Proceedings of the 2012 International Conference on Industrial Engineering and Operations Management
Istanbul, Turkey, July 3 – 6, 2012

Economic Lot Scheduling Problem Considering Deteriorating Items

Vahid Kayvanfar
Research Institute of Food Science and Technology
Mashhad 91735-139, Iran

Mostafa Zandieh
Department of Industrial Management, Management and Accounting Faculty, Shahid Beheshti University, G.C., Tehran, Iran

Abstract

The economic lot scheduling problem (ELSP) is tackled in this paper in which items deteriorate in an exponential rate. Shortage is allowable for all items and is completely backlogged. To solve the considered problem, extended basic period approach under power-of-two policy is employed. The goal of this research is specifying optimal batch size for a given product besides minimizing total imposed costs to production system. Setup cost, holding cost includes deteriorating factor as well as slack cost constitute three cost components of the considered problem. Since the ELSP is shown as an NP-hard problem, imperialist competitive algorithm (ICA) and genetic algorithm (GA) are employed to provide good solutions. Experimental results demonstrate that the proposed approach can competently solve such complicated problems.

Keywords
Economic lot scheduling problem, Deterioration factor, Slack cost, Imperialist competitive algorithm.

1. Introduction
The economic lot scheduling problem (ELSP) combines lot sizing and production scheduling decisions and is one of the most representative topics. The conventional ELSP is concerned with the scheduling of cyclical production of $n \geq 2$ products on a single facility in lots which are different in size and consequently are different in production times and cycles, over an infinite planning horizon, assuming deterministic demand for each product. Actually, the conventional ELSP is defined as the problem of finding the production sequence, production times and idle times of several products in a single facility in a cyclic schedule so that the demands are made without stock-outs or backorders and average inventory holding and setup costs are minimized (Moon et al. 2006). The ELSP occurs when one machine is used to meet deterministic and fixed demand of several products over an infinite horizon. Also, the issue of batching arises because the system usually requires a set-up cost and a set-up time when a machine switches from one product to the next. In addition to the discrete parts manufacturing, multi-products or multi-purpose processors are common features in many chemical plants such as those producing pharmaceuticals, biochemical, polymers, cosmetics, food and beverages, etc. Of ELSP applications, one may arises where the raw material is produced internally at a finite rate in the packaging of liquid medical products. The pharmaceutical industry employs common machines to bottle and package different products, and the bottled products are the finished items (Gallego and Joneja 1994).

A production plan in the ELSP schedules the items within 'basic periods', where a basic period (BP) is an interval of time that consists of setup and production of a subset (or all) of the products (Yao and Elmaghraby 2001). The solution of the ELSP is usually given in terms of a set of multipliers $\{k_i\}$; i =1, 2,…, n and the BP in which each product is produced. The BP's in the ELSP can be categorized as either the 'BP' or the *'extended basic period'* (EBP) approach. The researchers have demonstrated that the ELSP under the EBP approach, denoted as the ELSP (EBP), always yields better solutions (Elmaghraby 1978). In the literature, changing the policy of resolving from the BP approach to the EBP approach is for eliminating the wasted capacity of the production facility due to the restrictive feasibility condition. The EBP expands upon the BP by allowing different cycle times for different products. The power-of-two (PoT) policy necessitates that $k_i = 2^q$; $q \geq 0$ integer, for all k_i in the set of multipliers *K(B)*. Under such policy, researchers were able to derive some easy and effective heuristics to solve both incapacitated and capacitated lot sizing problems.

In the literature, Cooke et al. (2004) proposed a relatively simple MIP formulation for the ELS problem that creates a complete schedule, assuming a basic period value and production frequencies that have been predetermined. A special case of ELSP is studied by Jackson et al (1985) on the joint replenishment problem, where the capacity of the production facility is unlimited. Brander and Segerstedt (2009) modified the traditional cost function to include not only set-up and inventory holding cost but also a time variable cost for operating the production facility. Also, Zanoni et al. (2012) studied the multi-product ELSP with manufacturing and remanufacturing opportunities. They proposed a simple and easy to implement algorithm to solve the model using a basic period policy. A new inventory model in which products deteriorate at a constant rate and in which demand, production rates are allowed to vary with time has been introduced by Balkhi and Benkherouf (1996). There are some classifications for deterioration. Ghare and Schrader (1963) classified the inventory deteriorating into three categories: 1) direct spoilage, e.g., vegetable; 2) physical depletion, e.g., gasoline; and 3) deterioration in terms of loss of efficacy in inventory, e.g., medicine. Raafat (1991) categorized deterioration by the time-value of inventory: 1) utility constant: namely, its utility does not change significantly as time passes e.g., liquid medicine; 2) utility increasing: its utility increases as time passes, like some alcoholic drink. 3) Utility decreasing: its utility decreases as time passes, e.g., fresh foods, etc.

2. Problem Description

In the ELSP (with BP or EBP approach), the algorithm initially searches for an initial basic period B and its corresponding set of multipliers $K(B)$, and tries to obtain another basic period $B^{'}$ and its multipliers $K(B^{'})$ which improve the objective function value. Until obtaining no other basic period and its corresponding multipliers which improve the objective function value, the search continues. In intermediate steps of search, for sets of B and $K(B)$, one must either test its feasibility or obtain a feasible production schedule. This cyclical schedule is subject to the following assumptions related to the production facility:

At a time, only one item i among total n items can be produced. A deterministic given setup cost (a_i) and setup time (s_i) are defined for each item which are sequence-independent. Demand rate (d_i) and production rate (p_i) are predetermined. Holding costs (h_i) are determined by the quantity of the items held. Shortages are allowed for all items, but are completely backlogged. Each item deteriorates at an exponential rate θ_i and deterioration cost of per unit equals ξ_i . Each item has a due date (due_i) which must be delivered. Violating this assumption may cause a slack cost which equals π_i.

The solution of conventional ELSP consists of a set $T = \{T_1, T_2, T_3,\ldots, T_N\}$, such that each T_i is long enough to allow enough production of item i at the beginning of the cycle plus allow production of other items in the time left between the ends of production of item i and the start of the next cycle. If a set T is feasible and minimizes cost, it is optimal. Two terms of objective function in conventional ELSP are: 1) setup cost denoted by a_i, incurs whenever the production facility sets up to produce the other items, and 2) inventory holding costs h_i. In addition to these two cost terms, we include the deterioration cost for the deteriorating items and slack cost for items violate the due date.
The objective function of presented mathematical model according to EBP approach under PoT policy named Slack-Deter-ELSP (EBP-PoT) considering deteriorating items and slack cost are as follows:

$$\text{minimize } TC(\{k_i, B\}) = \sum_{i=1}^{n}\left\{\frac{a_i}{k_i B} + \frac{1}{2}H_i k_i B + \lambda_i \pi_i\right\} \qquad (1)$$

Where consists of setup cost, holding cost including deteriorating factor and slack cost. This objective function is subjected to the following constraints;

$$\sum_{i=1}^{n}\left[\left(s_i + \beta_i(k_i, B)\right)\right] w_{i\varphi(i,\tau)} \le B \qquad ;\tau = 1,2,3,\ldots, K = lcm\{k_i\} = 2^{\max_i\{\nu_i\}} \qquad (2)$$

Which states that the total occupancy must be less than the length of basic period in each basic period τ (capacity constraint for a feasible production schedule). Eq. (3) ensures capacity feasibility or load feasibility which states the load never must exceeds the facility's capacity.

$$\sum_{i=1}^{n} \frac{s_i + \beta_i(k_i, B)}{T_i} \le 1 \qquad ; i = 1,2,3,...,n \qquad (3)$$

Eqs. (4) and (5) together ensure that only one job can be processed at any instance in time. Actually, these precedence constraints state that start time of the job *i* is greater than or equal to the completion time of the job *k*, i.e. job *i* is latter in sequence rather than job *k* or vice versa. At any status, one of these constrains is redundant and the other one is active. Since by determining the sequence (using Constraints (4) and (5)), the completion time of each item can be specified. In these constraints M is a large enough positive number and y_{ik} = 0 or 1 and means if job *i* proceeds job *k* is the sequence, y_{ik} = 1, else y_{ik} = 0.

$$My_{ik} + t_i - t_k \ge p'_k \qquad ; i = 1,2,..,n \; ; i \le k \qquad (4)$$

$$M(1 - y_{ik}) + t_k - t_i \ge p'_i \qquad ; i = 1,2,...,n \; ; i \le k \qquad (5)$$

Eq. (6) shows the PoT policy which can be stated as follows:

$$k_i = 2^{\nu_i} \;\; ; \nu_i \in \{0,1,2,3,....,n\} \qquad ; i = 1,2,3,...,n \qquad (6)$$

Eq. (7) compels the production duration of job *i* must be scheduled among the first k_i basic periods. Constraint (8) identifies a basic period among the k_i basic periods belonging to product *i*. Actually Eqs. (7) and (8) represent the starting basic periods of the production batches for all of the items.

$$\sum_{\tau=1}^{k_i} w_{i\tau} = 1 \qquad ; i = 1,2,3,...,n \qquad (7)$$

$$\varphi(i,\tau) = \begin{cases} \tau \bmod k_i & ; if \; \tau \ne \gamma k_i \,, \gamma \in N \\ k_i & ; if \; \tau = \gamma k_i \,, \gamma \in N \end{cases} \qquad ; i = 1,2,3,...,n \qquad (8)$$

Eqs. (9), (10) and (11) are self-explained. Eq. (12) signifies the occupancy of each production batch for item *i*.

$$\begin{cases} w_{i\tau} = 1 \; ; \text{if product } i \text{ is produced in } i\text{th basic period} \\ w_{i\tau} = 0 \; ; \text{Other wise} \end{cases} \qquad ; \text{for all } i \text{ and } \tau \qquad (9)$$

$$\begin{cases} \lambda_i = 1 & ; \text{if item } i \text{ has tardiness} \\ \lambda_i = 0 & ; \text{Other wise} \end{cases} \qquad ; i = 1,2,3,...,n \qquad (10)$$

$$H_i = d_i(\theta_i \xi_i + h_i) \qquad ; i = 1,2,3,...,n \qquad (11)$$

$$\beta_i(k_i, B) = \frac{d_i}{p_i}\left(1 + \frac{k_i B \theta_i}{2}\right) k_i B \qquad ; i = 1,2,3,...,n \qquad (12)$$

In Eq. (13) C_i and p'_i denote completion time and total processing time of job *i* (setup time plus occupancy of time of item *i*), respectively and t_i is the start time of job *i*. Obviously, start time of each job depends on the previous jobs in sequence, i.e. it equals total spent time by prior jobs in sequence. Finally, T'_i in Eq. (14) is tardiness of job *i*.

$$C_i = t_i + p'_i = t_i + \left(s_i + \beta_i(k_i, B)\right) \qquad ; i = 1,2,3,...,n \qquad (13)$$

$$T'_i = \max\{0, C_i - due_i\} \qquad ; i = 1,2,3,...,n \qquad (14)$$

The solution of our Slack-Deter-ELSP (EBP, PoT) problem consists of a set of multipliers $\{k_i\}$, value of the basic period (B) as well as a set of $\{\lambda_i\}$. A feasible production schedule for the obtained solution must be generated. To minimize the objective function, our Imperialist Competitive Algorithm (ICA) explores in the solution space of $\{k_i\}$. Since for a given set of multipliers $\{k_i\}$, the objective function is convex with respect to B, so we have $\frac{\partial TC(\{k_i\}, B)}{\partial B} = 0$ to acquire the minimum of B value as follows:

$$B(\{k_i\}) = \sqrt{\frac{2\left(\sum_{i=1}^{n}\frac{a_i}{k_i}\right)}{\sum_{i=1}^{n} H_i k_i}} \tag{15}$$

Proc FT heuristic (Yao and Huang, 2005) is then employed to test feasibility of ($\{k_i\}$, B). If there exists a feasible production schedule for the set ($\{k_i\}$, B), this schedule will be held as a nominee of the optimal solution, otherwise another schedule as primal schedule is produced to set a special value of B, that makes possible ($\{k_i\}$, B) to obtain a feasible production schedule with the minimum cost for the set $\{k_i\}$.

3. The Proposed Imperialist Competitive Algorithm

ICA is a novel global search heuristic that uses imperialism and imperialistic competition process which uses the socio-political process of imperialism and imperialistic competition as a source of inspiration (Khabbazi et al. 2009). The ICA initiates with an initial population, like most evolutionary algorithms. Each individual of the population is called a '*country*' equivalent '*chromosome*' in Genetic Algorithm (GA). Some of the most powerful countries are chosen to be the imperialist states and the other countries constitute the colonies of these imperialists. All the colonies of initial countries are partitioned among the mentioned imperialists based on their power. Equivalent of fitness value in the GA, the power of each country, is conversely proportional to its cost. An empire is constituted from the imperialist states with their colonies.

By forming initial empires, the colonies in each of them start moving toward their relevant imperialist country. This movement is a simple model of assimilation policy which was pursued by some of the imperialist states. Then, the imperialistic competition begins among all the empires. Any empire which is not able to succeed in this competition and can't increase its power (or at least prevent decreasing its power) will be eliminated from the competition. The imperialistic competition will gradually result in an increase in the power of powerful empires and a decrease in the power of weaker ones. The total power of an empire depends on both the power of the imperialist country and the power of its colonies. This fact is modeled by defining the total power of an empire as the power of imperialist country plus a percentage of mean power of its colonies (Khabbazi et al. 2009).

The movement of colonies toward their relevant imperialists along with competition among empires and also the collapse mechanism cause all the countries to converge to a state in which there exists just one empire in the world and all the other countries are colonies of that empire. In this ideal new world, colonies have the same position and power as the imperialist.

The term *"country"* in ICA is equivalent to *"chromosome"* in GA. Here, a country is a $1 \times N_{var}$ array which is defined by

$$country = (g_1, g_2, g_3, ..., g_{N_{var}})$$

Where each g_i is a variable which should be optimized. Each of these variables can be interpreted as a socio-political characteristic of a *country*, such as religion, culture, language, etc. From optimization perspective the solution with least cost value is the best one. As in our problem, each multiplier must be represented as a specific section of a *country*, in order to encode the value of k_1, the first l_1 bits are employed to comply such a goal and the particular part of *country* from the (l_1 + 1)th bit to the (l_1 + l_2)th bit represents the value of k_2 and so on. Country representation is shown in Fig. 1 to illustrate how k_is encode.

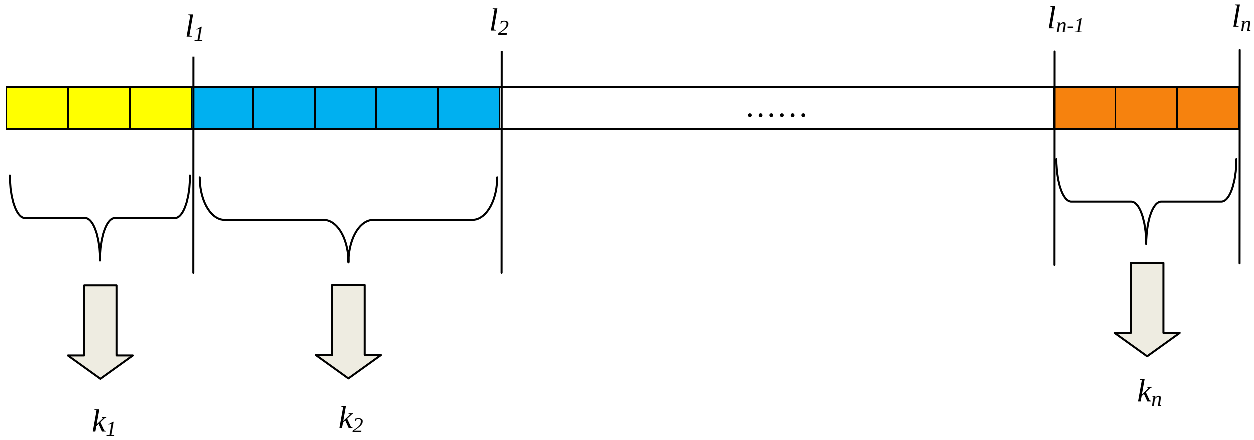

Figure 1: Country representation and coding scheme of multipliers for the proposed algorithm

As shown in Fig.1 as an example the first three bits are used to encode the k_1, the next five bits for k_2 and at last, the last three ones are employed to encode k_n. For scrutiny of ICA, one may refer to Khabbazi et al. (2009).

4. Generation of Feasible Solution

In order to obtain a feasibility testing procedure, we use Proc FT (Yao and Huang, 2005) with some changes. Suppose G signify a candidate schedule and $L(G)$ be the maximal load secured by G. Presume that a set of multipliers $\{k_i\}$ and B is given. The corresponding occupancy times $\beta_i(k_i, B)\}$ evidently will be determined. Use a random production schedule to acquire an initial schedule of production G, and calculate $L(G)$. Regard L^* as the best load secured up to now, and G^* its corresponding production schedule. (If G is the first production schedule then set $L^* = L(G)$ and $G^* = G$). When $L^* \leq B$, clearly the recommended assignment is a feasible production schedule. Here, indicator μ is defined as follows; if a feasible production schedule is acquired in Proc FT, $\mu = 1$; otherwise, $\mu = 0$. After generation a random production schedule as a random solution, if $\mu = 0$, i.e. no feasible production schedule is attained, use the "*Schedule Smoothing Procedure*" (Yao and Huang, 2005) to improve the maximal load secured by candidate schedule, L^* until $\mu = 1$ or L^* can no longer be improved. If L^* has not been improved for a constant consecutive iterations (*maxit*), Stop. Or else, select a subset of items for re-optimization randomly; fix the schedule for the rest of the items, and return to start another local search iteration. The constant *maxit* is arbitrary criterion for stopping the proposed heuristic algorithm which is defined by user opinion.

5. Computational Results

The Slack-Deter-ELSP (EBP, PoT) is implemented using GA and ICA in MATLAB 7.5.0 and has run on a PC with a 1.83 GHz Intel Core 2 Duo processor and 2 GB RAM memory. Three categories of test examples including three small, four medium and five large instances are randomly generated where input data sets are illustrated in Table 1.

Table 1: Problems data sets

Input variables	**Distribution**
Demand rate (d_i)	~DU[2000, 60000]
Produce rate (p_i)	~ DU[5000, 125000]
Holding cost (h_i)	~ DU[5, 120]
Setup cost (a_i)	~ DU[60, 600]
Setup time (s_i)	~ DU[5, 15]
Deterioration cost (ξ_i)	~ DU[10, 110]
Deterioration factor (θ_i)	~ U[0.3, 3]

Due date (due_i)	~ DU[30, 85]
Slack cost (π_i)	~ DU[80, 500]

Each test example has run 4 times on each algorithm in all categories so as to ensure the stable respond of the algorithms. Since we have three categories of problems, we compare the results for all small, medium and large problems on GA and proposed ICA separately. Table 2 shows average results for the best obtained parameters. As can be seen, the obtained results demonstrate high performance of ICA in respect of GA, i.e. the ICA outperforms GA in all instances in the considered characteristics. The best computational results of sample problems in all categories for Slack-Deter-ELSP (EBP, PoT) problem are shown in Table 3.

Table 2: Average results for the problems grouped by size

Problem Size	**Lingo**	**GA**			**ICA**		
	Best cost	**Best cost**	**Computational time**	**Best Time**	**Best cost**	**Computational time**	**Best Time**
Small	1275.35	1275.35	539.33	262.40	1275.35	179.69	4.24
Medium	1012.52	3417.32	912.11	601.42	707.64	355.53	14.64
Large	-	22791.34	1153.73	585.44	22280.82	515.33	65.43

Where the calculated times are computed to second. Also, in large cases, we compared only obtained results from GA and ICA, owing to being time consuming of lingo implementing.

Table 3: The best computational results of sample problems

Type& No		$\sum_{i=1}^{n}\rho_i$	**TC*($)**	**B(days)**	k_1	k_2	k_3	k_4	k_5	k_6	k_7	k_8	k_9	k_{10}	k_{11}	k_{12}	k_{13}	k_{14}	k_{15}
Small	1	0.84	4400.64	31.47	4	4	4	2	4	-	-	-	-	-	-	-	-	-	-
	2	2.44	175.09	21.04	8	8	4	2	64	-	-	-	-	-	-	-	-	-	-
	3	3.14	250.06	15.80	16	16	16	16	4	-	-	-	-	-	-	-	-	-	-
Medium	1	6.33	820.11	4.50	64	16	4	8	8	16	32	8	8	16	-	-	-	-	-
	2	2.86	722.57	20.31	16	32	8	8	8	32	16	16	4	8	-	-	-	-	-
	3	6.33	720.17	5.13	4	16	8	4	4	16	32	64	4	4	-	-	-	-	-
	4	6.69	520.11	6.17	8	8	16	8	4	8	16	8	8	16	-	-	-	-	-
Large	1	2.52	35000.83	33.25	8	4	8	4	4	16	16	32	8	32	16	8	16	8	32
	2	7.33	1350.15	26.38	8	8	4	4	16	8	8	16	16	16	8	8	8	8	4
	3	9.41	24001.68	26.89	8	4	4	4	4	8	8	8	16	32	16	64	16	32	8
	4	7.32	24300.64	12.31	16	8	16	16	32	16	16	16	16	8	64	16	16	32	32
	5	8.53	28000.79	20.91	8	4	4	16	16	8	32	16	16	16	8	32	8	32	8

Total cost (TC), basic period (B) according to day and all multipliers (k_is) are shown in Table 3. As could be implied all multipliers are power of two.

6. Conclusion

In this paper, an imperialist competitive algorithm (ICA) for solving multi-criteria economic lot scheduling problem (ELSP) considering deterioration items and slack costs regarding capacity constraint using the extended basic period (EBP) approach under the Power-of-Two (PoT) policy is proposed. To the best of author's knowledge, regarding such criteria in ELSP has not been mentioned yet in the literature. To solve the considered problem, we employ ICA which is equipped with a feasibility testing procedure. In order to evaluate the effectiveness and robustness of the proposed ICA, we accomplish a comparison between our ICA and GA. To do so, we utilize three different categories of test problems in small, medium and large sizes. Each instance has run 4 times so as to make certain for stability of the algorithms. The computational results reveal that our proposed ICA is an efficient solution approach for solving Slack-Deter-ELSP (EBP, PoT) and outperforms the GA.

References

Balkhi, Z., Benkherouf, L., A production lot size inventory model for deteriorating items and arbitrary production and demand rates, *European Journal of Operational Research*, vol. 92, pp. 302-309, 1996.

Brander, P., and Segerstedt, A., Economic lot scheduling problems incorporating a cost of using the production facility, *International Journal of Production Research*, vol. 47, pp. 3611-3624, 2009.

Cooke, D. L., Rohleder, T.R., and Silver, E.D., Finding effective schedules for the economic lot scheduling problem: A simple mixed integer programming approach, *International Journal of Production Research*, vol. 42, pp. 21-36, 2004.

Elmaghraby, S. E., The economic lot scheduling problem (ELSP): review and extension, Management Science, vol. 24, pp. 587–597, 1978.

Gallego, G., and Joneja, D., Economic Lot Scheduling Problem with Raw Material Considerations. *Operations Research*, vol. 42, no. 1, pp. 92-101, 1994.

Ghare, P. M., and Schrader, G. F., A model for an exponentially decaying inventory, *Journal of Industrial Engineering*, vol. 14, pp. 238–243, 1963.

Jackson, P.L, Maxwell. W.L, and Muckstadt, J.A., The joint replenishment problem with a power-of-two restriction, *IIE Transactions*, vol. 17, pp. 25-32, 1985.

Khabbazi. A., Atashpaz. E., and Lucas. C., Imperialist competitive algorithm for minimum bit error rate beamforming, *International Journal of Bio-Inspired Computation*, vol. 1, pp. 125-133, 2009.

Moon, I.K., Cha, B. C., and Bae, H. C., Hybrid genetic algorithm for group technology economic lot scheduling problem, *International Journal of Production Research*, vol. 44, pp. 4551-4568, 2006.

Raafat, F., Survey of literature on continuously deteriorating inventory models. *Journal of Operational Research Society*, vol. 42, pp. 27–37, 1991.

Yao, M. J., and Elmaghraby, S. E., The Economic Lot Scheduling Problem under Power-Of-Two Policy, *computers and mathematics with applications* (CMWA), vol. 41, pp. 1379-1393, 2001.

Yao, M. J., and Huang, j. X., Solving the economic lot scheduling problem with deteriorating items using genetic algorithms, *Journal of Food Engineering*, vol. 70, pp. 309-322, 2005.

Zanoni, S., Segerstedt, A., Tang, O., and Mazzoldi, L. Multi-product economic lot scheduling problem with manufacturing and remanufacturing using a basic period policy. *Computers & Industrial Engineering*, in press, 2012.