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A Model for Optimizing the Multi-Item Aggregated Order Quantity in a Two-Level Supply Chain: Centralized and Decentralized Perspective

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Abstract

Any industry has its own supply chain where it is not necessary to have all stages in the supply chain and the optimal ordering policies of those individual stages may be different. In this paper we want to extend the two–level supply chain inventory model discussed in related works. The main consideration of this paper is to determine the optimal ordering policy according to inventory and transportation costs in a two stage supply chain consisting of warehouse and retailer in case of multiple–items. The evaluation is based on centralized and decentralized decisions for a retailer and a single warehouse. A model is developed and finally a case study is presented to demonstrate the effectiveness of the proposed approach.

Keywords
Order quantity, Centralized & Decentralized decision, Multi–item, Ordering cost, Transportation cost.

1. Introduction
The major goal of every supply chain is to maximize its profit. But in most supply chain the main target of each stage is to maximize its own profit rather than total profit and this is the main theme of decentralized decision. But in centralized decisions all the stages try to increase the total supply chain profitability. Many researches are done for single item order quantity in centralized & decentralized decisions. Baboli et al. (2008) provided an algorithm for single-item ordering policy both in centralized & decentralized cases [3]. This paper represents an extension of their model where multi-items have been considered. The order quantity is determined by considering that all products are ordered & received jointly. So transportation cost from warehouse to retailer will be the multiplication of number of orders per year and transportation cost per order.

2. Literature Review
Many researches are provided by considering a supply chain as a single firm. Thus they investigate centralized model. Silver et al (1998) investigated a supply chain consisting of one warehouse and one retailer in which the external demand rate was known [6]. In supply chain in which the sites belong to independently owned companies decentralized decision is made.

Abdul–Jalbar et al. (2003) compared the effect of centralization versus decentralization on the total cost. They considered a two level supply chain consisting of one warehouse and a number of retailers. By numerical examples they show that as the number of instances in which the decentralized policy is better. In addition, given a number of retailers sensitivity analysis indicates that under specific conditions of the replenishment and holding cost at the warehouse the centralized policy is better [5].

Chen and Chen (2005) present a joint replenishment arrangement with a two echelon supply chain having one supplier or manufacturer and one buyer or retailer, facing a deterministic demand and selling a number of products in the marketplace. They considered a situation involving a major set up cost and minor processing cost. The retailer also has major set up costs due to economies of scale in transportation and distribution expenses and an item specific minor setup cost for each additional item involved in the order. They proposed both centralized and decentralized

decision models to determine the best solution to minimize costs and by a search algorithm they numerically illustrated the benefits generated from such an arrangement [7].
Andersson and Marklund (2000) studied decentralized inventory control in a two level distribution system. They consider a two level distribution system with a central warehouse and N non-identical retailers. All installations use continuous review installation stock (R, Q) policies. They present an approximate cost evaluation technique where the retailers replace their stochastic lead times by correct averages. They introduce a modified cost structure at the warehouse to decompose the multi-level inventory control problem into N+1single level sub problems, one problem per installation [8]. Li and Liu (2006) consider a supplier-buyer system with probabilistic customer demand. They show that quantity discount policy is a way to achieve coordination. They propose a method to divide the profit due to joint decision between the buyer and supplier and the optimal quantity discount policy is obtained by using this profit sharing method [11]. We are now presenting our model.

3. Model Formulation

Different costs concerning with warehouse and retailer are: holding cost, ordering cost, transportation cost, labor cost. Holding cost is the cost for keeping inventory for a certain time. It is denoted by H which is multiple of holding cost rate (h) and product unit cost(C). Fixed ordering cost consists of order receiving and shipping cost which is denoted by S. Transportation cost is the cost for transporting products from one stage to another.

3.1 Assumptions

- Demand rate is constant for the retailer.
- Ordering cost is fixed for a certain amount
- Transportation cost is considered from warehouse to retailer and the warehouse incurs this cost.
- Shortage is allowed neither at the warehouse nor at the retailer.
- The lead time for an order to arrive at the retailer from warehouse is constant.
- The lead time for the warehouse constant.
- There is no lot-splitting at the warehouse.
- Order quantity doesn't exceed the capacity of vehicle.
- EOQ is determined for multi-items individually.

3.2 Notations

- i: Index of an item, $i = 1, \dots \dots, m$
- D_i: Demand rate of the retailer for item i
- $\sum D$: Total demand rate of the retailer for all the items in total.
- S_{ri}: Ordering cost of the retailer for item i
- S_r: Total ordering cost of the retailer.
- S_{wi}: Ordering cost of the warehouse for item i
- S_w: Total ordering cost of the warehouse.
- h_r: Holding cost rate at the retailer
- h_w: Holding cost rate at the warehouse
- $H_r = h_r C_r$: holding cost at the retailer.
- $H_w = h_w C_w$: holding cost at the warehouse.
- C_r: Unit cost of items for retailer.
- C_w: Unit cost of items for retailer.
- T_r: Order cycle time of the retailer
- LT_r: Lead time for the retailer
- T_w: Order cycle time of the warehouse
- Q_{ri}: Order quantity of the retailer for item i
- $\sum Q_r$: Total order quantity of the retailer.
- Q_{wi}: Order quantity of the warehouse for item i
- $\sum Q_w$: Total order quantity of the warehouse.
- C_{tr}: Transportation cost from warehouse to retailer.
- TC_r: Total cost of the retailer

- TC_w: Total cost of the warehouse
- TC: Total system cost
- n: Integer multiple of order quantity.

3.3 Decentralized Case

In the decentralized case, the retailer and the warehouse intend to optimize their own costs independently. Demand rate at the retailer and transportation time to the retailer is supposed to be constant. Shortage is not allowed at the retailer therefore the retailer has a simple EOQ model and inventory level shown in figure 1.

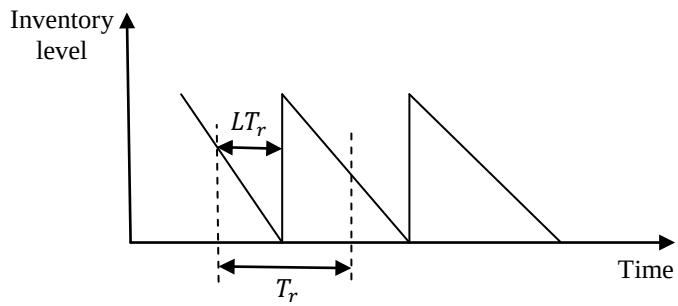

Figure 1: Inventory level of retailer

The total cost of retailer is the sum of holding and ordering costs. The total cost and economic order quantity of retailer are respectively as follows:

$$TC_r(\sum Q_r) = \left(h_r C_r \frac{(\sum Q_r)}{2}\right) + \left(\frac{\sum D}{\sum Q_r}\right) S_r \tag{1}$$

$$Q_{ri}^* = \sqrt{\left(\frac{2S_{ri}}{h_r C_r}\right) D}, \qquad Q_{ri}^* = Q_{r1}^*, Q_{r2}^*, \dots\dots$$

$$\sum Q_r^* = \sum Q_{ri}^* \tag{2}$$

Now for warehouse no shortage is allowed. We consider that the order quantity of warehouse is integer multiple of the order quantity of retailer. For warehouse the maximum level of inventory will be $(\sum Q_w - \sum Q_r)$. Nature of inventory levels at warehouse is shown in figure 2.

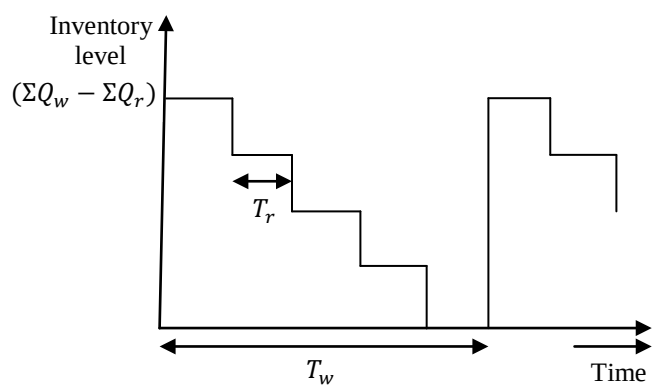

Figure 2: Inventory level of warehouse

In the decentralized case the total cost of warehouse is the sum of the holding and ordering costs plus the transportation cost for multi-items from warehouse to retailer. Thus the total cost of the warehouse is –

$$TC_w(\sum Q_w) = \left(\frac{\sum D}{n \sum Q_r}\right) S_w + \left(\frac{\sum D}{\sum Q_r}\right) C_{tr} + \left(\frac{h_w}{2} C_w\right) \cdot (\sum Q_w - \sum Q_r) \tag{3}$$

$$\text{As, } \sum Q_w = n \sum Q_r \tag{4}$$

$$TC_w(n, \sum Q_r) = \left(\frac{\sum D}{n \sum Q_r}\right) S_w + \frac{h_w C_w (n-1)(\sum Q_r)}{2} + \left(\frac{\sum D}{\sum Q_r}\right) C_{tr} \tag{5}$$

Now setting the derivatives of TC with respect to n equal to zero, the approximate optimal value of n,

$$n^* = \sqrt{\frac{h_r C_r S_w}{h_w C_w S_r}} \tag{6}$$

3.4 Centralized Case

In the centralized case the objective is to find the optimal cost for both the retailer and the warehouse. The total ordering cost for multi-items can be obtained by using following equation:

$$S_r = \sum_{i=1}^{m} S_{ri} \tag{7}$$

The total system cost is the sum of holding and ordering cost at both the retailer and warehouse plus the transportation cost from warehouse to retailer. Thus the total system cost will be –

$$TC(n, \sum Q_r) = \left(\frac{\sum D}{n \sum Q_r}\right) S_w + \frac{h_w C_w (n-1)(\sum Q_r)}{2} + \left(\frac{\sum D}{\sum Q_r}\right) S_r + \left(h_r \frac{C_r (\sum Q_r)}{2}\right) + \left(\frac{\sum D}{\sum Q_r}\right) C_{tr} \tag{8}$$

If we set the derivatives of TC with respect to $\sum Q_r$ and n equal to zero, we get

$$\sum Q_r^* = \sqrt{\frac{2D\left[\left(\frac{S_w}{n}\right) + S_r + C_{tr}\right]}{[h_w C_w (n-1) + h_r C_r]}} \tag{9}$$

$$\therefore n^* = \left(\frac{1}{\sum Q_r}\right) \sqrt{\frac{2DS_w}{h_w C_w}} \tag{10}$$

We can also write n as follows:

$$n^* = \sqrt{\frac{[(h_r C_r - h_w C_w) S_w]}{[(S_r + C_{tr}) h_w C_w]}} \tag{11}$$

From the above relation it is clear that n and $\sum Q_r$ is inversely related.

4. Algorithm for Centralized Case

The steps of algorithm for centralized decisions are listed below:

- Determine the total ordering cost of retailer by using equation 7.
- Set, $n = \sqrt{\frac{[(h_r C_r - h_w C_w) S_w]}{[(S_r + C_{tr}) h_w C_w]}}$ for determining the value of n.
- Then set, $\sum Q_r = \left(\frac{1}{n}\right) \sqrt{\frac{2DS_w}{h_w C_w}}$ for determining the value of optimum order quantity.
- Compare the total cost for centralized & decentralized decisions & find the optimum cost and economic order quantity for this cost.

5. Case Study

We carry out a case study implementing the developed model and the algorithm in a real situation. The study is done on six products of Unilever Bangladesh Limited considering one warehouse & one retailer. All the data are approximate values. The ordering cost for all products are collected from the company & based on this data the individual ordering cost is assumed for every product according to the volume & labor requirements. Ordering cost

consists of cost per order, labor cost for loading & unloading which is obtained from equation 7. The demand of retailer per year, holding cost, ordering cost, unit cost of each product are shown in table 1.

From warehouse of Unilever Bangladesh Limited some approximate costs are collected shown in table 2. Here it is found that holding cost is lower than retailer but ordering cost is twice than that of retailer. The transportation cost for these six products is 200 BDT which is then distributed over the products with respect to volume of each item.

Table 1: Approximate value of demand & different costs of retailer

Name of products	**Demand/year**	**Unit price, C_r (in BDT)**	**Holding cost rate, h_r (in BDT)**	**Ordering cost, S_r (in BDT)**
Soap	3000	50	0.5	15
Shampoo	1200	170	0.5	20
Toothpaste	2400	65	0.5	10
Deodorant	600	98	0.5	15
Cosmetics	2700	70	0.5	20
Washing powder	1200	70	0.5	10

Table 2: Approximate value of different costs of warehouse

Name of products	**Unit price, C_w (in BDT)**	**holding cost rate, h_w (in BDT)**	**Ordering cost, S_w (in BDT)**	**Transportation cost, C_{tr} (in BDT)**
Soap	46	0.15	30	50
Shampoo	167	0.15	40	40
Toothpaste	61	0.15	20	20
Deodorant	94	0.15	30	15
Cosmetics	65	0.15	40	15
Washing powder	66	0.15	20	60

Cost analysis is done by determining the value of ΣQ_r and n. the value of n is determined from equation (6) for decentralized case & equation (11) for centralized case. For these two values we get two values of ΣQ_r. For centralized case the ordering cost for retailer reduces because per order cost include loading & unloading costs of labor but total ordering cost increases highly as a result of increase in total cost of retailer. This comparison is shown in table 3.

Table 3: Cost analysis and comparison

Decisions	n	**Order quantity ($\sum Q_r$)**	**Annual cost of retailer (in BDT)**	**Annual cost of warehouse (in BDT)**	**Total System cost (in BDT)**
Decentralized	3	226	8767	39345	48112
Centralized	2	116	11005	28678	39683

Loss or savings for retailer & warehouse in centralized decision are shown in percentage in table 4.

Table 4: loss or savings in centralized decisions

For retailer	**For warehouse**	**Total System**
–25.53%	27.12%	1.59%

6. Results and Discussions

From the above numerical results the optimum order quantity and optimum cost is achieved. It is found that optimum order quantity is selected for centralized decisions. Because total supply chain profitability is obtained more in centralized decisions rather than decentralized decisions. For centralized decision the saving of warehouse is 27.12% which is more than decentralized decision and the total system saving is 1.59%. So this decision may be used by this company for practical proof.

7. Conclusions and Recommendations

This paper looked for the optimal ordering policy of multiple items in a two-level supply chain in two cases: centralized and decentralized. An algorithm was proposed to find out multiple item aggregated economic order quantities for both the retailer and the warehouse which minimize the total system cost in the centralized case. The case study indicated that the centralized case leads to savings for the warehouse and the whole system as compared to the decentralized case. But centralized decisions incur losses for the retailer. These losses can be covered and the retailers can be motivated if warehouse will share its saving through discount quantity.

This paper considers only two stages in a supply chain without the effect of other stages. The ordering cost of warehouse is considered twice than retailer and for centralized decision per order cost of retailer decreases for decreasing the order quantity but total ordering cost per year increases. The transportation cost is considered only from warehouse to retailer with less truck load. So there is a scope to extend this paper for multi-items, multi-suppliers condition. For multi-vehicle supply chain this paper will be one of the bases to do work. We will try to do these in future.

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