# Multi Item Two Stage EOQ Model from Single Source to Multiple Destinations under Varying Freight Policies

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## Abstract

Managing supply chain is a difficult task because of complex interrelations and integrations as various entities exist in it. The study describes the dynamics of two stage supply chain where there is single source of supply, an intermediate stoppage, single distributor and multiple destinations of consumption i.e. the integration of procurement and distribution decision making under two stage environment. Different discount policies are offered to procure and transport goods from the one stage to other stage when it is assumed that inventory carrying charge at the stoppage is very high after a pre-specified time. Model will benefit organizations in a long run by helping them determining optimal quantity to be ordered which not only reduces the cost of procurement and transportation costs A case is presented to testify and validate the procedure.

## Keywords
Two stage supply chain, Discount schemes, Truckload, Less- than- truckload

## 1. Introduction
Supply chain management has been both an important and a productive aim of corporations and companies in today's competitive environment as they are striving hard to minimize the cost component viz. procurement, production, distribution and inventory holding cost to increase their operation profit and to sustain it for longer period of time. In many industries, components and sub components are manufactured at variety of sites and the same are sold across the cities and continents, such a scenario necessitate companies to reduce various cost by adopting comprehensive supply chain policies. Effective supply chain management solves many problems encountered in the business today, first, the vendors involved in the chain will actually have a clearer idea of what the buyer needs and can then adequately provide for these needs. Slow response times and delays in project start dates also become less frequent because the automated supply chain helps shave the time off of the order placement and fulfillment process, furthermore problem like the one(intermediate stoppage in transportation) discussed in this paper can also be solved by applying wide-ranging supply chain policies of optimization.

It's no secret that department store retailing is no longer simply operating an anchor store at the mall—you need expertise across channels. And with private-label merchandise, your supply chain gets longer and more complex every day managing the overseas purchasing process. To complicate matters, discounters now offer the brands that have been your staple offerings. Top brands—and best sellers—compete directly with their own stores and Internet presence. How can you compete? In this research paper, our attention is focused on an integrated procurement-distribution two stage supply chain model incorporating the discounted policies on purchasing goods and transportation network with the healp of case study of a departmental store. A model is formulated which explains the flow of ordered quantity from single source to multiple destinations with one intermediate stoppage point. In the model buyer at destination with various retail stores avails quantity discounts on bulk order and freight discounts on bulk transported quantity. Quantity discounts are provided by the supplier, in which supplier has fixed the quantity level beyond which discount would be given, in particular, on the basis of all unit discount model. The mode of transporting the goods from source to destinations takes place in two stages, where in the first stage, goods are moved using cargo. The unloading point of goods is the intermediate stoppage. At the stoppage, unloading of goods and their further processing takes a specified time for which the holding cost is free. Since cargo points require space to unload goods of number of cargos, the holding cost at stoppage increases with very high rates after a preset

time. Keeping inventory for long time may not be beneficial for the buyer but sometimes the inventory has to be kept because of some undue causes like transportation facility is not available to move goods from cargo point to destination. As the buyer has received discounts from supplier on the ordered quantity mentioned above, Cargo Company is also giving discounts on weighted quantity from supply point to intermediate stoppage.

Movement of goods from intermediate stoppage to destination is the second stage of model which is completed through modes of transportation, which are categorized as truckload (TL) and less-than-truckload (LTL) transportation. In Truckload transportation, the cost is a fixed of one truck up to a given capacity. In this mode company may use less than the capacity available but cost per truck will not be deducted. However in some cases the weighted quantity may not be large enough to substantiate the cost associated with a TL mode. In such situation, a LTL mode may be used. LTL may be defined as a shipment of weighted quantity which does not fill a truck. In such case transportation cost is taken on the bases of per unit weight. The model shows some initial and ending inventory at source to fulfill the uncertainties, as shortages are not allowed at any cost. Also, products ordered in any period will reach to a retailer at the second stage of next period.

## 2. Literature Review

Supply chain optimization requires development of models that require integrating the process of procurement, processing, storing, shipping and getting them to the consumer. The overall objective of these models is to minimize total cost by obtaining the economic order quantity and transporting needed quantity in a timely and efficient manner. Levi and Kaminsky (2000) mentioned that, supply chain management is a set of approaches utilized to efficiently integrate suppliers, manufacturers, warehouses and stores, so that merchandise is produced and distributed at the right quantities, to the right locations and to the right time, in order to minimize system-wide costs while satisfying service level requirements.

Modeling of two stage models taking into considerations the procurement and distribution decisions for multi products has least been studied by researchers, though subjects have been studied separately and extensively . Quantity discounts is one of the most popular mechanism of coordination in business for a long time to entice the buyers for purchasing larger quantity (Munson and Rosenblatt 1998). However, unless enough care is given to the design of a discount scheme, much of the potential benefits could be lost (Altintas et. al. 2008). Cheung and Lee (2002) forced shipment co-ordination in order to have full truckload (TL) shipments. Corbett and Groote (2000) considered coordinating the supply chain when the buyer has some private information. Ertogral (2008) took a single stage multi incapacitated dynamic lot sizing problem (MILSP) with transportation cost and assumed finite planning horizon with dynamic demand. He considered all unit inventory management models to formulate the problem with piece wise linear transportation cost function. Mendoza and Ventura (2008) developed an unconstrained integrated inventory-transportation model to decide optimal order quantity for inventory system over a finite horizon. All the above integrations are on single staged supply chains. Below are few studies of two stage supply chains.

Subramanya and Sharma (2009) integrated two stage supply chain network of an automobile company, measured the performance parameters and established the priority decision and queuing rules for improving the utilization of resources. The study restricted to measuring operational processes in a two stage supply chain between the supplier - manufacturer - distributors. Wang et al. (2009) examined the dynamics of a two stage supply chain consisting of one retailer and one distributor with order-up-to control policy. Lee et al. (2000) examined the value of information in a two-stage supply chain under an autoregressive demand process. Riekstsa and Ventura (2010) discussed two staged inventory models over an infinite planning horizon with constant demand rate and two modes of transportation. These transportation options include truckloads and a less than truck- load carrier. An optimal algorithm is derived for a one-warehouse one-retailer system. A power-of-two heuristic algorithm is also proposed for a one-warehouse multi-retailer system. Capar et al. (2011) dealt with two stage supply chain with two distribution centres (DC) and two retailers. Each member of supply chain used a (Q,R) inventory policy and incurred standard holding, backlog, ordering and transportation costs. Each DC is able to serve retailer on a first-come-first- served basis, but the transportation cost is lower for retailers within the DC's service area. Their goal is to provide an effective retailer ordering policy and to compare the overall supply chain performance of their policy to the current policy used by the company. Hsiao (2008) investigated the integrated stochastic inventory problem for a two-stage supply chain consisting of a single retailer and a single supplier. By using batch shipment policy, the expected total cost can be significantly reduced. Equally sized batch shipment models, controlled by both the reorder and the shipping points

with sharing information, variable safety factors are constructed. The problem is solved optimally by the proposed algorithms that determine the economic lot size, the optimal batch sizes, number of batches, and safety factor.These studies barely included procurement decisions into analysis.

The rest of this paper is organized as follows: Section 3 presents the sets , symbols and initial analysis used in the problem description and formulation; Section 4 provides the formulation of the problem; Section 5 presents the case study which validated the formulation along with results of the study; and finally, Section 6 discusses some concluding remarks and directions for further research.

## 3. Sets, Symbols and Initial Analysis

The formulated mathematical model is based on the following sets and symbols.

### 3.1 Sets

- Product set with cardinality P and indexed by i.
- Period set with cardinality T and indexed by t.
- Item discount break point set with cardinality L and indexed by small l.
- Freight discount break point set with cardinality K and indexed by small k.
- Waiting time set at intermediate stoppage with cardinality Γ and indexed by τ.

### 3.2 Parameters

$C$ Total cost

$c_t$ Cost of unit weighted quantity to be shipped in period t

$D_{imt}$ Demand for product i in period t for $m^{th}$ destination

$O_t^{\tau}$ Holding cost incurred on the total weight in period t for τ days

$h_i$ Inventory holding cost per unit of item i per period t

$w_i$ Per unit weight of item i in kgs

$\phi_{it}$ Unit purchase cost for $i^{th}$ item in $t^{th}$ period

$\beta_{mt}$ Fixed freight cost for each truck load (TL) for $m^{th}$ destination in $t^{th}$ period

$d_{ilt}$ It reflects the fraction of regular price that the buyer pays for purchased items.

$a_{ilt}$ Limit beyond which a price break becomes valid in period *t* for product i for $l^{th}$ price break

$b_{kt}$ Limit beyond which a freight break becomes valid at period *t* for $k^{th}$ price break

$f_{kt}$ It reflects the fraction of regular price that the buyer pays for transported weights.

$IN_i$ Inventory level at the beginning of planning horizon for product *i*

$d_f$ Discount factor

$\omega$ Weight transported in each full truck

$s$ Cost per kg of shipping in LTL mode

### 3.3 Decision Variables

$X_{it}$ Amount of product i ordered in period t transported to $m^{th}$ destination ordered in period t.

$R_{ilt}$ If the $i^{th}$ ordered quantity in $t^{th}$ period falls in $l^{th}$ price break then the variable takes value 1 otherwise zero.

$$R_{ilt} = \begin{cases} 1 & if \;\; X_{it} \; falls \;\; \text{in} \;\; l^{th} \;\; pricebreak \\ 0 & \text{otherwise} \end{cases}$$

$I_{it}$ Inventory level for $i^{th}$ product at the end of period t.

$L_{1t}$ Total weighted quantity transported to intermediate stoppage at stage 1 in period t.

$L_{2mt}$ Total weighted quantity transported to $m^{th}$ destination at stage 2 in period t.

$\alpha_{mt}$ Total number of truckloads to $m^{th}$ destination in $t^{th}$ period.

$y_{mt}$ Amount in excess of truckload capacity (in weights) to $m^{th}$ destination in $t^{th}$ period.

$u_{mt}$ *(or, 1-$u_{mt}$)* The variable reflects usage of policies, either both TL and LTL policies or only TL policy or only LTL.

$$u_{mt} = \begin{cases} 1, \text{ if considering TL \& LTL both policies or only LTL} \\ 0, \text{ if considering only TL policy} \end{cases}$$

$Z_{kt}$ If the weighted quantity transported falls in $k^{th}$ price break then the variable takes value 1 otherwise zero

$$Z_{kt} = \begin{cases} 1 & \text{if } L_t \text{ falls in k pricebreak} \\ 0 & \text{otherwise} \end{cases}$$

$v_{\tau t}$ Time period for which quantity is stored at intermediate stoppage

$$v_{\tau t} = \begin{cases} 1 & \text{if } L_{1t} \text{ waits at halt for period t} \\ 0 & \text{otherwise} \end{cases}$$

**3.4 Initial Analysis**

The inventory level at period t is dependent upon $X_{it}$ and at fuzzy demand $\tilde{D}_{imt}$ in the following way:

$$I_{it} = I_{it-1} + X_{it} - \sum_{m=1}^{M} D_{imt} \quad where\ i = 1...P \ , \ t = 2...T$$

The inventory level at the end of the period 1for item type i is composed of the inventory level at the beginning of the planning horizon, and the net change at the end of period one

$$I_{i1} = IN_i + X_{i1} - \sum_{m=1}^{M} D_{im1} \quad where\ i = 1...P$$

Total fuzzy demand in all the periods is more than or equal to total ending inventory level and ordered quantity of all the periods.

$$\sum_{t=1}^{T} I_{it} + \sum_{t=1}^{T} X_{it} \geq \sum_{t=1}^{T} \sum_{m=1}^{M} D_{imt}, \quad where\ i = 1...P$$

The buyer will order minimum quantity i.e.

$$X_{it} \geq \sum_{l=1}^{L} a_{ilt} R_{ilt}, \ i = 1...P; t = 1...T$$

It shows that the order quantity of all items in period t exceeds the price break threshold.
In any period, exactly one level will be activated either discount or no discount situation,so

$$\sum_{l=1}^{L} R_{ilt} = 1 \quad i = 1...P; \quad t = 1...T$$

Transported quantity according to item weightage is :

$$L_{1t} = \sum_{i=1}^{P} \left[ w_i X_{it} \sum_{l=1}^{L} R_{ilt} \right], \ t = 1...T; m = 1...M$$

Above constraint is an integrated constraint for procurement and distribution.

The minimum quantity transported is $b_{kt}$ i.e.

$$L_{1t} \geq \sum_{k=1}^{K} b_{kt} Z_{kt} \qquad t = 1...T$$

It shows that the transported quantity of all items in period t exceeds the freight break threshold.
In any period, exactly one level will be activated depending on the weighted quantity transported

$$\sum_{k=1}^{K} Z_{kt} = 1 \quad t = 1...T$$

In any period, at the intermediate stoppage exactly one level will be activated of halt at one of the period *t* depending on the processing of units for next stage

$$\sum_{\tau=1}^{\Gamma} v_{\tau t} = 1 \quad t = 1...T$$

The total weighted quantity transported in stage 1 of period $t$ is equal to the total weighted quantity transported in stage 2 of period ($t$ + 1)

$$L_{1\,t} = \sum_{m=1}^{M} L_{2m(t+1)} \qquad t = 1\ldots T$$

The minimum weighted quantity transported is equal to:

$$L_{2mt} \le (y_{mt} + \alpha_{mt}\omega)u_{mt} + (\alpha_{mt} + 1)\omega(1 - u_{mt}) \quad t = 2...T + 1;\ m = 1....M$$

Overhead units from TL capacity in weights are:

$$L_{2mt} = (y_{mt} + \alpha_{mt}\omega) \ \ t = 2...T + 1; \ \ m = 1....M$$

**3.5 Construction of Objective Function and Price Breaks**

The optimization problem of minimizing the cost of purchasing, holding and transportation subject to procurement and distribution constraints is formulated as follows.

$$Min\ C = \sum_{t=1}^{T}\left[\sum_{i=1}^{P}\left\{h_i I_{it} + \sum_{l=1}^{L} R_{ilt} d_{ilt}\phi_{it} X_{it}\right\} + \sum_{k=1}^{K} Z_{kt} f_{kt} c_t L_{1t} + \sum_{\tau=1}^{\Gamma} L_{it} O_t^{\tau} v_{\tau t}\right] + \sum_{t=2}^{T+1}\sum_{m=1}^{M}\left[(sy_{mt} + \alpha_{mt}\beta_{mt})u_{mt} + (\alpha_{mt} + 1)\beta_{mt}(1 - u_{mt})\right]$$

As discussed above, variable $R_{ilt}$ specifies the fact that when the order size at period $t$ is larger than $a_{ilt}$ it results in discounted prices for the ordered items for which the price breaks are defined as:

$$d_f = \begin{cases} d_{ilt} & a_{ilt} \le X_{it} \le a_{il+1t} \\ d_{iLt} & X_{it} \ge a_{iLt} \end{cases} \qquad i = 1...P; \quad t = 1...T; \quad l = 1...L$$

Freight breaks for transporting quantity are:

$$d_f = \begin{cases} f_{kt} & b_{kt} \le L_{1t} \le b_{k+1t} \\ f_{kt} & \mathrm{L}_{1t} \ge b_{Kt} \end{cases} \qquad t = 1...T$$

where $b_{kt}$ is the minimum required quantity to be transported in cargo.

## 4. Mathematical Formulation

The following is the formulation for above described analysis:

$$Min\ C = \sum_{t=1}^{T}\left[\sum_{i=1}^{P}\left\{h_i I_{it} + \sum_{l=1}^{L} R_{ilt} d_{ilt}\phi_{it} X_{it}\right\} + \sum_{k=1}^{K} Z_{kt} f_{kt} c_t L_{1t} + \sum_{\tau=1}^{\Gamma} L_{it} O_t^{\tau} v_{\tau t}\right] + \sum_{t=2}^{T+1}\sum_{m=1}^{M}\left[(sy_{mt} + \alpha_{mt}\beta_{mt})u_{mt} + (\alpha_{mt} + 1)\beta_{mt}(1 - u_{mt})\right]$$

$$I_{it} = I_{it-1} + X_{it} - \sum_{m=1}^{M} D_{imt} \quad where\ i = 1...P\ ,\ \ t = 2...T \tag{1}$$

$$I_{i1} = IN_i + X_{i1} - \sum_{m=1}^{M} D_{im1} \quad where\ i = 1...P \tag{2}$$

$$\sum_{t=1}^{T} I_{it} + \sum_{t=1}^{T} X_{it} \ge \sum_{t=1}^{T}\sum_{m=1}^{M} D_{imt,} \quad where\ i = 1...P \tag{3}$$

$$X_{it} \ge \sum_{l=1}^{L} a_{ilt} R_{ilt} \quad where\ i = 1...P \quad t = 1...T \tag{4}$$

$$\sum_{l=1}^{L} R_{ilt} = 1 \quad where\ i = 1...P, t = 1...T \tag{5}$$

$$L_{1t} = \sum_{i=1}^{P}\left[ w_i X_{it} \sum_{l=1}^{L} R_{ilt} \right], \ where\ \ t = 1...T \tag{6}$$

$$L_{1t} \geq \sum_{k=1}^{K} b_{kt} Z_{kt} \qquad t=1...T \tag{7}$$

$$\sum_{k=1}^{K} Z_{kt} = 1 \quad t=1...T \tag{8}$$

$$\sum_{\tau=1}^{\Gamma} v_{\tau t} = 1 \quad t=1...T \tag{9}$$

$$L_{1t} = \sum_{m=1}^{M} L_{2m(t+1)} \qquad t=1...T \tag{10}$$

$$L_{2mt} \leq (y_{mt} + \alpha_{mt}\omega)u_{mt} + (\alpha_{mt} + 1)\omega(1 - u_{mt}) \quad \text{where m = 1...M; t = 2, ..., T + 1} \tag{11}$$

$$L_{2mt} = \alpha(y_{mt}, \omega) + \ where\ \ m = 1...M; t = 2...T + 1 \tag{12}$$

$$X_{it}, I_{it}, L_{1t}, L_{2mt}, \ _{mt}\ and\ integer; R \quad , u \qquad _{ilt} \quad 0, 1, v_{it} = 1\{.P;\} \qquad t = 1...T; l = 1...L; m = 1...M$$

## 5. Case Study

India, slowly and steadily becoming the hub for global BPO services, due to this rise, India already got a new nick name “Call center of the world”. Many non metros cities across India witnessing rapid growth in IT sector because of booming BPOs and Call centers with the availability of talented English speaking graduates. One such city is Jaipur, because of the beaming BPOs in its proximity with state capital, Delhi.

As the IT related services are increasing at the express speed, so is the need for various computer peripherals, especially storage devices such as CDs, DVDs and Pen drives. Mr Mohan ex procurement manager of one of the leading BPO in Jaipur decided to start the retail showrooms in Jaipur selling various computer peripherals and hardware. So he owned three shops in various parts of the city where he will sell computer peripherals as Sky Computers (names changes, due to the cutting edge competition). He got the good deal for the peripherals other than storage devices for the bulk purchase, in his own city but for the storage devices such as CDs, DVDs and Pen drives he decided to look for a supplier from other cities as he was not getting the best deal in Jaipur.

He contacted Amtek Ltd., Banglore who specializes in manufacturing of storage devices for the supply of devices for his shops in Jaipur thrice a year. Fortunately the deal struck with Amtek for the supply of CDs, DVDs and Pen drives and it was decided that each item will be supplied per carton from the warehouse of Amtek in Banglore. Supplies will have to be flown from Bangalore airport to a storage house at the Jaipur airport from where it will be transported to the retail shops of Sky Computers .

Keeping supplies for the first day at the storage house at Jaipur airport is free of charge but subsequent stay will cost Mr Mohan hefty charges. So to keep the transportation cost at bare minimum he hired one of the most efficient transporters in Jaipur ,who will supply the products from the Banglore airport to Jaipur airport then to the retail shops of Mr Mohan. Transporter offers Mr Mohan various discounts depending upon the mode of transport chosen. Amtek also offers some quantity discounts to Mr Mohan in order to encourage large orders from Sky Computers”. The relevant data is provided as under (Table 1 - 10) which are mentioning the data given by supplier (procurement discounts, purchase cost, holding cost, initial inventory of the planning horizon kept, per pack weight of $i^{th}$ product-Table 1-5), given by transporter(freight discounts, freight cost, cost of unit weighted quantity in TL mode-Table 6-8), halt cost provided at intermediate stoppage point(Table 9) and demand as forecasted by store itself (Table 10).

Table 1: Discount Factors $d_{ilt}$ corresponding to thresholds $a_{ilt}$ for i[th] Product in all three Periods (in $)

| | | Level 1 | Level 2 | Level 3 |
|---|---|---|---|---|
| Storage Devices | CD | $0 \le X_{it} < 400$ | $400 \le X_{it} < 800$ | $800 \le X_{it}$ |
| | Discount factor | 1 | 0.85 | 0.75 |
| | DVD | $0 \le X_{it} < 650$ | $650 \le X_{it} < 1200$ | $1200 \le X_{it}$ |
| | Discount factor | 1 | 0.90 | 0.85 |
| | Pen Drive | $0 \le X_{it} < 450$ | $450 \le X_{it} < 800$ | $800 \le X_{it}$ |
| | Discount factor | 1 | 0.78 | 0.68 |

Table 2:Purchase Cost $\phi_{it}$ for i[th] Product in t[th] Period (in $)

| | | Period 1 | Period 2 | Period 3 |
|---|---|---|---|---|
| Storage Devices | CD | 410 | 405 | 500 |
| | DVD | 625 | 650 | 605 |
| | Pen Drive | 450 | 440 | 430 |

Table 3: Holding Cost $h_i$ for i[th] Product (in $)

| Storage Devices | CD | DVD | Pen Drive |
|---|---|---|---|
| Cost | 100 | 43 | 60 |

Table 4: Maximum Initial inventory in the starting of planning horizon ($IN_i$) in cartons

| Storage Devices | CD | DVD | Pen Drive |
|---|---|---|---|
| Cost | 50 | 256 | 40 |

Table 5: Per Pack Weight $w_i$ for i[th] Product (in Kg)

| Storage Devices | CD | DVD | Pen Drive |
|---|---|---|---|
| Weight | 6 | 6 | 12 |

Table 6: Freight Discount Factors $f_{kt}$ corresponding to $b_{kt}$ in all three Periods at 1[st] stage of transportation (in $)

| | | Level 1 | Level 2 | Level 3 |
|---|---|---|---|---|
| Time Periods | Period 1 | $3500 \le L_{1t} < 7500$ | $7500 \le L_{1t} < 9500$ | $9500 \le L_{1t}$ |
| | | 1 | 0.95 | 0.90 |
| | Period 2 | $3500 \le L_{1t} < 7500$ | $7500 \le L_{1t} < 9500$ | $9500 \le L_{1t}$ |
| | | 1 | 0.96 | 0.86 |
| | Period 3 | $3500 \le L_{1t} < 7500$ | $7500 \le L_{1t} < 9500$ | $9500 \le L_{1t}$ |
| | | 1 | 0.92 | 0.82 |

Table 7: Freight Cost $\beta_{mt}$ for m[th] destination in each period in 2[nd] stage of transportation (in $)

| | | Period 2 | Period 3 | Period 4 |
|---|---|---|---|---|
| Three Shops | Shop 1 | 1500 | 1900 | 1500 |
| | Shop 2 | 1700 | 2000 | 1800 |
| | Shop 3 | 1900 | 1300 | 1700 |

Table 8: Cost $c_t$ of unit weighted quantity in TL mode in period t

| Storage Devices | Period 1 | Period 2 | Period 3 |
|---|---|---|---|
| Cost | 36 | 38 | 30 |

Table 9: Halt Cost $O_t$ for t[th] period (in $)

| Storage Devices | CD | DVD | Pen Drive |
|---|---|---|---|
| Cost | 42 | 36 | 36 |

Table 10: Demand $D_{imt}$ of i$^{th}$ product at m$^{th}$ destination in t$^{th}$ period

| | Period 1 | | | Period 2 | | | Period 3 | | |
|---|---|---|---|---|---|---|---|---|---|
| | Shop 1 | Shop 2 | Shop 3 | Shop 1 | Shop 2 | Shop 3 | Shop 1 | Shop 2 | Shop 3 |
| CD | 145 | 188 | 181 | 167 | 189 | 186 | 210 | 208 | 143 |
| DVD | 160 | 228 | 232 | 233 | 300 | 360 | 240 | 258 | 262 |
| Pen Drive | 170 | 202 | 228 | 180 | 360 | 174 | 181 | 168 | 167 |

Weight transported in each full vehicle ω= 1000 kgs;
Cost of shipping one unit in LTL mode of transportation, s=$2

**5.1 Solution Analysis**

The vital objectives, the firms are concerned about are how much to order and how to minimize the total cost. Here, we have tried to answer these questions with the help of case in procurement-distribution scenario of a two stage supply chain. The critical research findings in period 1 are: Ordered Quantity for CDs in period 1 is 464 packs with discount of 15% on purchase cost and remaining ending inventory would be nothing after fulfilling the demand. Distribution in first stage from supplier to intermediate stoppage is 19662 kgs with 10% discount on cargo. At intermediate stoppage Storage Devices will be halted for two days and then at second stage distributed through trucks at shops of Mr. Mohan. The quantity will reach at shop 1 at second stage of second period with load of 17662 kgs by using both TL & LTL policy, in 17 trucks and 662 extra kgs. Rest of the procurement and distribution policies of other periods for Storage Devices is given below (Table 11-14).

Table 11: Procurement Policy at 1$^{st}$ stage

| | Period 1 | | | Period 2 | | | Period 3 | | |
|---|---|---|---|---|---|---|---|---|---|
| | $X_{it}$ | Discount | $I_{it}$ | $X_{it}$ | Discount | $I_{it}$ | $X_{it}$ | Discount | $I_{it}$ |
| CD | 464 | 15% | 0 | 592 | 15% | 50 | 511 | 15% | 0 |
| DVD | 1213 | 15% | 849 | 44 | 0% | 0 | 760 | 10% | 0 |
| Pen Drive | 800 | 32% | 240 | 474 | 22% | 0 | 516 | 22% | 0 |

Table 12: Distribution Policy at 1$^{st}$ stage

| Period 1 | | Period 2 | | Period 3 | |
|---|---|---|---|---|---|
| $L_{1t}$ | Discount | $L_{1t}$ | Discount | $L_{1t}$ | Discount |
| 19662 | 10% | 9504 | 14% | 13818 | 18% |

Table 13: Distribution Policy at 2$^{nd}$ stage

| | Period 2 | | | | Period 3 | | | | Period 4 | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Destination | $L_{2mt}$ | $\alpha_{mt}$ | $y_{mt}$ | $u_{mt}$ | $L_{2mt}$ | $\alpha_{mt}$ | $y_{mt}$ | $u_{mt}$ | $L_{2mt}$ | $\alpha_{mt}$ | $y_{mt}$ | $u_{mt}$ |
| Shop 1 | 17662 | 17 | 662 | Both | 0 | 0 | 0 | - | 13000 | 13 | 0 | TL |
| Shop 2 | 1000 | 1 | 0 | TL | 0 | 0 | 0 | - | 818 | 0 | 818 | LTL |
| Shop 3 | 1000 | 1 | 0 | TL | 9504 | 9 | 504 | Both | 0 | 0 | 0 | - |

Table 14: Number of days weighted quantity waited at intermediate stoppage( $\tau$ )

| | Period 1 | Period 2 | Period 3 |
|---|---|---|---|
| Waiting Days | 2 days | 2 days | 2 days |

We have used well known programming tool of optimization, Lingo 13.0 (Appendix A) and found that total cost incurred by Mohan in the two stage problem is $5,323,678.

## 6. Conclusion

This study reports the stability of two staged supply chain model. The problem is to coordinate inventory and transportation decisions across the supply chain so that total cost of the system is minimized. The two stage distribution of the products can make already complex supply chain process more complex. Model discussed above emphasis on such situation where quantity of product to be ordered is ascertained keeping in mind the constraints it

impose, while trying to keep the cost associated with procurement and distribution at minimum by considering various discount factors. The model is validated with the help of case taken for a branded store.

Model will benefit organizations in a long run by helping them determining optimal quantity to be ordered which not only reduces the cost of procurement and transportation costs but in case of storage devices it also helps reducing the holding cost and hence maximizing the profits and efficiency of organizations. We believe that this supply chain model will act as a building block and strong foundation of further study to be made in the field of two staged supply chain model. Future research can investigate the extension of model to multiple stages or incorporation of uncertainty with replenishment policies.

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**Appendix A**

```
SETS:
PRODUCT/P1,P2,P3/:pdt_weight,HOLD_COST;
PERIOD_NO/T1..T3/:CT,L1T,HALT_COST;
PERIOD/T2..T4/;
DESTN/D1,D2,D3/;
Second_stage_transport(DESTN,PERIOD):L2T,OVERHEAD_WEIGHT, JT, UT,BT;
demand(PRODUCT,DESTN,PERIOD_No):Qty_demand;
QTY_LEVEL/L1,L2,L3/;
TPT_LEVEL/K1,K2,K3/;
HALT/TOW1,TOW2,TOW3/;
TPT_LEVEL_VAR(TPT_LEVEL,PERIOD_NO):ZKT,FKT,BKT;
HALT_SET(HALT,PERIOD_NO):VTOWT;
PURCH_DEMAND_ORDER_INVT(PRODUCT,PERIOD_NO):P_COST,ORDERSIZE,INVT,IN;
QTY_LEVELed_variables(PRODUCT,QTY_LEVEL,PERIOD_NO):AILT,RILT,DILT;
```

```
ENDSETS
DATA:
! Import holding cost and product weight from excel sheet (data.xlsx);
HOLD_COST,PDT_WEIGHT=@OLE('C:\Users\LAPI\Desktop\DATA.XLSX','A2:A6','B2:B6');
! Import demand and purchase cost from excel sheet (data.xlsx);
QTY_DEMAND, P_COST= @OLE ('C: \  Users\ LAPI\ Desktop\ DATA.XLSX' ,'D2:O18', 'Q2:AB18');
! Import leveled parameters from excel sheet (data.xlsx);
AILT, DILT= @OLE ('C:\ Users\ LAPI\ Desktop\ DATA. XLSX’,‘AE2:AH1150’, ‘AK2:AN1150’);
! Data of distribution;
! Cost on transportation at 1st stage;
CT =      36   38   30;
! Cost on transportation at 2nd stage;
FKT, BKT= @OLE ('C:\ Users\ LAPI\ Desktop\ DATA. XLSX’, ‘AO2:AO15’, ‘AP2:AP15’);
HALT_COST=    42   36   36;
!WEIGHT EACH TRUCK CARRIES;
TRUCKLOAD = 1000;
! PER KG COST IN LTL;
S =2;
!PER TRUCK COST IN TL IN 3 PERIODS FOR THREE DESTINATIONS;
BT= @OLE ('C:\ Users\ LAPI\ Desktop\ DATA. XLSX’, ‘AQ2:AQ15’);
ENDDATA
! Objective Function;
MIN=@SUM(PURCH_DEMAND_ORDER_INVT(I,T):HOLD_COST(I)*INVT(I,T))+@SUM(PURCH_DEMAN
D_ORDER_INVT(I,T): @SUM ( QTY_LEVEL (L) : RILT (I,L,T) * P_COST (I,T) * DILT (I,L,T) * ORDERSIZE
(I,T) ) ) + @SUM ( TPT_LEVEL_VAR(K,T) : ZKT (K,T) * FKT (K,T) * CT (T) * L1T(T) ) + @SUM
(HALT_SET(TOW,T) : L1T(T) * VTOWT(TOW,T) * HALT_COST(T)^TOW) + @SUM
(second_stage_transport(M,T): (S* OVERHEAD_WEIGHT (M,T) + JT(M,T) * BT(M,T) ) * UT(M,T) +
(JT(M,T)+1) * BT(M,T) * (1-UT(M,T)));
! Constraint on initial inventory;
@FOR(PRODUCT(I)|I#EQ# 1:@FOR(PERIOD_no(T)|T#EQ#1:IN(I,T)<=50));
@FOR(PRODUCT(I)|I#EQ# 2:@FOR(PERIOD_no(T)|T#EQ#1:IN(I,T)<=256));
@FOR(PRODUCT(I)|I#EQ# 3:@FOR(PERIOD_no(T)|T#EQ#1:IN(I,T)<=40));
! Net Change in inventory after fulfilling the demand;
@FOR(PURCH_DEMAND_ORDER_INVT(I,T)|T #EQ# 1:INVT(I,T)=IN(I,T)+ORDERSIZE(I,T)-
@SUM(DESTN(M):QTY_DEMAND(I,M,T)));
@FOR(PURCH_DEMAND_ORDER_INVT(I,T)|T#GE#2:INVT(I,T)=INVT(I,T-1)+ORDERSIZE(I,T)-
@SUM(DESTN(M):QTY_DEMAND(I,M,T)));
@FOR(PRODUCT(I):@SUM(PERIOD_NO(T):INVT(I,T))+@SUM(PERIOD_NO(T):ORDERSIZE(I,T))>=@SU
M(PERIOD_NO(T):@SUM(DESTN(M):QTY_DEMAND(I,M,T))));
! Discount constraint on procurement;
@FOR(PURCH_DEMAND_ORDER_INVT(I,T):ORDERSIZE(I,T)>=@SUM(QTY_LEVEL(L):AILT(I,L,T)*RIL
T(I,L,T)););
@FOR(PURCH_DEMAND_ORDER_INVT(I,T):@SUM(QTY_LEVEL(L):RILT(I,L,T))=1) ;
@FOR(PERIOD_NO(T):L1T(t)=@SUM(PRODUCT(I):@SUM(QTY_LEVEL(L):pdt_weight(i)*ordersize(i,t)*RIL
T(I,L,T))););
! Halt time constraint;
@FOR(PERIOD_NO(T):1=@SUM(HALT(TOW):VTOWT(TOW,T)));
! Discount constraint on 1st stage of transportation;
@FOR(PERIOD_NO(T):L1T(T)>=@SUM(TPT_LEVEL(K):BKT(K,T)*ZKT(K,T)));
!Only one level will be activated for discounts on transported quantity
;@FOR(PERIOD_NO(T):@SUM(TPT_LEVEL(K):ZKT(K,T))=1);
! Transported Quantity of 1st stage of tth period is equal to the quantity transported through trucks to second stage of
(t+1)th period;
@FOR(PERIOD_no(T):L1T(T)=@SUM(DESTN(M):L2T(M,T)));
! Transported Quantity to different stores through no. of trucks;
```

```
@FOR(Second_stage_transport(M,T):L2T(M,T)<(OVERHEAD_WEIGHT(M,T)+JT(M,T)*TRUCKLOAD)*UT(
M,T)+(JT(M,T)+1)*TRUCKLOAD*(1-UT(M,T)));
! Overhead Transported Quantity to different stores;
@FOR(Second_stage_transport(M,T):L2T(M,T)=OVERHEAD_WEIGHT(M,T)+JT(M,T)*TRUCKLOAD);
@FOR(PURCH_DEMAND_ORDER_INVT:@GIN(ORDERSIZE));
@FOR(PURCH_DEMAND_ORDER_INVT:@GIN(INVT));
@FOR(PURCH_DEMAND_ORDER_INVT:@GIN(IN));
@FOR(QTY_LEVELed_variables:@BIN(RILT));
@FOR(HALT_SET:@BIN(VTOWT));
@FOR(PERIOD_NO(T):HALT_COST>=0);
@FOR(PERIOD_NO(T) :@GIN (L1T));
@FOR(SECOND_STAGE_TRANSPORT(M,T):@gin(L2T));
@FOR(TPT_LEVEL_VAR(K,T):@BIN(ZKT));
@FOR(SECOND_STAGE_TRANSPORT:@gin(JT));
@FOR(SECOND_STAGE_TRANSPORT:@gin(OVERHEAD_WEIGHT));
@FOR(SECOND_STAGE_TRANSPORT:@BIN(UT));
END
```