machine scheduling and cell loading literature, to the best knowledge of authors, scheduling the tardy jobs in a multi-period environment has not been addressed. Three strategies are proposed to find the optimal cell loads and job sequences. Two mathematical models are used to experiment the proposed strategies. In addition, three types of due dates (tight, medium and loose) are used to study the impact of the frequency of due dates in a period. Besides, three different demand levels are used to study the impact of capacity requirements. Finally, two tardy job assignment methods are proposed to observe the impact on the performance measure. Case problems are solved based on the primary performance measure and proposed strategies are compared in terms of n_T , maximum tardiness (T_{max}) and total tardiness (TT). The first strategy, early start allowance and tardy job assignment after each period, performed better in terms of n_T . For the other objectives, tradeoffs are observed among different strategies depending on the type of due date, demand level and tardy job assignment method.

Keywords

Cellular manufacturing, number of tardy jobs, multi-period, cell loading, scheduling tardy jobs.

1. Introduction

Group Technology and Cellular Manufacturing have been widely used in the last decades in manufacturing design, planning and operation. Süer and Saiz (1993) defined Group Technology (GT) as an approach that identifies the items with either similar design characteristics or similar manufacturing process characteristics and groups them into families of like items [1]. As an application of GT philosophy, cellular manufacturing gives opportunity to produce high variety of products in a more efficient fashion. Cell loading drew attention in the literature as cellular manufacturing gained popularity in both industry and literature. There are several performance measures which have been used in the scheduling literature. Minimizing number of tardy jobs (n_T) is one of the most important performance measures in machine scheduling. A job is considered “tardy” when its completion time (c_i) is greater than its due date (d_i); i.e. ($c_i > d_i$). Due to globalized competition among manufacturing firms, companies work hard to satisfy customer expectations with respect to quality, cost and timeliness. Not only being a vital internal performance indicator in terms of timeliness but being a factor which has a crucial effect on creating loyal customers and avoiding penalty costs, minimizing the number of tardy jobs is even more relevant in today’s competitive industrial world.

In this paper, the primary objective is to minimize the number of tardy jobs. Once optimal schedules are obtained, tardy jobs are scheduled using the proposed assignment strategies. Later, their performances are evaluated with respect to (maximum tardiness) T_{max} and (Total tardiness) TT . Two mathematical models are used to minimize n_T and three different assignment strategies are proposed to deal with tardy jobs. Experimentation is carried out with varying demand levels and due dates.

2. Literature Review

According to Süer, there is a similarity between parallel machine and cell loading problems [2]. A job can be assigned to one of its feasible cells as in parallel machine scheduling. Therefore, literature review is divided into two parts: parallel machine scheduling and cell loading & job sequencing.

2.1 Parallel Machine Scheduling

There are several works in the literature which deal with parallel machine scheduling to minimize n_T . Süer, Czajkiewicz and Baez (1993) developed an integer programming model and proposed three heuristic procedures for minimizing n_T problem [3]. Ho and Chang (1995) proposed two heuristics for minimizing n_T in parallel machine scheduling problem [4]. Süer, Pico, and Santiago (1997) proposed four mathematical models for the identical machine scheduling problem to find optimal solutions for n_T when lot splitting is allowed [5]. Gupta, Ruiz-Torres, and Webster, (2003) worked on the hierarchical criteria identical parallel machine problem which have n_T as the primary objective, minimum flow time as the secondary objective [6]. M'Hallah and Bulfin (2005) proposed a branch and bound algorithm which minimizes weighted number of tardy jobs on identical and non-identical parallel machines [7].

2.2 Cell Loading & Job Sequencing

Despite the similarity, cell loading still differs from parallel machine scheduling. In many cases, cells cannot be designed to process all the jobs due to different families having different processing requirements. The other difference among them is the definition and usage of processing time. Processing times are used directly to build the schedules in parallel machine systems. However, in cell loading, first production rates are determined based on the routes and processing times information [2]. Greene and Sadowski (1983) discussed the advantageous and disadvantageous sides of cellular manufacturing systems and Group Technology [8]. Süer (1997) developed two mathematical models to minimize the number of tardy jobs in a multi-period time horizon [2]. Süer and Bera (1998) proposed a simultaneous solution of cell loading and cell size determination for labor intensive manufacturing cells [9]. Süer, Saiz, and Gonzalez (1999) presented manufacturing cell loading rules for multi-independent cells [10]. However, minimizing number of tardy jobs for multiple periods and assignment of tardy jobs to the optimized schedule has not been addressed in literature before. As a result, this paper focuses on minimizing number of tardy jobs in a multi-period environment by using mathematical models and then develops different heuristic methods to deal with the assignment of tardy jobs.

3. Problem Statement

A thermo-siphon body manufacturing system is taken as the case study for the problem. The manufacturing process consists of forming, welding and leak test. There are two cells equipped to perform all operations for all jobs. As a result the cells are identical and self-sufficient. Each planning period consists of one week with 40 hours of capacity, thus having a total production capacity of 80 hours per week. Each job has its own due date. Seven different products with different unit processing times are manufactured. In this study, a four-period horizon has been considered.

4. Methodology

Süer (1997) proposed two mathematical models to minimize the number of tardy jobs in a multi-period time horizon [2]. However, in their paper, there was no discussion on how to deal with tardy jobs. In this study, three different assignment strategies are proposed to deal with tardy jobs.

The first mathematical model is used to minimize n_T for all periods by allowing early start of jobs. On the other hand, the second mathematical model is used to minimize n_T when early start is not allowed. Indeed, these two mathematical models can be useful in different production planning systems. For example, in a system where equipment and people utilization is very critical and due dates are very tight, it would be desirable to process the jobs earlier than needed if resources are available. On the other hand, in a Just in Time (JIT) pull system, the timeliness of the production is more critical. Therefore, early start and early completion of a job is not desirable as it leads to higher inventory levels. As a result, these two mathematical models can be useful for different scheduling strategies.

4.1 Basic Information about Models

Both mathematical models are integer programming models. All of the jobs are sorted by Earliest Due Date (EDD) thus obtaining $d_{[1]} \leq d_{[2]} \leq d_{[3]} \leq \dots \leq d_{[n]}$ and then models are solved.

Notation

n	number of jobs
m	number of cells
$d_{[i]}$	due date of the job in the i^{th} order

$P_{[i]t}$	processing time of the job in the i^{th} order in period t
$X_{[i]j}$	1 if the job in the i^{th} order is assigned to cell j 0 otherwise
CA	Capacity available per cell per period
T	The number of periods

Indices

i	Job index
j	Cell index
t	Period index

The following assumptions are made in both mathematical models:

- i) A job can be assigned to only one cell.
- ii) Each job is equally important.
- iii) Cells are identical and independent.
- iv) Each cell is capable of producing all jobs.
- v) Overtime is not allowed.
- vi) Customers accept late delivery (no lost sales)

4.2 Mathematical Model 1: Early Start is Allowed

The objective of this mathematical model is to maximize number of early jobs which is equivalent to the minimization of the number of tardy jobs. The objective function is shown in Equation (1). The first constraint forces early jobs to be completed by their due date as shown in Equation (2). The second constraint (Equation 3) limits the assignment of a job to at most one cell. The decision variable takes only binary values as 0 or 1 which reflects the assignment of a job to a cell.

Objective function:

$$Max Z = \sum_{t=1}^T \sum_{i=1}^n \sum_{j=1}^m X_{[i]jt} \tag{1}$$

Subject to:

$$\sum_{t=1}^T \sum_{i=1}^k \{P_{[i]t} X_{[i]jt}\} \leq d_{[k]t} \quad j = 1, 2, 3, \dots, m \tag{2}$$

$$\sum_{t=1}^T \sum_{j=1}^m X_{[i]jt} \leq 1 \quad i = 1, 2, 3, \dots, n \tag{3}$$

$X_{[i]jt} \in (0, 1)$ for all i, j and t .

4.3 Mathematical Model 2: Early Start is not Allowed

In this strategy, jobs are not allowed to start early, which means every job is scheduled to the period they are due. The objective function is the same as in mathematical model 1. The first constraint in the previous mathematical model is modified to fit to this problem (Equation 4). A parameter, CA, is defined to denote the capacity available in a period which actually blocks the early start of jobs. Other constraints and assumptions are kept the same as in the previous mathematical model.

Subject to:

$$CA_{(t-1)} + \sum_{t=1}^T \sum_{i=1}^k \{I_{[i]t}\} X_{[i]jt} \leq d_{[k]t} \quad k = 1, 2, 3, \dots, n$$

$$j = 1, 2, 3, \dots, m$$

$$t = 1, 2, 3, \dots, T \tag{4}$$

4.4 Assignment of Tardy Jobs

Minimizing the number of tardy jobs in a cellular manufacturing system is vital. However, assignment of tardy jobs is another crucial task and is rarely addressed in the scheduling literature. Once early jobs are scheduled to cells, the next task is the assignment of tardy jobs. Three strategies are proposed to deal with this phase of the problem.

Strategy A: Early start is allowed and tardy jobs are added at the end of the period they were due. In this strategy, cells are loaded and job sequence is obtained based on “early start allowed” strategy. Early start allows processing a job in an earlier period. The tardy jobs are assigned after the last early job in the same period they were due.

Strategy B: Early start is allowed and tardy jobs are added at the end of planning horizon. In this strategy, early start approach is kept as the same as the previous strategy. However, the assignment of tardy jobs is different. Tardy jobs are added after the entire planning horizon, which is the 4th period in this study.

Strategy C: Early start is not allowed and tardy jobs are added at the end of the period they were due. In this strategy, early start is not allowed and mathematical model 2 is used to minimize the number of tardy jobs. The assignment of tardy jobs is performed using Strategy A.

Allowing of early start means that as long as capacity is available (current load is less than 40 hours for one cell), following periods’ jobs can be processed in advance. On the other hand, if early start is not allowed, a job can only be processed in the period it is due. Mathematical model 1 is used along with Strategy A and Strategy B. Mathematical model 2 is used for Strategy C. These strategies are illustrated Figure 1. Two rules are used to include the tardy jobs in the schedule, namely Earliest Due Date (EDD) and Shortest Processing Time (SPT).

4.5 A Numerical Example for Strategies A, B and C

A 9-job numerical example is used to illustrate the methodology. The batch production times and due dates of the jobs in periods P1, P2, P3 and P4 are shown in Table 1. The case problem is solved with strategies A, B and C.

Table 1: Example problem

Job	Due Date				Batch Production Time			
	Period 1	Period 2	Period 3	Period 4	Period 1	Period 2	Period 3	Period 4
1	12	14	13	12	12	14	13	12
2	6	8	10	8	3	4	5	4
3	14	14	12	12	7	7	6	6
4	10	6	6	8	5	3	3	4
5	12	18	15	12	4	6	5	4
6	6	4	6	8	3	2	3	4
7	8	10	8	9	8	10	8	9
8	10	10	10	10	5	5	5	5
9	20	24	20	24	5	6	5	6

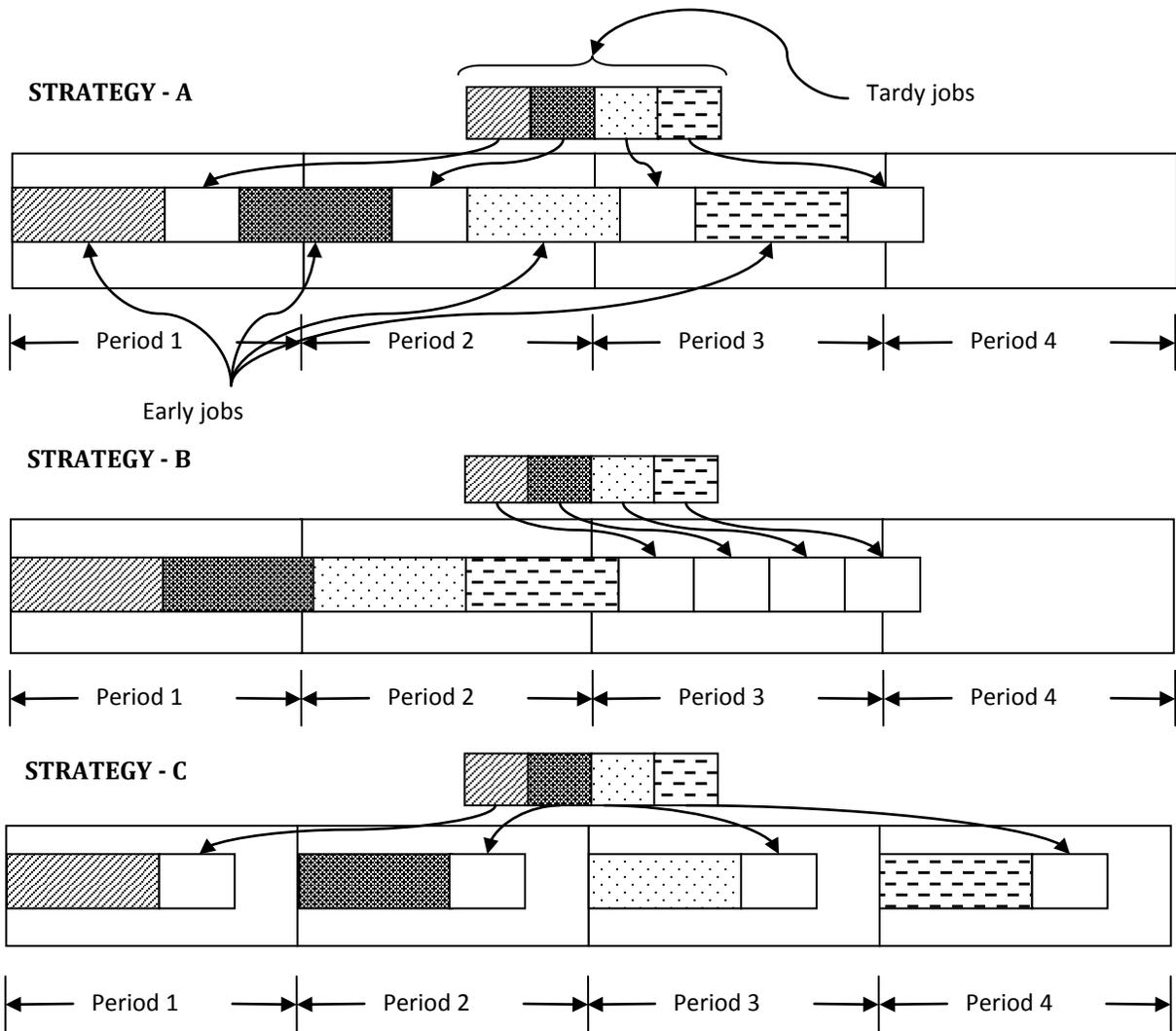


Figure 1. The illustration of three strategies

Solution with Strategy A: In this strategy, mathematical model 1 is used to solve the problem and the results are summarized in Table 2. Tardy jobs were added to the schedule after the period they were due. Tardy jobs are identified, T_{max} and TT values are also computed.

Table 2: Results of Strategy A for the Example Problem

Period	Period 1	Period 2	Period 3	Period 4
Cell 1 – Early Jobs	4 8 9	2 1 9	4 6 2 8 3 1 5 9	2 4 6 7 8 1 3 5 9
Cell 2 – Early Jobs	2 6 3	6 4 7 8 3 5	7	
Tardy Jobs	1 5 7			
n_T	3	0	0	0
$\sum n_T$	3			
T_{max}	25			
TT	63			

Solution with Strategy B: In this strategy, mathematical model 1 was used to optimize the number of tardy jobs. The tardy jobs are added at the end of planning horizon and the results are shown in Table 3.

Table 3: Results of Strategy B for the Example Problem

Periods	Period 1	Period 2	Period 3	Period 4
Cell 1- Early Jobs	4 8 9	1 2 9	1 2 3 4 5 6 8 9	1 2 3 4 5 6 7 8 9
Cell 2- Early Jobs	2 3 6	3 4 5 6 7 8		
Tardy Jobs	1 5 7			
nT	3	0	0	0
$\sum nT$	3			
T_{max}	66			
TT	182			

Solution with Strategy C: Mathematical model 2 is used since early start is not allowed in this strategy. The results are shown in Table 4.

Table 4: Results of Strategy C for the Example Problem

Period	Period 1	Period 2	Period 3	Period 4
Cell 1- Early jobs	3 8	4 5 8 9	3 4 6 9	2 3
Cell 2- Early jobs	2 4 5 9	2 3 6	2 5 8	4 5 9
Tardy jobs	1 6 7	1 7	1 7	1 6 7 8
n_T	3	2	2	4
$\sum n_T$	11			
T_{max}	20			
Total Tardiness	144			

5. Experimentation

The experimentation includes two sections as the initial data generation and the experimentation. It is carried out on a computer with 2-Gb memory and 1.7- Ghz dual core processor.

5.1 Data Generation

The processing times and due dates of jobs are generated randomly. Customer orders are generated for each period. Lot size (order quantities) for each job is generated from uniform distributions as shown in Table 5. Each product has a unit processing time as given in Table 6. According to table 1, products 1,2,3,4 and 5 used in 60 %, products 3,4,5 and 6 used in 80 % and products 4,5,6 and 7 used in 100 % capacity requirement level data generation. Capacity requirements for 25 jobs are generated from multiplying products' unit processing times and lot sizes. Capacity requirements are used as job processing times in cell loading and job sequencing. Job processing times are calculated by multiplying lot sizes and unit process times. Due dates are generated from uniform distributions as shown in Table 7.

Table 5: Statistical distributions used in data generation

Capacity Requirement	Lot sizes for each job ordered	Unit Process Times (Hrs)	Products Used In Generation
60%	Uniform Dist. (10, 30)	Uniform Dist. (0.07, 0.1)	Products 1,2,3,4,5
80%	Uniform Dist. (15, 35)	Uniform Dist. (0.09, 0.11)	Products 3,4,5,6
100%	Uniform Dist. (25, 35)	Uniform Dist. (0.1, 0.12)	Products 4,5,6,7

Table 6: Unit processing times

Product	Unit Process Time (Hrs)
1	0.07
2	0.08
3	0.09
4	0.1
5	0.1
6	0.11
7	0.12

Table 7: Distribution parameters used in due date generation

Due Date Type	Distribution Used (Hrs)
Tight	Uniform(0, 17)
Medium	Uniform(10, 27)
Loose	Uniform(20, 37)

5.2 Experimentation

Twenty seven cases are experimented with that includes three strategies (Strategies A, B and C), three types of due dates (tight, medium and loose) and three capacity requirement levels (60%, 80% and 100% of the system capacity) are experimented. Mathematical models are solved by using ILOG-OPL CPLEX optimization software. Earliest Due Date (EDD) and Shortest Processing Time (SPT) rules are used to schedule tardy jobs.

6. Results

The consolidated results are shown in Table 8. The results are grouped in four sections; comparison of strategies, comparison of due dates, comparison of demand levels, and comparison of scheduling rules for tardy jobs.

6.1 Comparison of Strategies

Based on the objective of minimizing nT, strategy A consistently provided the best solution for all demand levels and types of due dates. Occasionally, other strategies also matched Strategy A. In terms of Tmax, strategy B provided the best solutions for all due dates and for 60% and 80% demand levels. In the case of 100% demand level, the results were not conclusive. In terms of TT, Strategy B did well at 60% and 80% demand levels. However, results varied at 100% demand level depending on the type of due date, demand level and tardy job assignment strategy. The dominant strategies are summarized in Table 9.

Table 8: Results of the Experimentation

Strategy A	Strategy	Due Date Type	Periods				Total	EDD	SPT	EDD	SPT
			1	2	3	4	n_T	T_{max}		TT	
60%	Early start is allowed	Tight	6	0	0	0	6	40.95	44.45	235.83	234.46
		Medium	0	0	0	0	0	0	0	0	0
		Loose	0	0	0	0	0	0	0	0	0
80%	Early start is allowed	Tight	9	0	0	0	9	112.78	122.97	958.02	953.19
		Medium	1	0	0	0	1	97.58	97.58	97.58	97.58
		Loose	0	0	0	0	0	0	0	0	0
100%	Early start is allowed	Tight	13	0	0	0	13	141.6	147.86	963.92	949.49
		Medium	5	0	0	0	5	136.3	141.51	667.47	667.1
		Loose	6	0	0	0	6	122.46	126.92	722.48	722.16
Strategy B	Strategy	Due Date Type	Periods				Total	EDD	SPT	EDD	SPT
			1	2	3	4	n_T	T_{max}		TT	
60%	Early start is allowed	Tight	6	0	0	0	6	15.67	18.23	79.2	78.37
		Medium	0	0	0	0	0	0	0	0	0
		Loose	0	0	0	0	0	0	0	0	0
80%	Early start is allowed	Tight	9	8	5	3	22	17.54	26.56	227.97	231.57
		Medium	1	0	0	0	1	19.4	19.4	19.4	19.4
		Loose	0	0	0	0	0	0	0	0	0
100%	Early start is allowed	Tight	13	22	22	21	57	102.49	102.56	1056.03	1051.14
		Medium	5	9	11	10	25	17.29	24.8	213	212.53
		Loose	6	6	5	1	17	11.02	11.04	90.45	93.39
Strategy C	Strategy	Due Date Type	Periods				Total	EDD	SPT	EDD	SPT
			1	2	3	4	n_T	T_{max}		TT	
60%	Early start is not allowed	Tight	6	6	7	6	25	17.15	19.96	349.12	344.78
		Medium	0	0	0	0	0	0	0	0	0
		Loose	0	0	0	0	0	0	0	0	0
80%	Early start is not allowed	Tight	9	7	11	9	36	22.97	28.26	623.23	610.97
		Medium	1	3	4	1	9	32.6	32.6	144.59	143.65
		Loose	0	0	0	0	0	0	0	0	0
100%	Early start is not allowed	Tight	12	11	12	13	48	28.07	34.05	970.58	981.85
		Medium	4	5	5	4	18	17.22	20.42	235.27	222.68
		Loose	5	1	2	2	10	14.1	14.1	47.08	42.3

6.2. Due date based comparison

When due dates are tight, Strategy A was the dominant one in terms of n_T (equivalency with strategy B occurred at 60% demand level). Strategy B was the dominant one in terms of T_{max} for 60% and 80% demand levels. Strategy C outperformed others at 100% demand level. When due dates are medium, Strategy A was the dominant one in terms of n_T (equivalency with Strategy B occurred at 60% and 80% demand levels). Strategy B was better strategy in terms of T_{max} however, there was no dominant strategy. Strategy B dominated other strategies in terms of TT for all demand levels and assignment strategies. When due dates are loose, tardy jobs were observed only at 100% demand level. In terms of n_T , strategy A dominated other strategies. In terms of T_{max} , strategy B dominated other strategies. In terms of TT , strategy C dominated other strategies.

Table 9: Analysis of Results

	Strategy	Due Date Type	Total	EDD	SPT	EDD	SPT
			n_T	T_{max}		TT	
60%	ES IS ALLOWED	TIGHT	A,B	B	B	B	B
		MEDIUM	A,B,C	A,B,C	A,B,C	A,B,C	A,B,C
		LOOSE	A,B,C	A,B,C	A,B,C	A,B,C	A,B,C
80%	ES IS ALLOWED	TIGHT	A	B	B	B	B
		MEDIUM	A,B	B	B	B	B
		LOOSE	A,B,C	A,B,C	A,B,C	A,B,C	A,B,C
100%	ES IS ALLOWED	TIGHT	A	C	C	A	A
		MEDIUM	A	C	C	B	B
		LOOSE	A	B	B	C	C

6.3. Demand level-based comparison

As the demand level increased, number of strategies performing equally was reduced. For example Strategy A was the only best strategy when demand level reached to 100%. Strategy B dominated others in terms of T_{max} and TT . However, when demand level is 100%, Strategy A dominated others in terms of n_T . Both of Strategies B and C dominated A in terms of T_{max} . However, there was no dominant strategy among B and C. Both Strategies B and C dominated A in terms of TT . There was no dominant strategy observed when demand level increased to 100%.

6.4. Tardy job assignment strategy-based comparison

In terms of T_{max} , EDD performed slightly better than SPT for all strategies (as indicated in bold). In terms of TT , SPT performed slightly better than EDD for strategy A. However, no dominance occurred among strategies B and C.

7. Conclusion and Future Work

In machine scheduling, it is important to minimize number of tardy jobs. However, how we deal with tardy jobs is also important. This is critical especially in multi-period context. In this paper, an attempt is made to address these issues. According to the results of the experimentation, strategy A provided best solution for n_T in all cases. However, there was no dominant strategy observed for the objectives of T_{max} and TT . Strategy B consistently did well in 60% and 80% demand levels. However, as the demand level increased to 100%, the results varied depending on the type of due date. EDD performed slightly better as a tardy job assignment strategy than SPT for performance measure T_{max} and SPT did better when TT was considered.

In conclusion, there is no unique strategy that works best in all situations. Based on the planning strategy, whether it's forward or backward planning, early start can be applied. If a forward planning strategy is in use, early start should be allowed in planning as long as the BOM of the corresponding product does not prevent an early start or there is a supplier restriction. Overall, smaller number of tardy jobs with higher maximum tardiness and total tardiness values were obtained from strategy A. On the other hand, greater number of tardy jobs with lower maximum tardiness and total tardiness were obtained with strategy B and C. Therefore, the selection of the desired strategy strongly depends on the scheduler's viewpoint (whether less backorder with higher tardiness or more backorders with lower tardiness is preferred). In this case, it is strongly believed that customer profiles might have a crucial impact on the planning which is not studied in this paper. The problem studied can be extended with customer profiles which have different preferences among the number of tardy jobs, maximum tardiness and total tardiness as a future work.

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