Fuzzy Multi-Objective Parallel Machines Scheduling Problem through New Solution Method

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Abstract

One of the basic and significant problems that a shop or a factory manager encounters is a suitable scheduling and sequencing of jobs on machines. One type of scheduling problem is just-in-time scheduling. Motivated by just-in-time manufacturing, we consider the multi-objective problem. These objectives minimize total weighted tardiness, earliness and minimize total flow time with fuzzy parameters on parallel machines, simultaneously with respect to the impact of machine deterioration. A number of different methods such as branch and bound, cutting plane, and other heuristic methods can solve the above problem. In recent years, researches have used genetic algorithms, simulated annealing, and machine learning methods for solving such problems. This problem is known as NP-hard. We discuss several dominance properties of optimal solutions, then present mathematical model and propose one method for defuzzification.

Keywords
Scheduling, Fuzzy multi-objective, parallel machines, Machine deterioration, Total flow time, Earliness and Tardiness

1. Introduction

In classical scheduling problems, the processing time of a job is assumed a constant. However, there are many situations that the processing times of jobs may be subject to change due to deterioration and/or learning phenomena McKay et al. (2002). Scheduling with earliness and tardiness costs has received considerable and increasing attention from the new researches. In many practical situations, it is required to guarantee that as many jobs as possible to meet their due dates (i.e., to minimize the number of tardy jobs) since in such cases, having a job missing its due date is very costly. Thus, minimization of the number of tardy jobs should be the primary concern. On the other hand, it is desirable to minimize the job earliness to minimize the inventory cost. Early/tardy scheduling problems are compatible with the concepts of just-in-time production and supply chain management, which have been adopted by many organizations. Indeed, these production strategies view both early and tardy deliveries as undesirable. By the machine deterioration effect, we mean that each machine deteriorates at a different rate. This deterioration is considered in terms of cost that depends on the production rate, the machine operating characteristics and the kind of work done by each machine. Moreover, job-processing times are increasing functions of their starting times and follow a simple linear deterioration. Browne and Yechiali (1990) first introduced it. Since then, deteriorating job scheduling problems have been widely discussed. Ruat et al. (2008) considered the problem of scheduling a given number of jobs on a single machine with time deteriorating job values and capacity constraints while the objective function is to maximize total revenue. Gawiejnowicz et al. (2006) considered a single machine time-dependent scheduling problem. They introduced two scenarios for a given sequence of job deterioration and formulated a greedy polynomial time approximation algorithm for each scenario.

One of the familiar objective functions in parallel machine scheduling is the minimizing of weighted tardiness penalties. Bilge et al. (2004) used a Tabu search method to schedule parallel machines with total weighted tardiness penalties. YI and Wang (2003) introduced a model for scheduling grouped jobs on identical parallel machines. In their model a set-up time is incurred when one-machine changes from processing one type of component to a different type of component. The objective function here is to minimize the total earliness–tardiness penalties. Radhakrishnan et al. (2000) emphasized the JIT production philosophy, and used simulated annealing for parallel machine scheduling with earliness–tardiness penalties and sequence dependant set-up times.

The fuzzy approach represents an alternative way to model imprecision and uncertainty, which is more efficient than the latter, especially when no historical information is available Anglani et al. (2005). First, it was introduced as are presentation scheme and calculus for uncertain or vague notions. The fuzzy set theory provides a conceptual framework Wu and Lee (2008) that performs so efficiently in decreasing the scheduling problem computational complexity with respect to the same problem formulated by the probability theory. It should be noted that such an imprecision is due to the subjective and qualitative evaluations, rather than the effect of uncontrollable events. The use of the fuzzy sets theory in treating different scheduling problems has been so successful, particularly where judgment and intuition play an important role Petrovic and Petrovic (2001), such as customer demand Petrovic and Petrovic (2001), processing times Kuroda and Wang (1996), production due dates Hong and Chuang (1999), or job precedence relations Ishii and Tada (1995). Prade et al. (1979) published the earliest paper in fuzzy scheduling. Ishii and Tada (1995) considered a single-machine scheduling problem minimizing the maximum lateness of jobs with fuzzy precedence relations. Han et al. (2000) considered single-machine scheduling problem with fuzzy due dates. Ishi- buchi and Murata (2000) presented a flow shop-scheduling problem with fuzzy parameters, such as fuzzy due dates and fuzzy processing times. Kuroda and Wang (1996) analyzed the fuzzy job shop-scheduling problem. Konno and Ishii (2000) discussed an open shop scheduling problem with fuzzy allowable time and fuzzy resource constraint. Itoh and Ishii (1999) proposed a single-machine scheduling model dealing with fuzzy processing times and due dates. Litoiu and Tadei (2001) present some new models for real-time task scheduling with fuzzy deadlines and processing times.

In classical parallel machine scheduling problems, all the parameters and variables are considered deterministic. However, since multiple sources of uncertainty and complex interrelation-ships at various levels between diverse entities exist in these kinds of problems, it is quite unreliable to set them as precise values. Some research Piersma and Romeijn (1996) has modeled parallel machine problem by Probability distribution that is usually predicted from historical data. However, whenever statistical data is unreliable or even unavailable, stochastic models may not be the best choice. Fuzzy set theory may provide an alternative approach for dealing with the uncertainty. Fuzzy set theory has been found extensive applications in various fields. A limited amount of the literature has been devoted to fuzzy parallel-machine scheduling problems. Peng and Liu (2004) proposed a practical application. In addition to single-objective scheduling models, they considered the multi-objective FPMSPs and formulated as three-objective models, which not only minimize the fuzzy maximum tardiness, but also minimize the fuzzy maximum completion time (i.e., make span) and the fuzzy maximum idleness. In this paper, we consider Fuzzy multi-objective parallel machines scheduling problem to minimize total weighted tardiness, earliness, and so minimize total flow time and machine deteriorating cost which is an extension of the problem studied in Mazdeh et al. (2010). The job scheduling problem to minimize weighted tardiness in parallel machines is NP-hard in the strong sense (Cao D., et al. 2005; Pfund et al. 2008), then the combinational problem in which the objective functions are minimizing the total tardiness, earliness, flow time and machine deteriorating cost will also be NP-hard.

The remainder of this paper is organized as follows. In Section 2, we define the problem and introduce the objective functions in detail. Next, the mathematical formulation for the model is developed. In Section 3, we presented new solution method for it and describe an approach in order to consider the four objectives as a single objective.

2. Problem Definition

The following notation and definitions are used to describe the multi-objective on parallel machines scheduling problem that is an extension of the problem studied in Mazdeh et al. (2010): Consider the problem of scheduling a set of N independent jobs, $J_1,J_2,. . . ,J_n$ on a number of parallel machines selected from a set of M potential machines that each of the jobs needs exactly one operation on one machine. Each job J_i has a processing time $\tilde{p}_j$ and a due date $\tilde{d}_j$ that all processing times and due dates are fuzzy numbers. Each machine is supposed to deteriorate at a different rate. This deterioration is a function of production rate, machine's operating characteristics and the kind of work accomplished by each machine and considered in terms of cost. A job is early if its completion time is smaller than the common due date. On the other hand a job is tardy if its processing ends after due date. As it is not known in advance whether a job will be completed before or after the due date. The notation and other assumptions we used in mathematical formulation is given below.

2.1. Problem assumptions

We consider a scheduling problem with n jobs to be processed on m machines with the following assumption:

1) Each machine has the ability for processing each job;
2) The machine can process at most one job at a time;
3) No processing is allowed;
4) Associated with each job j (j=1, … ,n) there are a processing time $\tilde{p}_j$ and a due date $\tilde{d}_j$;
5) Job processing time may be different if processed by a different machine;
6) Job processing time is described by a function of the starting time and fixed part of the processing time of the job ($\tilde{P}_{jm} = a_{jm} + \tilde{b}_j\tilde{S}_{jm}$);
7) The growth rate of the processing time ($\tilde{b}_j$) is independent on machine;
8) Jobs deterioration are independent of machines deterioration;
9) The jobs are independent of each other;

2.2. Parameters

Input parameters

N: total number of jobs to be scheduled,
M: total number of machines available,
i,j ϵ I={0,1,…,N}: designate the job, where job 0 is a dummy job, which is always at the first position on a machine,
α_i: earliness weight of job i, i ϵ I,
W_i: tardiness penalty of job i, i ϵ I,
$\tilde{d}_i$: due date of job i, i ϵ I,
$\tilde{C}_i$: finished time of job i, i ϵ I,
$\tilde{p}_{im}$: processing time of job i on machine m, i ϵ I, m ϵ M,
$\tilde{S}_{im}$: starting time of job i on machine m, i ϵ I, m ϵ M,
a_{im}: fixed part of the processing time for job i on machine m, i ϵ I, m ϵ M,
$\tilde{C}_{jm}$: machine deteriorating cost,
$\tilde{b}_i$: the growth rate of the processing time of job i on machine m, i ϵ I,
r_i: arrived time of job i to queue,
E_i: the earliness value of job i, i ϵ I,
T_i: the tardiness value of job i, i ϵ I,

Decision variables.

$$X_{ijm} \triangleq \begin{cases} 1 & \textit{if job j immediately follows job i in sequence on machine m,} \\ 0 & \textit{otherwise} \end{cases}$$

$$Y_{jm} \triangleq \begin{cases} 1 & \text{if job j assigned to machine m,} \\ 0 & \textit{otherwise} \end{cases}$$

Each machine deteriorates at a different rate. This deterioration is described in terms of cost. C_{jm} is a function of machine's operating characteristics and the kind of work done by each machine. The mathematical formulation is presented below:

2.3. Mathematical formulation

Model:

$$MinZ1 = \sum_{i=1}^{N} W_i\tilde{T}_i \tag{1-1}$$

$$MinZ2 = \sum_{i=1}^{N} \alpha_i\tilde{E}_i \tag{1-2}$$

$$MinZ3 = \sum_{i=1}^{N} \tilde{C}_i - r_i \tag{1-3}$$

$$MinZ4 = \sum_{m=1}^{M} \sum_{j=1}^{N} \tilde{C}_{jm} Y_{jm} \tag{1-4}$$

$$\sum_{j=1}^{N} X_{0jm} \leq 1 \qquad m = 1,2, \dots, M, \tag{1-5}$$

$$\sum_{i=0, i \neq j}^{N} \sum_{m=1}^{M} X_{ijm} = 1 \qquad j = 1,2, \dots, N, \tag{1-6}$$

$$\sum_{j=1, i \neq j}^{N} X_{ijm} \leq Y_{im} \qquad i = 1,2, \dots, N, \qquad m = 1,2, \dots, M, \tag{1-7}$$

$$\sum_{i=0, i \neq j}^{N} X_{ijm} = Y_{im} \qquad j = 1,2, \dots, N, \quad m = 1,2, \dots, M, \tag{1-8}$$

$$\sum_{m=1}^{M} Y_{im} = 1 \qquad i = 1,2, \dots, N, \tag{1-9}$$

$$\tilde{C}_j \geq \tilde{S}_{jm} + \tilde{P}_{jm} \quad j=1,2,\dots,N, \ m=1,2,\dots,M, \tag{1-10}$$

$$\tilde{P}_{jm} + M(1-Y_{jm}) \geq a_{jm} + \tilde{b}_j * \tilde{S}_{jm} \qquad j=1,2,\dots,N, \ m=1,2,\dots,M, \tag{1-11}$$

$$\tilde{S}_{jm} + M(1-X_{ijm}) \geq \tilde{C}_i \qquad j=1,2,\dots,N, \ i \neq j, \ i=0,1,2,\dots,N, \ m=1,2,\dots,M, \tag{1-12}$$

$$\tilde{C}_0=0 \tag{1-13}$$

$$\tilde{S}_{0m}=0 \qquad m=1,2,\dots,M, \tag{1-14}$$

$$\tilde{E}_i=Max\{0,\tilde{d}_i-\tilde{C}_i\} \qquad i=1,2,\dots,N, \tag{1-15}$$

$$\tilde{T}_i=Max\{0,\tilde{C}_i-\tilde{d}_i\} \qquad i=1,2,\dots,N, \tag{1-16}$$

$$\tilde{C}_i,\tilde{S}_{im},\tilde{T}_i \in R^+ \qquad i,j=1,2,\dots,N, \ m=1,2,\dots,M, \tag{1-17}$$

$$X_{ijm},Y_{jm} \in \{0,1\} \qquad i,j=1,2,\dots,N, \ m=1,2,\dots,M. \tag{1-18}$$

In the model (1), Eq. 1-4 are the objective functions, namely minimizing total weighted tardiness, total weighted earliness, total flow time and minimizing machine deteriorating cost, respectively. More precisely, Eq. 1 states that, if C_i-d_i > 0 then delivering job i has been delayed and it causes tardiness. Otherwise, if C_i-d_i < 0 it causes earliness that showed in Eq. 2. Eq. 4 states that if a job is processed on one machine, machine-deteriorating cost will happen. Constraint 5 ensures that for each machine selected to assign jobs; only one real job follows the dummy job 0. Sometimes, in a job scheduling scheme, it is possible to schedule only one job on a machine, in this situation a dummy job (i = 0) helps us to define X_{ijm}. Also sometimes, to minimize objective functions, no job is scheduled on one machine, in this situation, no job follows the dummy job 0 and Eq. 5 is equal to zero. Eq. 6 determines that if a job is assigned to a machine, it will be immediately preceded by one job and a job must be processed only at one position on a machine. Eq. 7 and 8 state that N jobs are assigned over M machines and if job i is immediately followed by j on machine m then both jobs i and j belong to machine m. Eq. 9 states that each job is assigned to exactly one machine. Eq. 10 relates the processing time of each job to its start time and completion time. Eq. 11 expresses the relation between the processing time of each job, its start time and fixed part of the processing time. On the other hand, this equation bounds the amount of processing time. Eq. 12 expresses that the job starting time is at least equal to the completion time of the preceding job. Eq. 13 completion time of job 0 is equal to zero. Eq. 14 states that the starting time of the dummy job on each machine is equal to zero. Eq. 15 expresses the relation between the completion time of each job, its due date and earliness variable and Constraint 16 specifies the tardiness of each job. Eq. 17 expresses that input parameters are positive and Eq. 18 shows that X_{ijm} and Y_{jm} are binary parameters.

3. Solving Method

Model 1 is a fuzzy nonlinear programming. To solve this model, at first the fuzzy numbers are converted into the interval numbers by α-cut approach. In other words, α-cut approach converts the nonlinear programming with fuzzy numbers into the nonlinear programming with interval numbers. Then, the nonlinear programming with interval number converts into the nonlinear programming with deterministic number by applying the convex conversion presented in appendix A. In this approach, decision maker must determine a fix value for α, this value determines the decision maker's risk. For larger α, decision maker has accepted more risk because he/she hasn't consider the uncertainty of fuzzy numbers and vice versa.

3.1. Membership function of fuzzy numbers and conversion them into interval numbers

Fuzzy numbers used in this paper are the processing time of job *i* on machine *k* ($\tilde{P}_{Ki}$), transportation time between place *r* and *s* ($\tilde{t}_{rs}$), set-up time job *i* to *j* on machine *k* ($\tilde{S}_{ijk}$) assumed to be triangular fuzzy number, and due date ($\tilde{d}_i$) has been introduced like due date in Sakawa and Kubota (2000) that has been illustrated in figure 3. Fuzzy number $\tilde{P}_{Ki}$ with triangular membership function is noted as $\left(P_{ki}^l, P_{ki}^m, P_{ki}^u\right)$, and this number is shown in figure 1.

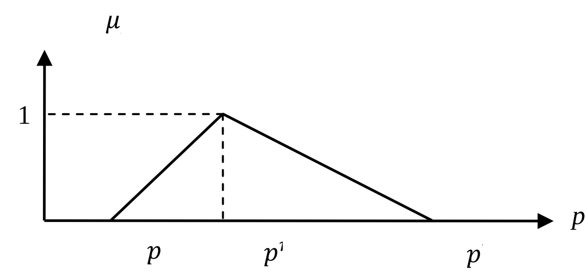

Figure 1: membership function of triangular fuzzy number

The membership function of this number is as follows:

$$\mu_{\tilde{P}_{Ki}}(P) = \begin{cases} \dfrac{P - P_{ki}^l}{P_{ki}^m - P_{ki}^l}, & P_{ki}^l \le P \le P_{ki}^m \\ \dfrac{P - P_{ki}^u}{P_{ki}^m - P_{ki}^u}, & P_{ki}^m \le P \le P_{ki}^u \\ 0 \quad , & P \le P_{ki}^l \ , P \ge P_{ki}^u \end{cases}$$

Lemma 1: The α-cut on this membership function for $\alpha \in [0,1]$ presents closed interval $[P_{ki}^l, P_{ki}^u]$ in which: $\tilde{P}_{ki_\alpha} = \left[P_{ki}^l \, , P_{ki}^u\right] = [\alpha . P_{ki}^m + (1-\alpha)P_{ki}^l \ , \ \alpha . P_{ki}^m + (1-\alpha)P_{ki}^u]$

(We define $\tilde{P}_{ki_\alpha}$ as Interval number (1))

Proof: Consider the α-cut on membership function of number $\tilde{P}_{Ki}$ shown in figure 2; by considering this α-cut, we have upper bound and lower bound for this number.

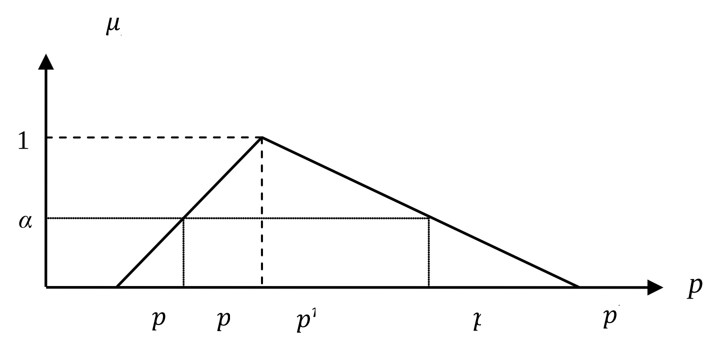

Figure 2: α-cut on membership function of triangular fuzzy number

According to figure 2, we can result:

$$\frac{\alpha}{P_{ki}^{L}-P_{ki}^{l}}=\frac{1}{P_{ki}^{m}-P_{ki}^{l}} \quad \Rightarrow \alpha\left(P_{ki}^{m}-P_{ki}^{l}\right)=P_{ki}^{L}-P_{ki}^{l} \Rightarrow P_{ki}^{L}=\alpha.P_{ki}^{m}+(1-\alpha)P_{ki}^{l}$$

$$\frac{\alpha}{P_{ki}^{u}-P_{ki}^{U}}=\frac{1}{P_{ki}^{u}-P_{ki}^{m}} \quad \Rightarrow \alpha(P_{ki}^{u}-P_{ki}^{m})=P_{ki}^{u}-P_{ki}^{U} \Rightarrow P_{ki}^{U}=\alpha.P_{ki}^{m}+(1-\alpha)P_{ki}^{u}$$

Fuzzy numbers $\tilde{t}_{rs}$ and $\tilde{S}_{ijk}$ are noted as $(t_{rs}^{l}, t_{rs}^{m}, t_{rs}^{u})$ and $\left(S_{ijk}^{l}, S_{ijk}^{m}, S_{ijk}^{u}\right)$, respectively. They are converted into the interval numbers by α-cut on these numbers as follows:

$\tilde{t}_{rs_{\alpha}}=[t_{rs}^{L}, t_{rs}^{U}]=[\alpha.t_{rs}^{m}+(1-\alpha)t_{rs}^{l} \;\;,\;\; \alpha.t_{rs}^{m}+(1-\alpha)t_{rs}^{u}]$ Interval number (2)

$\tilde{S}_{ijk_{\alpha}}=\left[S_{ijk}^{L}, S_{ijk}^{U}\right]=[\alpha.S_{ijk}^{m}+(1-\alpha)S_{ijk}^{l} \;\;,\;\; \alpha.S_{ijk}^{m}+(1-\alpha)S_{ijk}^{u}]$ Interval number (3)

Membership function of due date is as follows and is shown in figure 3.

$$\mu_{\tilde{D}_{Ki}}(d)=\begin{cases} 1 , & d \le d_i^{*} \\ \dfrac{d-d_i^{m}}{d_i^{*}-d_i^{m}} , & d_i^{*} \le d \le d_i^{m} \\ 0 , & d \ge d_i^{m} \end{cases}$$

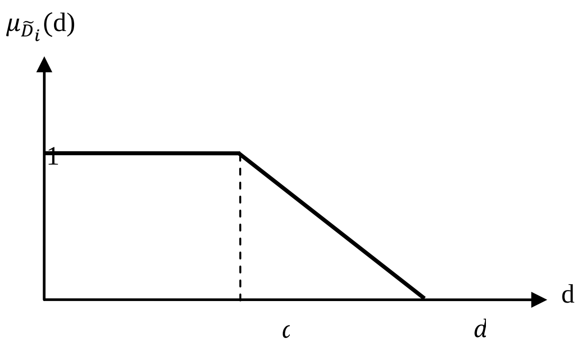

Figure 3: Membership function of due date

Due date $(\tilde{d}_i)$ is a fuzzy number that can be converted into interval number by applying α-cut as follows:

$\tilde{d}_{i_{\alpha}}=[d_i^{L}, d_i^{U}]=[0 \;\;,\;\; \alpha.d_i^{*}+(1-\alpha)d_i^{m}]$ Interval number (4)

3.2. Conversion fuzzy programming to deterministic programming

If we substitute interval numbers (1)-(4) for fuzzy numbers in Model 1, fuzzy nonlinear programming converts into interval programming. Then, to solve interval programming, we convert interval number into deterministic number by applying convex conversion; hence the interval programming converts into deterministic programming.

Follows, interval numbers (1)-(4) convert into deterministic number by applying convex conversion context presented in appendix A.

3.3. Proposed method for solving multi-objective linear programming

We consider a general form of multi-objective linear programming as follows:

$$\begin{aligned} &\max \quad f_1(x), f_2(x), ..., f_n(x) \\ &\min \quad g_1(x), g_2(x), ..., g_m(x) \\ &\text{s.t}: \quad x \in X \end{aligned}$$

That $f_1(x), f_2(x), ..., f_n(x)$ and $g_1(x), g_2(x), ..., g_m(x)$ are the objective functions and X is the feasible region. First, an ideal solution for each objective function separately will be obtained by following problems solving:

$$g_i^{*}=\min \quad g_i(x) \quad (i=1,...,m) \qquad f_i^{*}=\max \quad f_i(x) \quad (i=1,...,n)$$
$$\text{s.t}: \quad x \in X \qquad\qquad \text{s.t}: \quad x \in X$$

Then, will be obtained without unit function by multiply any function in its optimal value inverse that their range are $[0,1]$. Thus, multi-objective programming problem can satisfactorily solve by following the LP problem:

$$\max \ \beta$$

$$\begin{aligned} s.t: \quad & \beta \le \frac{f_i(x)}{f_i^*} \quad i = 1,\dots,n \\ & \beta \le -\frac{g_i(x)}{g_i^*} \quad i = 1,\dots,m \\ & x \in X \end{aligned}$$

We use of following auxiliary objective function in new model

max β

Also add following constrains to past constrains:

$$\beta \le -\frac{\sum_{i=1}^{N} W_i \times Max\{0, \left(S_{im} + P_{im}^L + \lambda_{\ im} \ (p_{im}^U - p_{im}^L)\right) - \gamma_i d_i^U\}}{Z_1^*} \tag{3-1}$$

$$\beta \le -\frac{\sum_{i=1}^{N} \alpha_i \times Max\{0, \gamma_i d_i^U - \left(S_{im} + P_{im}^L + \lambda_{\ im} \ (p_{im}^U - p_{im}^L)\right)\}}{Z_2^*} \tag{3-2}$$

$$\beta \le -\frac{\sum_{i=1}^{N} S_{im} + P_{im}^L + \lambda_{\ im} \ (p_{im}^U - p_{im}^L) - r_i}{Z_3^*} \tag{3-3}$$

$$\beta \le -\frac{\sum_{m=1}^{M} \sum_{j=1}^{N} \left(C_{jm}^L + \omega_{jm}(C_{jm}^U - C_{jm}^L) \right) \times Y_{jm}}{Z_4^*} \tag{3-4}$$

4. Evaluation of Model Performance with an Illustrative Example

A small illustrative example has been developed to evaluate the performance of the model. Tables 1, 2 and 3 summarize the data used for two numerical examples of 10 jobs. We consider the following assumptions for problem parameter generation:

1. The tardiness weights (W_i) and earliness penalties (α_i) are uniformly generated from discrete uniform distribution on [1,3].
2. The arriving times (r_i) are integers and are generated from a uniform distribution on [0,20].
3. The growth rate of the processing times (b_i) are random numbers greater than or equal to 0 and less than 1, evenly distributed.

For solving the example problem, gave the common data of due dates, fixed part of the processing times and deteriorating cost. The model has been solved by the Lingo 13.0 solver. The experiments were run in an Intel(R) core(TM) i3 CPU, at 2.13GHz and with 4.00 GB of RAM memory.

Table 1: Some parameters for generation of problem instances (weights, arrival times and due dates)

Jobs	W_i	α_i	r_i	$\tilde{d}_i$
1	2	1	11	(40, 44, 46)
2	3	1	7	(39, 42, 43)
3	1	1	2	(53, 58, 60)
4	1	2	5	(57, 62, 64)
5	1	3	10	(50, 54, 56)
6	1	1	9	(32, 33, 34)
7	3	3	6	(57, 60, 62)
8	2	2	0	(47, 51, 53)
9	2	1	9	(23, 26, 27)
10	2	3	11	(50, 54, 56)

Table 2: The growth rate and fixed part of the processing time

Jobs	$\tilde{b}_i$	a_{i1}	a_{i2}	a_{i3}
1	(0.55, 0.60, 0.62)	22	18	20
2	(0.24, 0.26, 0.27)	28	26	29
3	(0.75, 0.75, 0.78)	46	50	50
4	(0.39, 0.42, 0.44)	48	50	48
5	(0.32, 0.33, 0.34)	36	34	35
6	(0.17, 0.18, 0.19)	18	18	18
7	(0.60, 0.65, 0.66)	46	46	46
8	(0.03, 0.04, 0.04)	44	46	43
9	(0.09, 0.10, 0.10)	12	11	11
10	(0.14, 0.15, 0.16)	37	34	34

Table 3: Machines deteriorating cost

Jobs	$\tilde{C}_{i1}$	$\tilde{C}_{i2}$	$\tilde{C}_{i3}$
1	(1.1, 1.2, 1.2)	(1.7, 2.0, 2.1)	(1.6, 1.7, 1.8)
2	(5.2, 5.4, 5.6)	(4.0, 4.3, 4.3)	(3.1, 3.2, 3.5)
3	(2.4, 2.4, 2.5)	(2.0, 2.2, 2.3)	(2.2, 2.4, 2.5)
4	(0.7, 0.8, 0.8)	(1.2, 1.3, 1.4)	(0.9, 1.1, 1.1)
5	(8.7, 9.5, 9.9)	(7.1, 7.9, 8.2)	(8.0, 8.7, 9.0)
6	(5.9, 6.0, 6.2)	(4.3, 4.4, 4.7)	(6.5, 7.1, 7.4)
7	(0.7, 0.8, 0.8)	(0.7, 0.8, 0.8)	(0.9, 1.0, 1.0)
8	(5.6, 6.1, 6.3)	(4.4, 4.8, 5.2)	(4.5, 4.8, 5.2)
9	(8.4, 8.7, 8.8)	(8.6, 9.6, 10.0)	(11.6, 12.2, 12.7)
10	(3.4, 3.7, 3.8)	(2.9, 3.1, 3.2)	(2.9, 3.1, 3.2)

Table 4 summarizes the evaluation results of deterministic and fuzzy model solutions, according to a group of parameters defined in Table 1 to Table 3.

Table 4. Evaluation of results

	Deterministic model solution	Fuzzy model solution
Machine 1	Jobs: {4, 6, 9, 1, 10}	Jobs: {4, 2, 5}
Machine 2	Jobs: {3, 2, 5}	Job s: {3, 6, 9, 1}
Machine 3	Jobs: {8, 7, 10}	Jobs: {8, 7, 10}
β value	-1.53672	-1.36380

Table 4 shows that in average fuzzy model reaches the better results with respect to objective values.

5. Conclusion

We have studied a fuzzy multi-objective parallel machines scheduling problem, i.e., to minimize total weighted tardiness and earliness, minimum machine deteriorating cost. First, we developed mathematical formulation for the model. Then we presented new solution method for it and proposed a method to include these objective functions in a single objective function. The final model was solved with commercial optimization software Lingo for an illustrative example. Computational results show that consideration of fuzzy parameters can result in better performances with respect to deterministic modeling. Time complexity is not addressed in this paper, since the computational time increases significantly when the size of problem increase, therefore developing efficient exact or heuristic solution methods is a critical need in this area.

Appendix A: (Noor approach)

Consider general forms of interval programming:

Model A-1:

$$Min \qquad Z=\sum_{j=1}^{n}\left[c_j^L,c_j^U\right]x_j$$

$s.t.$

$$\sum_{j=1}^{n}\left[a_{ij}^L,a_{ij}^U\right]x_j \ge [b_i^L,b_i^U] \quad i=1,\dots,m$$

$$x_j \ge 0 \qquad\qquad j=1,\dots,n$$

With applying convex conversion as following:

$$c_j^L \le c_j \le c_j^U \Rightarrow c_j=\lambda_j c_j^U+(1-\lambda_j)c_j^L=c_j^L+\lambda_j(c_j^U-c_j^L) \qquad 0\le\lambda_j\le 1 \quad j=1,\dots,n$$

$$a_{ij}^L \le a_{ij} \le a_{ij}^U \Rightarrow a_{ij}=\beta_{ij}a_{ij}^U+(1-\beta_{ij})a_{ij}^L=a_{ij}^L+\beta_{ij}(a_{ij}^U-a_{ij}^L) \quad 0\le\beta_{ij}\le 1 \quad j=1,\dots,n \quad i=1,\dots,m$$

$$b_i^L \le b_i \le b_i^U \Rightarrow b_i=\beta_i b_i^U+(1-\beta_i)b_i^L=b_i^L+\beta_i(b_i^U-b_i^L) \qquad 0\le\beta_i\le 1 \quad i=1,\dots,m$$

And with substitute preceding number in model A-1 we result:

$$Min \qquad Z=\sum_{j=1}^{n}\left[c_j^L+\lambda_j(c_j^U-c_j^L)\right]x_j$$

$s.t.$

$$\sum_{j=1}^{n}\left[a_{ij}^L+\beta_{ij}(a_{ij}^U-a_{ij}^L)\right]x_j \ge b_i^L+\beta_i(b_i^U-b_i^L) \qquad i=1,\dots,m$$

$$x_j \ge 0 \qquad j=1,\dots,n$$

$$0\le\beta_{ij}\le 1 \qquad i=1,\dots,m \quad j=1,\dots,n$$

$$0\le\beta_i\le 1 \qquad i=1,\dots,m$$

$$0\le\lambda_j\le 1 \qquad j=1,\dots,n$$

And or:

Model A-2:

$$\mathrm{Min} \qquad Z=\sum_{j=1}^{n}c_j^L x_j+\sum_{j=1}^{n}\lambda_j x_j(c_j^U-c_j^L)$$

s.t.

$$\sum_{j=1}^{n}a_{ij}^L x_j+\sum_{j=1}^{n}\beta_{ij}x_j(a_{ij}^U-a_{ij}^L)-\beta_i(b_i^U-b_i^L)\ge b_i^L \qquad i=1,\dots,m$$

$$x_j \ge 0 \qquad j=1,\dots,n$$

$$0\le\beta_{ij}\le 1 \qquad i=1,\dots,m \quad j=1,\dots,n$$

$$0\le\beta_i\le 1 \qquad i=1,\dots,m$$

$$0\le\lambda_j\le 1 \qquad j=1,\dots,n$$

Model A-2 is a problem programming with deterministic numbers.

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