Modeling Capacity Investment under Competition using Game Theory

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Abstract

Globalization and competitiveness in current market environments force companies to explore new means to make better, safer and more complete decisions. In times of crisis or normal conditions, firms opt for collaboration or aggressiveness to ensure their safety and enhance their market position. Game Theory evolved from a simple mathematical tool to a strategic decision-making support. This paper uses game-theoretic principles to model and investigate the rational decision making process of two gas companies operating in an emerging market. A mathematical model is proposed for the non-cooperative strategic decision making process with a reflection on the numerical results.

Keywords
Game Theory, capacity expansion, gas industry, industrial engineering, supply chain management.

1. Introduction
The industrial gas sector has witnessed a fast evolution process towards the end of the last century. Crises have affected decisions taken by companies operating in the sector and limited them to either enforcing one's market position by being more aggressive or collaborating by sharing knowledge and technologies. Studies cited by Moorthy in his report show that an increasing number of companies started to consider their competitor's decisions when selecting their own (Moorthy 1985). This aspect is added to the usual economic studies firms conduct when developing a new product, planning a strategic decision as capacity expansion or any tactic that might be affected by a competitor's choices. Game theory is used for investigating these situations. It is defined by Arsenyan et al (2011) as a branch of mathematics that explores the aspects of conflict and collaboration arising among companies or "players".

In general, capacity expansion is considered as an investment decision that might be conducted by analogical decisions taken by the competitors and for which the company needs to analyze the impacts on its strategic planning. Capacity expansion might touch production, warehousing or transportation. An investment strategy entails a sequence of tactical investment projects, for which the selection decision is based on the return generated and made following the strategies adopted or projected by the competitors.

In the gas industry, strategic planning usually involves taking into considerations the characteristics of certain markets in terms of demand increase, entry of new players, and so on. In advanced economies, oligopolies tend to encourage collaboration among firms to share a common vision for a specific path. This leads to reducing their mutual impacts on each other and maximizing their revenues. In emerging markets however, companies still follow a competitive approach. This paper presents a binary integer model through which the decision making of an industrial gas company is analyzed, and then applies game-theoretic principles to mathematically model the decision making process of two firms in the industrial gas sector of an emerging market. This paper is organized as follows: the next section gives an outlook on the available literature on competitive game theory. Section 3 introduces the mathematical models used in this paper's application. Section 4 presents the application of the models to the two companies operating in the industrial gas sector and discusses the results of the models, before concluding the paper with some remarks on future studies.

2. Literature Review

Game theory was formally introduced by Von Neumann and Morgenstern in 1944 and started showing its efficiency in various domains by the late 1980s. A typical game, as briefly described by Cachon and Netessine (2003) is composed of three elements: the *players*, who represent the agents having stakes in the game, their (set of) *strategies*, and their corresponding *payoffs*. A player's strategy is "the complete instructions for which actions to take in the game" (Cachon & Netessine 2003) in a way that each player aims at successfully achieving its strategy knowing that other players are acting the same way. The payoff received by a player depends on the set of strategies that each player chooses, in a way that each one of them is presented with optimal strategies (Bratvold & Koch 2011). A game, however, can be played in two different forms. In the normal form, players can choose their strategies concurrently and implement the actions corresponding to the selected strategies. In the extensive form, players take actions in a sequential way, learning therefore from the actions taken previously by other players. The majority of games reviewed in this section concerns players who are fully aware of their competitors, their strategies and the corresponding payoffs. This form is called a game of perfect information. Additionally, the example presented in this paper is a zero-sum game where the gain of a player implies a loss for the others.

Aumann (1959) and Shubik (1962) were the first to differentiate between *cooperative* and *non-cooperative* game theory. When power and politics create situations in which businesses or players are prompt to form coalitions, we deal with cooperative game theory (Turocy & Von Stengel 2001). In such setting, the players are able to commit to choose a set of strategies that will maximize their payoffs. In a non-cooperative game however, the players cannot take such decisions; they act independently to maximize their own payoffs (Cachon & Netessine 2003).

In a cooperative game, players usually view collaboration as a mean to share knowledge, resources, standards or technologies in order to alleviate a certain obstacle (Arsenyan et al 2011). Extensive research has been done in this area where, for example, Cai and Kock (2009) have investigated the possibility and the amount of collaboration for R&D and product development firms, using the prisoner's dilemma with social punishment. Xiao et al shed light on the decision making issues that occur between engineering teams in product development where they used principally a game theoretical approach to model the relationship between the teams (Xiao et al 2005). Applications in health care, accounting, manufacturing, and much more have proven useful to stakeholders by providing insights on matters other economical tools failed to enlighten. Arsenyan et al (2011) inspected the dimensions to consider for collaboration on product development. They produced a mathematical model to capture the aspects of trust, coordination, co-learning and co-innovation as parameters, the revenue generated by the innovation as well as the cost of collaborating. Another application in accounting concerned the cost allocation for insurance companies. Lemaire (1984) examined how cooperative game theory principles could be used to allocate costs between departments in insurance companies, using the Shapley value and the Nucleolus principle.

In a non-cooperative game, players try to choose their best payoff, which constitutes a candidate solution for the game. When all the players choose their best payoffs, the solution becomes a Nash Equilibrium (Cachon & Netessine 2003).

On another hand, non-cooperative game theory is concerned with the strategic choices that rational, intelligent firms make when competing autonomously (Moorthy 1985). Firms' rationality comes from the fact that they choose to maximize their revenues, while their intelligence is illustrated in their acceptance of other firms' rationality (Moorthy 1985). He applies non-cooperative game theory principles to two airline companies and justifies the need for businesses to go for equilibrium points. He asserts that a Nash equilibrium is necessary for firms to be comfortable with their strategies and the assumptions it foregrounds concerning other players of the game (Moorthy 1985). An interesting application was done by Nadeau (2003) who explored the dynamics that prevailed in the health care sector, especially between managers and workers. She uses non-cooperative game theory to model conditions for cooperation between managers and workers at the health and safety sector (Nadeau 2003). A Pareto equilibrium was found for the game and asked for both players to improve their efforts as a mean to enhance cooperation (Nadeau 2003).

Han and Ankum (1993) used the real options approach for project timing to analyze aspects of competition. Using game-theoretic principles, they propose various investment tactics in an oligopolistic market. The proposed numerical examples illustrate solving the timing problem under competition.

Murphy and Smeers (2005) consider three models of investments in capacity assuming respectively, a perfect competitive equilibrium, an open-loop Cournot game extended to include investments in capacity, and a last model based on closed-loop Cournot game. The authors pointed out that one of the results derived from the comparative analysis between the three models is that the prices and quantities produced in the closed-loop game, when the solution exists, fall between the prices and quantities in the open-loop game and the competitive equilibrium. Reynolds (1987) uses a model based on Cornaot-Nash equilibrium to model capacity investment in competitive market.

In this paper, we use game theory for setting a capacity investment strategy and, based on the reaction of competitors, we propose various investment tactics.

Our main contribution is, first, the development of a binary integer decision making model for capacity investment. Second, we develop a game theoretical model for the strategic choice of a firm operating in the industrial gas sector which investigates strategies of the capacity investment to follow given a set of constraints. To the best of our knowledge, a decision making model based on game theory has not been developed for capacity investment for which the industrial gas sector serves as an application to this point of time. The next section discusses the aforementioned mathematical models.

3. Mathematical models

3.1 Modeling the Strategy Choice

The main concern of a company in the industrial gas sector who is considering a set of strategies is to unveil the best decision/strategy to follow. In other words, from a set of strategies indexed by *i*, the company needs to know which one(s) will generate the biggest profits. We propose the following binary integer model which tries to maximize the revenues generated by the strategies *i*:

$$\text{Max } Z = \sum_i R_i * X_i \tag{1}$$

Where X_i is the binary variable taking value 1 if strategy *i* is selected and value 0 otherwise, and R_i represents the revenues generated by strategy *i*. The model is subject to the following constraints:

$$\sum_i C_i * X_i \leq I \tag{2}$$

$$\sum_i X_i \leq 1 \tag{3}$$

$$\sum_i K_i * X_i \geq \hat{K} \tag{4}$$

Equation (2) regards the costs C_i incurred when investing in strategy i. The sum of these costs should be limited to the budget *I* allowed for this decision. Equation (3) ensures that only one of the strategies from the whole set will be applied. The last equation (4) ensures that the sum of capacities K_i increased by strategy *i* meets at least the desired capacity $\hat{K}$ that the company wants to possess. We shall not forget to include the non-negativity constraint to this model.

3.2 The Game Model

According to Moorthy (1985), competition in the real world is more of an oligopoly. In other words, markets are the scene of close competition among a few firms, which eventually leads to the concern among firms of what the other is planning to do. In a non-cooperative zero-sum game, 2 or more players compete to obtain the largest payoffs. This paper adopts the strategic form to represent the game. As reported by Moorthy (1985), this form allocates a single set of strategies to each player, with its subsequent payoffs. It is important to note though that since the player of interest has the right to act first. The general form of the model is presented in equation (5):

$$\varphi = (N, (C_j)j \in N, (u_j)j \in N) \tag{5}$$

Where N denotes the set of players in the game such that N = {Player1, Player2 …}, and j designates a single player belonging to the players' set. C_j represents the set of strategies that player j can choose from, where C_j = {Strategy1, Strategy2…}. Finally, u_j designates the payoff that player j acquires when an outcome is possible, such that u_j = {Payoff1, Payoff2…}. The possible outcomes of a game are generally summarized in a graphical or tabular format, as shown in figure 1, where the intersection of two players' strategies represents a payoff for the two players. Yet this only holds in the case of zero-sum games where a gain for one player implies a loss for the other.

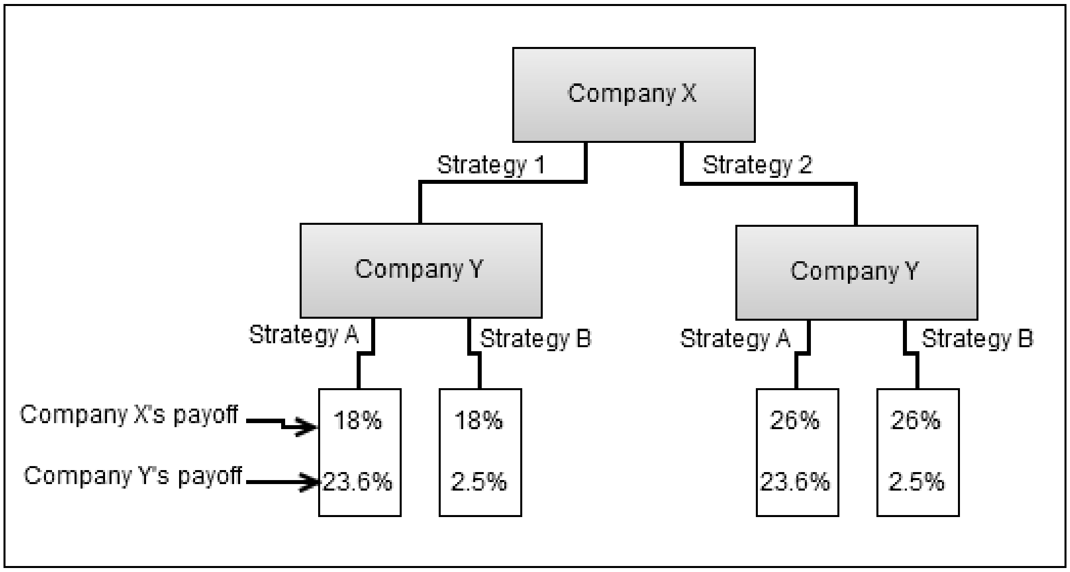

Figure 1: with Companies X and Y

4. Numerical Application

4.1 General Framework of Gas Industry

The main products offered by companies operating in the industrial gas sector are commonly grouped under three categories. First, industrial gases comprise hydrocarbon gases, which include acetylene, propane, butane and others, and compound gases which involve carbon dioxide, nitrous oxide and hydrogen chloride as the most prominently known ones. The second category, medical gases, includes nitrogen, oxygen and argon. The last set, specialty gases, is concerned primarily with noble gases such as neon and xenon. For this sector, customers vary from hospitals to food processing companies and other firms operating in the aeronautics, glass, paper or automobile industries.

The companies often have various means of distributing their products; that are categorized as continuous, bulk or unit transportation. Pipelines are the principle mean for continuous transportation. It allows distribution of massive amounts of gases on a constant basis. Bulk distribution regards the use of railroads, ships or trucks to transport gas tanks. The unit transportation considers the use of cylinders or bottles to distribute gases, principally to final customers. To accompany the distribution process, a set of technological installations are required on filling centers. The latter represent an intermediary unit that links conditioning centers with final customers. A basic filling center needs to be equipped with a storage capacity (tanks and cylinders), high-pressure pumps for extracting gas from cylinders and filling it into bottles using filling ramps.

In the gas industry, important clients such as hospitals or soft drink companies demand high quality products. In emerging markets, products tend to follow one of many standards; local or international, with the latter being more adulated.

4.2 General Presentation of the Application

Company X is facing a downsizing in its market share because its direct competitor, company Y, has recently adopted products with higher standards and undertaken a series of plant enlargements. The market in which both players are competing is an emerging one with an increase in annual demand forecast for industrial gases of 17%, and medical gases of 15%. Company X recognizes the urgent need for increasing its capacity to face the competition. The company has two strategies:

- *First strategy*: Company X can acquire a smaller company operating in the same market at 8.3 million US $. The company can benefit from the already existing infrastructure and the time saved from constructing new sites.
- *Second strategy*: Company X can increase its available capacity for the main gases that are directed to the health care and agribusiness sectors; namely oxygen, nitrous oxide and carbon dioxide.

Company X owns two production sites. The first one is called HD and the second is BC. For the second strategy, company X can only expand in one of the two sites because of limitations in its investment capabilities. It is important to note that each site has certain regulations that prevent it from expanding more than a certain limit. Additionally, company X can increase capacity of a certain gas in both sites. The following table explains the required resources for investing in each site along with the revenues generated by each expansion:

Table 1: Company X capacity expansion options

Site	Carbon dioxide		Oxygen		Nitrous Oxide		Revenues generated (annually)
	Capacity increase	Investment cost	Capacity increase	Investment cost	Capacity increase	Investment cost	
HD	14%	2.9 million US $	18%	117,000 US $	23%	1.4 million US $	1.5 million US $
BC	12%	3.5 million US $	20%	1.45 million US $	17%	1.64 million US $	2.3 million US$

In Table 1, we notice that company X can only invest in two sites: HD and BC. Investing in Carbon Dioxide is limited to an increase of 14% in HD and 12% in BC with respective investment costs of 2.9 and 3.5 million dollars. Investing in Oxygen is limited to a maximum increase of 18% in HD and 20 in BC with investments reaching 117,000 and 1.45 million dollars. The revenues generated by the total investments in each site are 1.5 for HD and 2.3 million dollars for BC, on average and annually.

The first question to investigate concerns the best option to select from the two strategies. The second question, which touches the essence of this paper, is similar to the first but takes into consideration the strategies of company Y. The following section will answer the two questions.

4.3 Application of the Strategic Choice Model

For answering the first question, we refer to the binary integer model in equations (1) to (4). Using the same notation, we identify the following strategies:

- X_{1ONC}: investing in acquiring the small company,
- $X_{2O,N,C}$: investing in capacity increase on site BC in oxygen or nitrous oxide or carbon dioxide,
- $X_{3O,N,C}$: investing in capacity increase on site HD in oxygen or nitrous oxide or carbon dioxide.

Parameters for the model are described as follow:

- R_i: revenues generated annually by investment i,

- C_i: cost incurred after investment i,
- K_j: required capacity increase in gas j, where j=O, N, C or ONC for the case of acquisition.

The required capacity parameter relates to the level of gas production that company X wishes to reach in order to level up to the competition. A market study was performed by the company to forecast the demand in prominent medical and industrial gases; namely oxygen, nitrous oxide and carbon dioxide. The following table summarizes the findings of the study:

Table 2: required capacity increase per gas

Gas	Oxygen	Nitrous Oxide	Carbon Dioxide
Forecasted demand increase	11.72%	12.34%	12.98%
Required overall capacity increase	15%	15%	13%

From Table 2, we can see that company X predicts that demand for Oxygen will augment by 11.72% over the coming year and that accordingly a 15% increase in its corresponding capacity is necessary to satisfy this demand. The same applies to Nitrous Oxide and Carbon Dioxide. We need however to change the model to take into consideration the fact that the problem's strategies are related to the company's sites, thus equation (1) becomes

$$\text{Max } Z = \sum_i \sum_j R_i * X_{ij} \tag{6}$$

Subject to the following constraints:

$$\sum_i \sum_j C_i * X_{ij} \leq I_j \tag{7}$$

$$X_{1ONC} + X_{2j} \leq 1 \tag{8}$$

$$X_{1ONC} + X_{3j} \leq 1 \tag{9}$$

$$\sum_i \sum_j K_j * X_{ij} \geq \hat{K}_j \tag{10}$$

$$X_{ij} = 0 \text{ or } 1 \tag{11}$$

Equation (7) ensures that the cost of the strategies chosen does not exceed the maximum investment budget. Equations (8) and (9) make sure that company X either acquires the small company or invests in capacity increase on the HD or BC site. Equation (10) ensures that the overall investment of company X in the chosen strategies should meet the desired capacity increase presented in table 2.

4.4 Results of the Strategy Choice Model

After running the model using excel premium solver, an optimal solution was found with the following considerations:

- Company X has to construct an additional oxygen unit and nitrous oxide unit in both sites (HD and BC), and one carbon dioxide unit in HD.
- Capacity will be increased optimally by 30% for oxygen, 40% for nitrous oxide and 14% for carbon dioxide, which meets by far the desired capacity increase.
- Total revenues attain 9.1 million US $ with a total investment cost of 8.6 million US $.

The next step is to investigate whether taking into consideration the strategies of company Y would affect the results of our model. The next section discusses the use of non-cooperative game theory to model this situation.

4.5 Game Model and Assumptions

There exists other companies in the market where companies X and Y compete, yet their market share is by far smaller than those owned by our two players. This leads to assumption 1 that this is a non-zero sum game. As for the information possessed by the two companies, we assume that the following conditions apply to both players:

- Each player is aware of the rules of the game; that is the strategies it has to decide for along with the strategies of its competitor,
- Players possess what Moorthy (1985) asserted as rationality and intelligence,
- Finally, players are aware of the history of their past choices and decisions.

Consequently, this game is of perfect information. Concerning the strategies for each player, company X can choose one of the following:

- *Strategy 1*: acquiring a smaller company. This will engender an estimated market share increase of 18%.
- *Strategy 2*: expansion by building additional production units, to improve the market share by 26%.

On another hand, company Y is considering the possibility of engaging in a strategic expansion by opening new production sites and increasing its current capacity. This strategy will be named Strategy A and is supposed to engender a market share increase of 23.6%. Strategy B happens if company Y does not succeed in undertaking Strategy A, especially if the market faces a demand decrease because of the cancellation of some of the medical and industrial projects that were promising a bigger demand. Still, company Y will be able to undertake the capacity increase project with a potential 2.5% increase in its market share. In the context of this paper,

- N = {1, 2} = {Company X, Company Y}
- C_1 = {Strategy 1, Strategy 2} = {Acquire small company, build additional production unit}
- C_2 = {Strategy A, Strategy B} = {Expand sites and units, expand on units only}

The possible outcomes for this game are summarized in Table 3. Payoffs for each combination of two strategies are shown inside their corresponding cells.

Table 3: Game in tabular form

Company X Strategy set	Company Y Strategy set	
	Strategy A	**Strategy B**
Strategy 1	-23.6 + 18= -5.6	18 - 2.5 = +15.5
Strategy 2	-23.6 + 26 = +2.4	26 – 2.5 = +23.5

Efforts for a company to pursue a path may become vain if it does not take into consideration the actions of its competitors. Such a tactic may turn out to be effective for companies that operate in close competition. For this paper's case, there are four possible outcomes that arise as presented in Table 3:

- If company X decides to acquire a small company while company Y opts for its strategic expansion decision, company X will lose at least 5.6% of its current market share.
- If company X selects to change its strategy for the expansion by building supplementary production units, while company Y still holds for its Strategy A, company X will be able to level up to its competitor and acquire an additional 2.4% of the market share.
- If company X chooses to acquire the small company while company Y is unable to complete its full strategy (chooses Strategy B), company X will achieve an additional 15.5% market share.
- Otherwise, if company X expands by building additional production units with company Y still being unable to fully expand (chooses Strategy B); the former will realize an additional 23.5% market share.

Thus, we recommended company X to choose its second strategy (building supplementary production units) in the given context, with a recommendation to reassess the quality and number of potential strategies.

4.6 Discussion of the Results

From first sight, company X seriously needs its competitor to go for Strategy B in order for it to increase its market share significantly; otherwise, it will not be able to achieve more than 2.4% increase in its market share. Yet company Y 's strategy B is purely governed by random events and might not be affected by any tangible decision.

On another hand, company X can still overthrow any strategy that company Y decides to follow by opting for building additional production units on its sites. These results confirm the findings of the previous model that did not consider the actions of competition. The binary integer model only presents the requirements for company X to meet certain constraints and to achieve a certain profit, regardless of what is happening in the market. The second model takes this analysis one step further and takes into consideration the strategies undertaken by company X's most direct competitor, company Y. The game model permits to behold the additional threats that a normal decision making model cannot unveil. This way, company X can consider other strategies or even collaborating with smaller companies.

There are still ambiguities that this game holds, mainly in terms of its assumption and the potential technology that the competitor possesses. Since it is not question of cooperation or collaboration, this model advises to select the strategy that guarantees the best payoff possible, in the worst scenario cases. This is known as the Max-Min criterion, where company X ought to maximize its own minimum payoffs (Turocy & Von Stengel 2001). There is however no possibility for the two companies to cooperate for a Nash Equilibrium based on the available information, even if such kind of cooperation could yield better payoffs. This type of cooperation is similar to the cases described by Nadeau (2003) and Arsenyan et al (2011).

5. Conclusion

In this paper, we demonstrate the need for business firms to consider competitor's actions in oligopolies. The aims and strategies of a company might interfere with its competitors', leading to conflict and lower returns. The second model presented embodies the non-cooperative game played by the two companies operating in the industrial gas sector. This application considered the different strategy sets that each player is able to select, in the context of a zero-sum game with perfect information. Results showed that the company of interest has to either invest in expanding its current production capacity, or to investigate other strategies. These results were taken from both models as the second one proved that it is threatening to make strategic decisions without taking into consideration the potential strategies of the direct competitors. In case of an economic crisis, this model can be tuned to provide an understanding of the dimensions of collaboration between the two firms, either in research and development, promotions or the like, as investigated by Arsenyan et al (2011). Further study will provide insight for the same aspects on expansion decisions.

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