A Model for Multilevel Location-Allocation for Market Capture in Congestion System

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Abstract

In this paper, a firm's plan is to locate the multi-level and multiple-server facilities in a competitive region. In this regard, one hierarchical multi-objective model is developed, in which lower-level servers attract requests that are not allocated to other competing servers and higher –level servers attract all of customers in the region. Here, a multi-objective model is presented to consider service congestion, the first objective function is to find maximum number of low level facilities that can be located and the second objective function is to minimize the maximum distance and waiting time among allocated customers of high level servers. Each customer can obtain the service or goods from several (rather than only one) facilities. The important trait our model is that All of the characteristics (multi-servers, multi-level, multi-objective, hierarchical facilities, congested systems, queuing system) use in a single model. Finally, there is a summary and a discussion together with possible future research direction.

Keyword
Location-allocation problem, queuing system, competitive region, hierarchical modes, heuristic algorithm.

1. Introduction
When customers want to obtain service, competition between firms involves 'capturing' or 'attracting' as many customers as possible through closeness, shorter waiting times, better service, price and so on. This paper uses queuing models; the queuing models are one of the most important systems that represent the behavior of customers and suppliers in location issues. Variety of real-world issues, including industrial production, telephone systems, computer networks, location and traffic have been modeled using queuing theory (Addabbo and Kocarev, 2009). Revelle and Eiselt (2003) represented four elements that specify location models, namely: customers in region, equipments that to locate, region and a decision parameter for customers. In this paper we want to do the facilities locating with regard to the other three elements. In this paper the new firm has the high-level facilities that didn't locate by competing firms and the low-level facilities. All of the firms in this region located low-level facilities. This location system represents a hierarchical structure. Systems with a hierarchical structure are common both in public and private sectors. Examples of this structure can be found in public health services, school systems, and so on (Marianov and Serra, 2001). In hierarchical systems, facilities at different levels provide different types of services. Hierarchical services can be classified according to their general structure; Narula (1985) defined the hierarchical models for example nested hierarchy, non-nested hierarchy, coherent hierarchy. A good review of the models for design location-allocation of hierarchical systems is given in (Gerrard and Church , 1994). Lee et al (2010) applied the Tabu based heuristics for the generalized hierarchical covering location problem. Marianov and Serra, (2001) published one paper in this category that there were two types of facility which designate 'low' and 'high' level. Chan et al (2004) presented the Hierarchical maximal-coverage Location- allocation model.

In this paper we apply the hierarchical models in a competitive region. Location under competition has attracted attention of researchers for a long time. The generalization of the competitive environment to a network was addressed first by Hakimi, (1983), who presented many paper related this issue An exhaustive review of the subject of competition on a line can be found in the paper was published by Eiselt and Laporte (1989). Serra et al, in 1992 applied hierarchical location in a competitive environment. In most paper assume that competitors capture customers mainly by distance, in this paper we consider waiting time that a customer has to face when arriving to a facility has also an influence on her/his choice of the facility to patronize. Time waiting at a facility before receiving service was calculated by Queuing formulas. Baron et al (2002) presented the problem of optimal location of a set of facilities in the presence of stochastic demand and analyses waiting time in the models. In designing the service networks such as medical centers, fire fighting facilities, police stations, and so on, location of service facilities and allocation of service calls to servers, drastically affect the congestion of demand, there will be times of heavy demand. Such a facility will be said to be congested and most of these papers will be concerned with congested systems. Congestion happens when a service center is not capable to serve all the simultaneous requests for service that are made to it (Marianov and Serra, 2001). An example of a congested system is provided by the accident and emergency department of a hospital which may be overwhelmed when there has been a serious accident involving many casualties. Marianov and Serra, (1998) presented several models to consider service congestion. Boffey and et al (2007) presented a review of apply congested systems in location problem. Berman et al (1989) developed some models using queuing theory for congested networks. Shavandi and Mahlooji (2006) presented a fuzzy location–allocation model for congested systems. Marianov and Serra (2001) had formulated new models for the location of congested facilities. Most of papers assume a single server in the region under study, because of the difficulty in analyzing multi-server queuing systems. Berman, Larson and Chiu (1985) in their paper on Stochastic Queue Median, developed a heuristic algorithm for the optimal location of one server in a congested network. Weber (1983) presented the paper to consider single server. Cohen (1982) analysed queuing systems with single server. Batta (1989) presented a model for studying the effect of using expected service time dependent queuing disciplines in optimal location of a single server. We use a model that locates several servers in each facility. Aboolian et al (2008) presented the model for multiple server center location problems. In a paper Berman and Drezner (2007) introduced a multiple server location problem in a stochastic environment. The distinct our paper with other papers is to incorporate all of the mentioned characteristics (multi-servers, multi-level, multi-objective, hierarchical facilities, congested systems, queuing system) within a single model.

The remainder of the paper is organized as follows: in next section we present the development of the mathematical relations of the model and in third section we conclude the paper with summary and suggestion for future research. The last section is dedicated to the references.

2. Development of the Model

We consider region like a network with nodes, where between any nodes there is a possible path, and nodes present either candidate location for facilities or demand concentrations, or both. In this case there are two levels of demand at all nodes, low-level demands and high-level demands. These two types of demand in each node have a specific rate. Competitors try to attract or capture as large a proportion of the demand as possible. In the model we present in this paper customers are assumed to choose the closest facility to obtain service. We believe that this assumption is very reasonable since for many systems there is no central authority that assigns customers to facilities and customers usually do not have a priori information on the waiting time at the different facilities. In this case, different percentage of the demand at each demand node exists that chose the server with the required level. Customers will choose the facility that minimizes travel and waiting times. We consider $N \cup N'$ nodes in network. N and N' are, respectively, the sets of candidate locations for the low-level facilities entering firm(nodes that are vacant of facilities competing firms), and the sets of locations occupies by the incumbent firm or firms, $|N'| = q.$ nodes $N \cup N'$ are set of candidate locations for the high-level facilities entering firm. Two kind of demands exist in any node i, i=1, 2,..., $N \cup N'$, demand for high-level facilities and demand for low-level facilities that respectively are h_i' and h_i. the high-level demand generation rate at each demand node i is the Poisson process with average demand rate h_i' and low-level demand generation rate at each demand node u with average demand rate h_i.

The entering firm wants to locate p high-level facilities and n low-level facilities in the region, represented as a network, where q competing low-level facilities are already located. We assume that all the nodes in the network are candidates to the location of high-level facilities, as well as nodes containing demand, but only vacant nodes of competing facilities are candidates to location of low-level facilities. A location problem involves users travelling to a facility for service, or servers travelling from facilities to the users in the case of mobile servers such as ambulances. In this paper we shall be assumed that when customer calls for service, servers after recognize level of service will be moved to customer.

We assume that each low–level facility behaves as a M/M/1 queue, that is, Poisson arrivals with a mean rate λ, exponentially distributed service time with mean μ, 1 server. For low-level facilities assume a single server in the region under study. Each high-level facility behaves as an M/M/m/k, where m is number of servers in each high-level facility and because of space considerations at the facilities the queue is limited to k customers. In practice, the assumption of infinite queues and to restrict the number in the queue to no more than a fixed number never applies

(Marianove, 2003). Note that for both levels, each services center can have one or more servers. We develop constraints for both situations 1 for low-level facility, m for high level facility. Entering firm access to M servers (consist of both low-level and high-level). To use the queuing approach it is necessary to know the below equations. In this model n and m are unknown, that can calculated with below relation:

The equations for an M/M/m/K queuing system are the following ((Hillier and Lieberman, 1986) :

$$P_0 = \left[1 + \sum_{n=1}^{m} \frac{\rho^n}{n!} + \frac{\rho^m}{m!} \sum_{n=1+m}^{k} \left(\frac{\rho}{m}\right)^{n-m}\right]^{-1} \tag{3}$$

P_n is the probability of a number n of customers being at the facility and $\rho = \lambda/\mu$ is the utilization. One set of similar equations can be written for each high-level facility. The probabilities P_{nj} at each facility j are a function of ρ_j, which is in turn a function of λ_j' .

$\lambda_j' = \sum_{i \in N \cup \acute{N}} h_i' X_{ij}'$ Also,

$$L_j = \sum_{n=m}^{k} (n-m) P_{nj} \tag{4}$$

$$w_j = \frac{L_j}{\left(\sum_{i \in N \cup \acute{N}} h_i' X_{ij}'\right) \times \left(1 - P_{kj}\right)} \tag{5}$$

Where L_j is the average length of the queue in the facility j, w_j the average waiting time in the facility j (excluding service time) given by little's formula. h_i' is rate of demand for high –level facility in node i, X_{ij}' is allocation variable that takes value 1 if population at demand nod I is allocated to a high-level server located at the candidate node j and zero otherwise.

In this paper present a two-objective model, the first objective Z1 maximizes the number of low-level facility that can under defined constraints located. Low-level facilities can located in node i if in this node didn't occupied by other company and sum of total demands for this low-level facility that located in node i shouldn't be less than amount standard α, this amount in Known. The second objective in this model Z2 is to minimize the maximum travel time plus the average waiting time spent at the service high-level facility for all customers. Note that this amount calculated for all of demand rate h_i'. The model can be formulated as:

$$Max \quad Z_1 = \sum_{j \in N} y_j \tag{6}$$

$$Min \quad Z_2 = \underset{i \epsilon N \cup N'}{Max} \left\{\left(w_j + t_{ij}'\right) y_j' h_i' X_{ij}'\right\} \ \forall j \ N \cup N' \tag{7}$$

S.t.

$$\sum y_j' = p \qquad \forall j \in N \cup N' \tag{8}$$

$$\sum_{j \in N \cup N'} X'_{ij} = 1 \qquad \forall i \in N \cup N' \tag{9}$$

$$\sum_{j \in N} X_{ij} = 1 \qquad \forall i \in N \tag{10}$$

$$X'_{ij} \le y'_j \qquad \forall\, i,j\ \in\ N \cup\ N' \tag{11}$$
$$X_{ij} \le y_j \qquad \forall\, i,j\ \in N \tag{12}$$
$$\sum_{i \in N} h_i\, X_{ij} y_j \ge\ \alpha\, y_j \qquad \forall\, j \in N \tag{13}$$
$$X_{ij}\ \times\ t_{\mathrm{ij}} \le \mathrm{s} \qquad \forall\, i,j\ \in N \tag{14}$$
$$m = \frac{M - \sum y_j}{p} \tag{15}$$
$$w_j = \frac{L_j}{\left(\sum_{i\,\in\,N \cup\,N'} h'_i X'_{ij}\right) \times \left(1 - \mathrm{P}_{k\mathrm{j}}\right)} \tag{16}$$
$$\mathrm{L_j} = \sum_{\mathrm{n}=m}^{\mathrm{k}} (n - m)\mathrm{P_{nj}} \qquad \forall j \in\ N \cup\ N' \tag{17}$$

$$P_{nj} = \begin{cases} \dfrac{P_0}{n!} \times \left(\dfrac{\sum\ h'_i X'_{ij}}{\mu}\right)^n & for\ n < m \\ \left(\dfrac{\sum\ h'_i X'_{ij}}{\mu}\right)^n \times \dfrac{P_0}{m\,!\,m^{\,n-m}} & for\ n \ge m \end{cases} \tag{18}$$
$$P_0 = \left[1 + \sum_{n=1}^{m} \frac{\left(\frac{\sum\ h'_i X'_{ij}}{\mu}\right)^n}{n!} + \frac{\left(\frac{\sum\ h'_i X'_{ij}}{\mu}\right)^m}{m\,!} \sum_{n=1+m}^{k} \left(\frac{\sum\ h'_i X'_{ij}}{\mu \times m}\right)^{n-m}\right]^{-1} \tag{19}$$
$$X_{ij} \in \{0{,}1\} \quad \forall i\ \in N\,, \forall j\ \in N \tag{20}$$
$$h_i \ge 0 \quad \forall i \in\ N \cup\ N' \tag{21}$$
$$y_j \in\ \{0{,}1\} \quad \forall j \in N \tag{22}$$
$$y'_j \in\ \{0{,}1\} \quad \forall j \in\ N \cup\ N' \tag{23}$$
$$X'_{ij} \in \{0{,}1\} \quad \forall i,j\ \in\ N \cup\ N' \tag{24}$$

Where

y_j location variable which takes value 1 if a low-level facility is located at nod j, and zero otherwise,
y'_j location variable which takes value 1 if a high-level facility is located at nod j, and zero otherwise,
w_j the average waiting time in queue facility located in nod j,
t'_{ij} shortest network distance between nodes i and j for high-level servers,
t_{ij} shortest network distance between nodes i and j for low-level servers,
h'_i rate of high-level demand in node i,
h_i rate of low-level demand in node i,
X'_{ij} represent the customer–facility assignments, that takes value 1 if population at demand node i is allocated to a high-level server located at the candidate node j and zero otherwise,
X_{ij} represent the customer–facility assignments, that takes value 1 if population at demand node i is allocated to a low-level server located at the candidate node j and zero otherwise,
N the set of low-level candidate nodes,
N' the set of nodes in network that located by competing facilities,
p number of high-level facility that firm access to them,
α standard minimum demand for each located low-level facility,
s standard distance from demand to low-level server,

L_j length of queue high-level servers in node j,
m number of servers in high-level facilities,
M all of servers that entering firm have,
P_{nj} probability of a number n of customers being at the facility j,
P_0 probability of the queuing system being in state 0 (0 users in the system),
μ_j service rate at server located in node j,
λ_j arrival rate of requests to low-level server located in node j,
λ'_j arrival rate of requests to high-level server located in node j,

The objective function, (6), maximizes number of low-level facilities that can be located. The objective function, (7), minimizes maximum distance and waiting time from demand to high-level servers. Constraint (8) specifies the number of facilities to be located by the entering firm. Constraint (9) and (10) ensures that a 100% of customers at a demand node i will be served somewhere. Constraints (11) and (12) force customer capture only by open facilities. Constraint (13) forces low-level facilities located at nodes that in worst situation have minimum demands. Constraint (14) forces customer allocated to low-level facilities that distance between them isn't more than standard distances. Constraint (15) defines number of server in each high-level facility, in this instance may get value m isn't integer and in this paper we round down it. Constraints (16)-(19) are the definitions of all auxiliary parameters. Constraints (20) to (24) ensure non-negativity, integrality and bounds on the variables. After, formulating the model, some numerical examples can be solved to test it.

3. Conclusions

In this paper, a new model for locating service centers in a congested situation is presented, in which every customer (demand) needs to be satisfied. This model, being linear in nature, explicitly includes a constraint on the waiting time or queue length. Applications for such a model include the location of medical facilities where at each location the number of medical care personnel must be determined and location of post offices, bank branches etc. The problem is to find location of facilities for both at high and low levels, and recognize the number of servers assigned to each high-level facility to minimize the maximum travel time plus average waiting time spent at the service facility. Of course, the model can be tested and approved by numerical examples and solved by heuristic algorithms like Tabu search, Genetic algorithm, SA and etc. One interesting future research in this regards, is to combine the model presented here with fuzzy location–allocation model and solve it with a heuristic algorithm.

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