Inventory Control for Defective Goods Supply Chain by Determination of Production Period Length

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Abstract

In today's competitive market, time and cost are two essential factors that manufacturers should take into consideration in their supply chain management. Therefore, the selection of appropriate length of each time period is critical to ensure competitive delivery time. In our study, a mathematical model for defective products supply chain that determines the most appropriate length of each period is proposed. In addition the model also optimizes costs and economic production quantity (EPQ). In this model, defective goods are repairable and some of them are considered as scrap, and after reworking the scrap goods are sold at a low price. The model presented is novel and we used LINGO to solve problems. The correctness and the fine function of the model has been ascertained using sample problems.

Keywords
Supply chain management, defective goods, economic production quantity, LINGO

1. Introduction
In any global market competition, time and cost are two important factors that manufacturers should take into consideration in their supply chain. Therefore, the selection of an appropriate length of each time period is an important aspect in a competitive market which many researchers do not pay much attention to. In a production system many defective goods were also produced by manufacturers, due to various factors such as imperfect quality and defective raw materials. So, they need facilities for reworking the defective goods produced. In most cases, reworking can be very profitable if the cost of production is too high or the raw materials are limited and costly.

In recent years, numerous methods have been proposed for improvement in efficiency of the supply chain. For instance, Vijay and Keah (2005) investigated the correlation between three features, Total Quality Management (TQM), Supply Chain Management (SCM) and Just in Time (JIT) to see their impact on the efficiency of business. They observed that simultaneous application brings synergy and enhanced the effectiveness of companies. Ruilin and Tang (2008) proposed a unified model for the optimization of the supply chain based on JIT. The results of their research confirmed the accuracy of optimization of the costs.

On the other hand, studies on inventory control models that oversee the basic concepts of warehouses and imperfect products after manufacturing had also attracted researchers (Kun-Jen et. al 2009; Jef et. al 2005). Simme and Ruud (2004) considered defective products by single item manufacturers who used the same facilities and production machines for reworking. Abdollah and Gultekin (2007) developed a model that considered defective goods and the ordered amounts which were returned as shortages. However, their model only solved numeral examples which showed that defective and useless goods lowered the overall profits. On the other hand, Hong et al. (1993) had studied the relationship between process quality and the investment to reduce set up time in a dynamic state. Another model that is regularly adopted is the economic production quantity (EPQ) model for defective goods (Jamal et.al 2004, Lee et. al 1997). However, such models do not consider reworking function of the defective goods. Salameh and Jabber (2000) developed an EPQ model, assuming that some of the goods produced are defective; but they did not pay explicit attention to the sale of defective goods in their model. Papachristos and Konstantaras (2006) modified Salameh's model, while Goyal and Cardenas-Baron (2002) presented a model simpler than Salameh's model in their research and observed improved results. Wahab and Jaber (2010) presented an economic order quantity (EOQ) model for defective goods, which showed that the defective goods and shortage reduced the overall profit.

In our paper, we propose a mathematical model for defective goods supply chain that has comprehensively included total costs such as cost of production, maintenance, transport, defective goods, scrap products, shortage in retailers and indirect cost. In addition, the uniqueness of our model is that it can be used to determine the most appropriate length of time period and economic production quantity which will enable manufacturers to decide between cost and length of time period.

2. Supply Chain Model

Our supply chain model aims to optimize the global costs of the defective goods. The supply chain model is divided into three levels (Figure 1). The first level (**Level 1)** is manufacturers. The manufacturers can receive multiple types of incoming parts, process them in mixed model or batch mode production and after that send the goods to the next level. During the manufacture of the products, some of the products may not be properly manufactured and are considered as defective goods. In most cases, some of the defective products can be repaired, while others are considered as scrap.

Receiving the goods produced by manufacturers are the distributors (**Level 2**). The distributors will keep the goods in their warehouse before sending them to various retailers. During storage in the warehouse, the longer the storage time the higher the rent or increase in the unnecessary inventory level. So, it is important to schedule the appropriate time to process each time period so that these cost can be minimized. The third level (**Level 3**) comprises of the retailers. In this paper, the proposed model is adapted to optimize costs including production, maintenance, shipping, reworking on the defective goods, scrap products and shortage in retailers due to the production of defective goods, indirect costs and income due to sale of scrap goods. It also determines the most appropriate length of the production period.

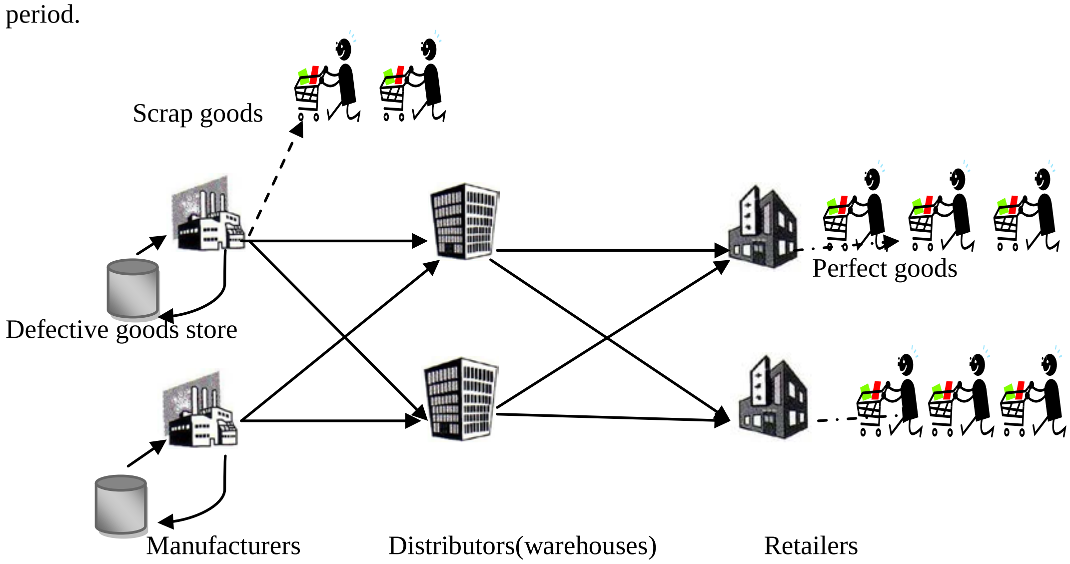

Figure 1: Three level of supply chain network

2.1 Assumptions

Since our model has three levels of supply chain network and multi products, we used the following assumptions to facilitate the model:

- The amount of demand was given to the manufacturers at the beginning of the period and duration of each period was fixed and clear.
- The duration of each period was equal to total of production time and rework time and shortage was allowed.
- The model was designed for multi-manufacturer, multi-distributor and multi-retailer.
- The parameters used were fixed for the demand rate, reworking time, production time, percentage of defective products, percentage of scrap products and prices.
- Inspection operation was perfect and inspection time was intended to be zero.

2.2 Model constraints

In our model, there were several constraints or limitations. First, the manufacturers supply capacities and the total warehouse capacities are limited. Second, for each product the storage allocated capacities are limited. Third, for each defective goods the store capacities and the storage allocated capacities are limited. Fourth, for each period, the customer demand should be provided. Finally, the limitation of production time and reworking time were also considered.

2.3. Decision variables and parameters

In this model, we defined the decision variables and parameters that will be used in cost optimization by linear programming software called LINGO as follows:

Q_{ilt} : Economic production quantity of product *l* by factory *i* during period *t*.
Def_{ilt} : Defective products produced *l* by factory *i* during *t*.
Sc_{ilt} : Scrap products produced *l* by factory *i* during *t*.
Co_{ilt} : Perfect products produced *l* by factory *i* during *t* before reworking.
TCo_{ilt} : Perfect products produced *l* by factory *i* during *t* after reworking.
pc_{ilt} : Production cost of per item by factory *i* during period *t*.
p_{ilt} : Percentage of defective goods *l* produced by factory *i* during period *t*.
γ_{ilt} : Rework cost of per defective goods *l* by factory *i* during period *t*.
h_{ljt} : Holding cost of product *l* in distributor *j* during period *t*.
h'_{ilt} : Holding cost of product *l* in defective goods store inside the factory *i* during period *t*.
θ_{lit} : Time required to production of per goods *l* by factory *i* during period *t*.
$T\theta_t$: Total of production time during period *t*.
μ_{lit} : Time of reworking required for goods *l* by factory *i* during *t*.
$T\mu_t$: Total of reworking time during period *t*.
X_{ijlt} : Amount of product *l* which is transported from factory *i* to distributor *j* during period *t*.
Y_{jklt} : Amount of product *l* which is transported from distributor *j* to retailer *k* during period *t*.
Tc_{lijt} : Shipping or transportation cost each product *l* from factory *i* to distributor *j* during period *t*.
$T'c_{ljkt}$: Shipping or transportation cost each product *l* from distributor *j* to retailer *k* during period *t*.
$T''c_{ilt}$: Shipping or transportation cost each defective goods *l* to defective goods store inside the factory *i* during period *t*.
α_{ilt} : Percentage of scrap product *l* produced by factory *i* during period *t*.
f_{ilt} : Production cost of per scrap goods *l* by factory *i* during period *t*.
β_{klt} : The shortage cost for each product *l* in retailer *k* during *t*.
B_{ilt} : The amount of the shortage of product *l* in retailer *k* during period t,
Ind_t : Indirect costs such as stagnant capital, maintenance,
M_{ilt} : The sale price of each scrap goods *l* produced by factory *i* during period *t*.

2.4. Objective function of the model

In this paper, we defined that the objective function of the model was to minimize the total cost of supply chain including the cost of production, maintenance in the distributors and defective goods stores, defective goods reworking, shipping from manufacturers to distributors, shipping from distributors to retailers, shipping from manufacturers to defective goods stores, also from defective goods stores to manufacturers, the cost of scrap goods production, shortage in retailers because of defective goods production, indirect costs and income due to sale of scrap goods. The objective function is shown as the following equation.

$$Z_{min} = \sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{t=1}^{T} Pc_{ilt} \,.\, Q_{ilt} + \sum_{j=1}^{J}\sum_{l=1}^{L}\sum_{t=1}^{T} h_{jlt} \,.\left(\sum_{t'=1}^{t'\neq T}\sum_{i=1}^{I} X_{ijlt'} - \sum_{t'=1}^{t'\neq T}\sum_{k=1}^{K} Y_{jklt'}\right) + \sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{t=1}^{T} h'_{ilt} \,.\, Def_{ilt} + \sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{t=1}^{T} \gamma_{ilt} \,.\, Def_{ilt} + \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{l=1}^{L}\sum_{t=1}^{T} TC_{ijlt} \,.\, X_{ijlt} + \sum_{j=1}^{J}\sum_{k=1}^{K}\sum_{l=1}^{L}\sum_{t=1}^{T} T'C_{jklt} \,.\, Y_{jklt} + \sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{t=1}^{T} T''C_{ilt} \,.\, Def_{ilt} + \sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{t=1}^{T} f_{ilt} \,.\, Sc_{ilt} + \sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{t=1}^{T} \beta_{ilt} \,.\, B_{ilt} + \sum_{t=1}^{T} Ind_t . T_t - \sum_{i=1}^{I}\sum_{l=1}^{L}\sum_{t=1}^{T} M_{ilt} . Sc_{lit} \quad (1)$$

The constraint of Equation (1) is as follows:

$$\sum_{i}^{I}\sum_{l}^{L} X_{ijlt} \leq W_{jt} \quad \forall\, j,t \quad \& \quad \sum_{i}^{I} X_{ijlt} \leq W_{jlt} \quad \forall\, j,l,t \quad (2)$$

Equation (2) represents the limitation of delivery capacity of distributors.

$$\sum_{j}^{J}\sum_{l}^{L} Y_{jklt} \leq W_{kt} \quad \forall\, k,t \;\&\; \sum_{j}^{J} Y_{jklt} \leq W_{klt} \quad \forall\, k,l,t \quad (3)$$

Equation (3) represents the limitation of delivery capacity of retailers.

$$Def_{ilt} = p_{ilt} . Q_{ilt} \quad \forall\, i,l,t \quad (4)$$

Equation (4) represents the amount of defective goods.

$$Sc_{lit} = \alpha_{ilt} . Def_{ilt} \quad \forall\, i,l,t \quad (5)$$

Equation (5) represents the amount of scrap goods.

$$Co_{lit} = (1 - p_{ilt}) . Q_{ilt} \quad \forall\, i,l,t \quad (6)$$

Equation (6) shows the amount of perfect goods before reworking.

$$TCo_{lit} = Co_{lit} + Def_{ilt} - Sc_{lit} \quad \forall\, i,l,t \quad (7)$$

Equation (7) represents the total of perfect goods after reworking.

$$Q_{ilt} = Co_{ilt} + Def_{ilt} \quad \forall\, i,l,t \quad (8)$$

Equation (8) shows the total of production.

$$\sum_{k=1}^{K} d_{klt} + \sum_{i=1}^{I} Sc_{ilt} \leq \sum_{i=1}^{I} Q_{ilt} \;,\; TCo_{lit} \geq \sum_{k=1}^{K} d_{klt} \quad \forall\, l,t \quad (9)$$

Equation (9) describes the supply of the all demand.

$$\sum_{k=1}^{K} B_{klt} = \sum_{k=1}^{K} d_{klt} - \sum_{i=1}^{I} Co_{lit} \quad \forall\, l,t \;,\; \sum_{i=1}^{I} Co_{ilt} \leq \sum_{k=1}^{K} d_{klt} \quad \forall\, l,t \quad (10)$$

Equation (10) investigates the amount of shortage because of defective products.

$$\sum_{i}^{I} X_{ijlt} = \sum_{k}^{K} Y_{jklt} \quad \forall\, j,l,t \quad (11)$$

Equation (11) explains that the inventory of warehouse is zero at the last period.

$$\sum_{j=1}^{J} Y_{jklt} = d_{klt} \quad \forall\, k,l,t \quad (12)$$

Equation (12) considers that the inventory of retailers is not more than demands.

$$\sum_{i=1}^{I}\sum_{t'=1}^{t'\neq T} X_{ijlt'} \geq \sum_{j=1}^{J}\sum_{t'=1}^{t'\neq T} Y_{jklt'} \quad \forall\, j,l,t \neq T \quad (13)$$

Equation (13) shows the inventory of warehouse before the last period.

$$\sum_{j=1}^{J} X_{ijlt} \leq TCo_{ilt} \quad \forall\, i,l,t \quad (14)$$

Equation (14) shows the amount of scrap products that exits from system.

$$\sum_{i=1}^{I}\sum_{l=1}^{L} \theta_{ilt} . Q_{ilt} \leq T\theta_t \quad \forall t_1 \quad (15)$$

Equation (15) explains the limitation of available times for all production.

$$\sum_{i=1}^{I}\sum_{l=1}^{L} \mu_{ilt} \,.\, Def_{ilt} \leq T\mu_t \quad \forall\, t \quad (16)$$

Equation (16) shows the limitation of available times for reworking.

$$T\theta_t + T\mu_t = T_t \quad \forall\, t \quad (17)$$

Equation (17) represents the length of each period.

$$X_{ijlt} \,, Y_{jklt}, Q_{ilt} \,, B_{klt} \geq 0 \quad \forall\, i,j,k,l,t$$

(18)

Equation (18) explains the amount of productions, delivery to warehouses and retailers, shortage in retailers should all have positive values.

3. Results and Discussions

The proposed mathematical model was investigated using the linear programming tool LINGO. It is a comprehensive tool designed to solve linear, nonlinear, quadratic, stochastic, and integer optimization models. The aim of using LINGO was because it can solve faster, easier and was efficient in terms of calculation time. The findings of model were the most appropriate length of each time period, the amount of scrap goods, defective goods, perfect goods before repair, perfect goods after repair which are equal to amount of demand, shortage in retailers, economic production quantity, and global optimized costs.

LINGO was used to analyze 9 sample problems with the same dimensions (3 manufacturers, 2 warehouses, 3 retailers, 1 kind of product and 1 period) and the same parameters, except production time ($T\theta_1$) and repair time ($T\mu_1$) (Table 1). The results illustrate production time of period 1 ($T\theta_1$), repair time of period 1 ($T\mu_1$), length of period 1 ($T\theta_1 + T\mu_1 = T_1$), total of scrap goods for each problem ($\sum SC$), total of defective goods ($\sum Def$), tot al of perfect goods before reworking ($\sum CO$), total of perfect goods after reworking ($\sum TCO$), shortage in retailer ($\sum$ B), economic production quantity ($\sum Q$) and total of global optimize costs (Z_{min}).

Table 1: Model output for the nine sample problems with same dimensions

No. problem	$T\theta_1$	$T\mu_1$	T_1	$\sum Sc$	$\sum Def$	$\sum Co$	$\sum TCo$	$\sum$ B	$\sum Q$	Z_{min}
1	51950	1730	53680	302	865	16437	17000	562	17302	589951
2	55000	2000	57000	307	902	16405	17000	594	17307	589354
3	59000	2500	61500	317	970	16347	17000	653	17317	588136
4	63000	3000	66000	325	1031	16294	17000	705	17325	587095
5	66307	3307	69614	332	1080	16251	17000	748	17332	586254
6	69000	4000	71000	332	1080	16251	17000	748	17332	586523
7	73000	4500	77500	332	1080	16251	17000	748	17332	586923
8	77000	5000	82000	332	1080	16251	17000	748	17332	587323
9	81000	5500	86500	332	1080	16251	17000	748	17332	587723

The main objective of this model was to optimize the total costs with consideration to the most appropriate length of each time period. The model output showed that the total costs (Z_{min}) were correlated with the length of period (T_1). The model, considering the internal parameters, optimized the total cost (Z_{min}) with an increase in the length of the time period. In order to supply the demands of retailers we need the least time for production that is shown in sample problem No. 1. So, with increment in time the total cost can be decreased, with respect to the internal parameters of this study the length of period in problem No. 5 was the best time for length of time period, because model had enough time to select the best way to find the optimize cost (Z_{min}). Any further increase in the time increased total costs because of indirect cost like cost of stagnant capital, and maintenance. From the relationship between total cost and length of period shown in Figure 2, it can be ascertained that the best length of period was between [T_a , T_b] which is dependent on the significance of demand order time.

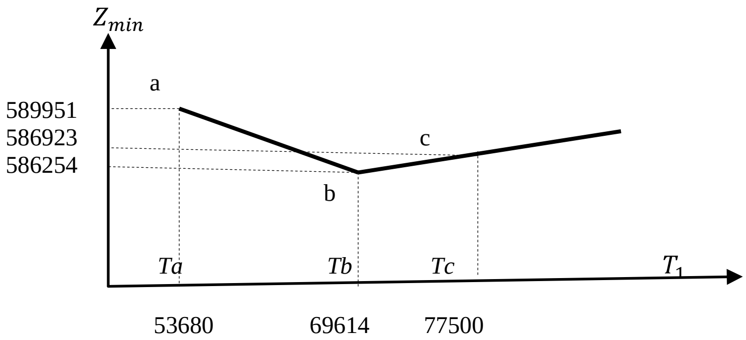

Figure 2: Relationship between length of time period and Z_{min}

5. Conclusion

A solvable mathematical model using LINGO as the linear programming software is proposed for the optimization of costs, economic production quantity and to determine the most appropriate length of time period for defective goods supply chain. The proposed model was able to optimize costs such as, production, maintenance, shipping, reworking on the defective goods, scrap products, shortage in retailers due to the production of defective goods. The model was able to determine the active manufacturer, distributor and the appropriate period of production. The results of the model output for the nine sample problems showed correctness and fine function of the model. The model is applicable for all producers who faced with problem of defective goods.

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