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Incorporating Price Decisions in a Profitable Multiple-Tour Problem

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Abstract

Classical profitable tour problems typically do not consider price decisions and usually seek to maximize the profit by serving a selective set of customers with known demands by a fleet of vehicles. This paper for the first time incorporates price decisions in a capacitated profitable multiple-tour problem with price-sensitive demands. The problem is formulated as a mixed-integer linear program that can be solved optimally in reasonable time for medium-sized instances of the problem by commercial optimization solvers. The numerical study indicates that the integration of price and routing decisions considerably improves the overall profit.

Keywords
Vehicle routing problem, pricing, maximum profit, price-sensitive demand

1. Introduction
Vehicle Routing Problems (VRPs) are among the most widely studied combinatorial optimization problems that deal with optimally visiting customers from a central depot. These problems are very important in the fields of transportation, distribution and logistics. A VRP was first proposed by Dantzig and Ramser (1959) and then much progress has been made and several variants of VRPs have been put forward. Among the various surveys on VRPs are the book by Toth and Vigo (2001) and the more recent update by Cordeau et al. (2007). The objectives of classical VRPs are to find the minimum total distance that is required to serve a fixed set of customers with known demands by a fleet of vehicles, where every customer has to be serviced. However, in recent studies, some variants of VRPs propose to select customers depending on the total profit that is gained when they are visited by the vehicles. Such VRP is called *Vehicle Routing Problem with Profit* (VRPP).

VRPPs are generalizations of *Traveling Salesman Problems with Profit* (TSPPs), surveyed by Feillet et al. (2005), to the case of multiple vehicles. Two opposite objectives are considered in TSPPs, one maximizing the total collected profit and the other minimizing the travel cost (with the right to drop vertices). Three generic problems are addressed, depending on the way the two objectives are incorporated:

- Profitable tour problem
- Orienteering problem
- Prize collecting travelling salesman problem.

In a *Profitable Tour Problem* (PTP), both objectives are combined in the objective function, i.e., a subset of the potential customers has to be selected with the objective of maximizing the difference between the total collected profit and the cost of the total traveled distance (Dell'Amico, 1995). In an *Orienteering Problem* (OP), a subset of the potential customers has to be selected with the objective of maximizing the total collected profit such that travel costs do not exceed a preset value (Laporte and Martello, 1990). In a *Prize Collecting TSP* (PCTSP) a subset of the potential customers should be selected with the objective of minimizing the total traveled distance such that the total collected profit does not be less than a predetermined amount (Balas, 2002).

The extension of an OP to the case of multiple-tours is known as a *Team Orienteering Problem* (TOP). A problem of this kind was first proposed by Butt and Cavalier (1994) under the name *Multiple-Tour Maximum Collection Problem*, but the definition of TOP was actually introduced by Chao et al. (1996). Vansteenwegen et al. (2011) provided a complete survey on this type of problems. Similarly, a *Prize-Collecting VRP* (PCVRP) is the extension of a PCTSP to the multiple-tours, which is derived from the hot rolling production of the iron and steel industry. This type of problems is appeared in the literature in Tang and Wang (2006) and Zhang et al. (2009). An extended version of PTPs for the case of multiple-tours is investigated by Archetti et al. (2009), that may be called *Profitable Multiple-Tour Problem* (PMTP). The number of papers published in this area is so limited compared to the two other problems.

Most of the papers in the literature of VRP do not consider the impact of price decisions on demands. In other words, in all of the VRPPs mentioned above, a predetermined demand is being considered for each customer. In fact, higher pricing causes lower demand and lower pricing causes higher demand. However, demands and process decisions are actually dependent. Geunes et al. (2007) took the first step in incorporating price decisions in a VRPP by proposing a set of approximation models for understanding the economics of delivery pricing in a routing-based environment. However, their models did not determine routing decisions.

This paper considers a PMTP with price-sensitive demands that optimizes routing and pricing decisions simultaneously. The objective is to maximize the total profit associated with these decisions. The problem is modeled as a *Mixed-Integer Linear Program* (ILP) that determines price and routing decisions.

The remainder of this paper is organized as follows. In Section 2, the problem is described and formulated as an MILP. Section 3 presents the computational experiment. Finally, Section 4 concludes the paper.

2. Problem Description and Formulation

This section describes our proposed PMTP. In this problem, a fleet of homogenous vehicles with uniform capacity and a set of potential customers are considered in a supply chain network with one distribution center and a set of potential customers (or retailers). The objective is to select a subset of the customers such that the total collected profit is maximized. The product price (retail price) includes two main components; one is the final cost (or wholesale price) which includes the manufacturing cost, labor cost, overhead cost etc. and the other is the profit which must be determined. The profit is the variable section of the product price and is what the proposed PMTP is actually going to determine. In other words, the PMTP determines this profit component of product price for each customer as well as routing decisions simultaneously. A set of profit percentage levels is considered, and for each customer the demand corresponding to each profit percentage level is known. The assumptions, notation and formulation are presented below.

2.1 Assumptions

- The potential customers are scattered in a region and their locations are already identified.
- A set of identical vehicles with the same capacity is considered. Each vehicle starts and ends its route at the distribution center and can visit any subset of customers with a total demand that cannot exceed the vehicle capacity.
- A single product is considered with a known set of profit percentage levels, which are advantageous for the company.
- The distances between customers are symmetric.
- Each customer’s demand is price sensitive and depends on the profit percentage level considered for retailing the product.
- The objective is to maximize the total collected profit.

2.2 Decision Variables

$$x_{ijk} = \begin{cases} 1 & \text{vehicle } k \text{ visits node } j \text{ immediately after node } i \\ 0 & \text{other wise} \end{cases}$$

$$y_{ikl} = \begin{cases} 1 & \text{node } i \text{ is visited by vehicle } k \text{ at profit level } l \\ 0 & \text{other wise} \end{cases}$$

$$\alpha_l = \begin{cases} 1 & \text{profit percentage level } l \text{ is selected} \\ 0 & \text{other wise} \end{cases}$$

2.3 Parameters

c_{ij} Travel distance between customers i and j

s Traveling cost per unit distance

Q Maximum capacity of each vehicle

K Maximum number of available vehicles
W Final cost (wholesale price) per unit of product
p_l Profit percentage per unit of product at level l
P_l Price per unit of product at level l, which is computed as $P_l = W(1+p_l)$
d_{il} Demand of customer i when profit percentage level l is selected $(\forall l : d_{1l} = 0)$
d_{il} A large positive number

2.4 Formulation

The proposed PMTP can be formulated as the following MILP:

$$\text{Max} \sum_{k=1}^{K}\sum_{i=1}^{N}\sum_{l=1}^{L} P_l\, d_{il}\, y_{ikl} - \sum_{i=1}^{N}\sum_{j=1, j\neq i}^{N}\left(s \times c_{ij} \sum_{k=1}^{K} x_{ijk} \right) \tag{1}$$

$$\sum_{k=1}^{K}\sum_{j=2}^{N} x_{1jk} \le K \tag{2}$$

$$\sum_{k=1}^{K}\sum_{i=2}^{N} x_{i1k} \le K \tag{3}$$

$$\sum_{k=1}^{K}\sum_{l=1}^{L} y_{ikl} \le 1 \qquad \forall i = 2,\dots,N \tag{4}$$

$$\sum_{j=1}^{N} x_{jik} = \sum_{j=1}^{N} x_{ijk} = \sum_{l=1}^{L} y_{ikl} \qquad \forall i = 2,\dots,N \qquad \forall k = 1,\dots,K \tag{5}$$

$$\sum_{i=1}^{N}\sum_{l=1}^{L} d_{il}\, y_{ikl} \le Q \qquad \forall k = 1,\dots,K \tag{6}$$

$$\sum_{i=1}^{N}\sum_{k=1}^{K} y_{ikl} \le \alpha_l\, M \qquad \forall l = 1,\dots,L \tag{7}$$

$$\sum_{l=1}^{L} \alpha_l = 1 \tag{8}$$

$$2 \le u_{ik} \le N \qquad \forall i = 2,\dots,N \qquad \forall k = 1,\dots,K \tag{9}$$

$$u_{ik} - u_{jk} + N\, x_{ijk} \le N - 1 \qquad \forall i, j = 2,\dots,N \qquad \forall k = 1,\dots,K \tag{10}$$

$$x_{ijk} \in \{0,1\} \qquad \forall i, j = 1,\dots,N \qquad \forall k = 1,\dots,K \tag{11}$$

$$y_{ikl} \in \{0,1\} \qquad \forall i = 1,\dots,N \qquad \forall k = 1,\dots,K \qquad \forall l = 1,\dots,L \tag{12}$$

$$\alpha_l \in \{0,1\} \qquad \forall l = 1,\dots,L \tag{13}$$

The objective function (1) is to maximize the total profit, i.e., the difference between the total collected revenue and the cost of the total distance traveled by all incorporated vehicles. Constraints (2) and (3) guarantee that each vehicle starts and ends its route at the distribution center. Constraints (4) ensure that all customers can be visited at most once. Constraints (5) imply the connectivity of each path. Constraints (6) are vehicle-capacity constraints. Constraints (7) and (8) state that only one profit percentage level can be selected for the product, and it is the same for all of the customers. Sub-tours are prohibited by constraints (9) and (10). Finally, Constraints (11), (12) and (13) set integral requirements on the decision variables.

3. Computational Experiments

The ILP (1)-(13) is coded in GAMS and solved by CPLEX 12.1. Computations are performed on a computer with 2.80 GHz CPU and 4GB RAM, running Windows operating system.

We used 5 instances taken from the VRP library with some modifications in the capacity limit, the number of available vehicles and the number of vertices. The number N of vertices ranges from 21 to 31. We take the first vertex in the data file (original instances) to be the distribution center and the next $N-1$ vertices to constitute the set of potential customers. For each case, we generated two different instances with $Q = 80{,}100$ and $K = 2$. The customers' demand values at each profit level are computed using an exponential function of the product price (retail price) as follows:

$$d_{il} = 244.7 \times d_i e^{-0.25 \times P_l} \quad (14)$$

where d_i is the demand value for customer i taken from the VRP library considered for the original instances and P_l is the product price at level l computed as $P_l = W(1 + p_l)$. In this equation, W is the final cost (wholesale price) of the product considered to be equal to 20$ and p_l is the profit percentage considered at level l which is defined as $(0.05 + 0.05l)$, $l = 1,...,5$. Values of c_{ij} were computed using the Euclidean distance formula, and $s = 0.1$.

Table 1: Numerical results for 30 instances of proposed PMTP that is modeled as MILP (1)-(13)

No.	*N*	*K*	*Q*	l^*	*CPU* (*seconds*)	*RI* (%) over the case l=1	*RI* (%) over the case l=5
1	31	2	100	4	2.691	11.66	6.49
2	31	2	80	5	0.863	15.17	0.00
3	31	2	100	5	0.876	17.46	3.07
4	31	2	80	5	0.771	17.07	1.13
5	31	2	100	4	1.610	9.45	19.47
6	31	2	80	4	1.406	11.62	1.91
7	31	2	100	3	3.213	7.53	2.26
8	31	2	80	5	0.765	14.95	0.00
9	31	2	100	5	0.890	18.94	0.00
10	31	2	80	5	0.660	15.99	0.00
11	26	2	100	2	2.131	33.47	38.50
12	26	2	80	2	2.060	36.49	25.81
13	26	2	100	5	2.221	10.65	0.11
14	26	2	80	5	0.707	13.27	0.76
15	26	2	100	3	0.763	6.15	33.03
16	26	2	80	3	5.475	8.34	17.72
17	26	2	100	4	0.779	12.15	18.38
18	26	2	80	5	1.090	13.49	0.00
19	26	2	100	3	1.883	9.70	1.04
20	26	2	80	5	0.484	16.81	0.00
21	21	2	100	3	2.264	7.07	25.91
22	21	2	80	4	0.656	12.22	14.99
23	21	2	100	4	0.666	11.75	18.04
24	21	2	80	5	0.438	9.91	0.12
25	21	2	100	2	0.586	2.45	41.87
26	21	2	80	3	0.440	5.88	33.07
27	21	2	100	3	0.538	5.44	27.16
28	21	2	80	4	0.446	14.25	11.23
29	21	2	100	4	0.794	7.89	21.62
30	21	2	80	5	0.338	13.07	0.09

A set of 30 test problems is considered with the characteristics explained above, and then is solved under three scenarios. In the first scenario, we consider the five profit percentage levels for the product while in the second and third scenarios, only one profit percentage level is considered; the lowest and highest levels, respectively. The results are reported in Table 1. For each instance, the number of vertices (N), the number of available vehicles (K) and the capacity limit (Q) are given. The optimal profit percentage levels are indicated under the columns l^* . Table 1 also reports the relative improvements (RIs) obtained from the proposed PMTP, which integrates price and routing decisions, over the cases that the profit percentage level is fixed before determining routing decisions as $l = 1$ and $l = 5$. The average RIs are 13.01% and 12.13% over the second and the third scenarios. This shows that the integration of price and routing decisions considerably improves the overall profit.

4. Concluding Remarks

This paper proposes an integrated profitable multiple-tour problem with price-sensitive demands that optimizes routing and pricing decisions simultaneously. The objective of the problem is to maximize the total profit associated with these decisions. The problem is formulated as a mixed-integer linear program that can be solved optimally in reasonable time for medium-sized instances of the problem by commercial optimization solvers. The numerical results indicate a significant increase in total profit collected by applying the proposed integration approach compared to the traditional approach that considers a fixed price for the product, and consequently fixed demands for customers, and then solves the associated routing problem.

As a future research direction, we are planning to focus on developing efficient algorithms to solve real-world instances of the problem with large sizes under different pricing policies.

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