Design Evaluation of Mechanical Brakes at System Conceptual Design Stage: A MADM Approach

Ankush Anand
School of Mechanical Engineering
SMVD University, Katra, Jammu and Kashmir, India

Mohd. Farooq Wani
Department of Mechanical Engineering
National Institute of Technology, Hazratbal, Srinagar, Kashmir, India

Abstract

This paper presents a methodology based on digraph and matrix method for the evaluation and selection of Mechanical Brakes at system conceptual design stage. The sustainability attributes of a mechanical brake are identified and are used to evaluate the design sustainability of mechanical brakes. The ideal value of this index is also obtained, which is useful in assessing the relative sustainability value of Mechanical brake design alternatives at conceptual design stage. A step-by-step procedure for design evaluation of mechanical brakes through sustainability is presented and illustrated with the help of an example.

Keywords: Sustainability, digraph, Mechanical Brakes, Sustainability Index.

1. Introduction. The automobile industry is changing at a very fast rate, the continuous research process has contributed a lot in automobile sector, which finds a major share with respect to the ever changing scenario of automobiles. The research activities which are being carried out in today's competitive world involve billions of dollars which are invested in the design and development of automobiles or systems, which are safe, cheap and are also delivering better performance. In other words, the traditional mechanical systems are being replaced by improved electromechanical systems that are able to do the same tasks faster, more reliably and more accurately. In a brake design process, the overall objective is to design and develop the braking systems, so as to reduce the accidents .Design and Manufacturing engineers are under tremendous pressure to develop new design methodologies to accommodate increasing customer requirements/demands. These include high performance at minimum cost, minimum service costs and reuse of the systems. In the classical approaches of design, conceptual design, detailed design, process planning, prototype manufacturing and testing are considered as sequential processes (Li and Li, 2000).

In order to take into account all aspects of system sustainability during the system design stage, it is inevitable that the designer needs an appropriate and efficient methodology/tool that aid the designer in the design and evaluation of a system from life cycle engineering point of view. Various methodologies have been developed by the researchers for sustainability of systems. Fitch and Cooper (2004) have proposed a methodology for life cycle analysis of systems at design stage, based on cost of energy for materials used in the systems. In this research study, authors have evaluated total energy required for reinforcing beam material throughout life cycle. Fitch and Cooper (2005) have developed life cycle design model for adaptive and variant design. The authors have used LCDM (life cycle design model) to evaluate the material substitution opportunities to minimize the resource consumption and life cycle air emissions, increase the recyclable mass for a Ford C- class sedan. These researchers have also identified the vehicle design scenarios that offer modest improvements in environmental performance and related cost trade-offs. Ishii (1995) has proposed a methodology for evaluation of life cycle cost of systems. In this methodology, the author has estimated cost of serviceability during ownership period of a system. The author has also obtained labor step cost based on labor time, labor penalty (hours) and logistic support. Zhu and Deshmukh (2003) proposed the application of Bayesian decision networks to study the impact of design decisions on the life cycle performance of systems. In this research study, the authors have demonstrated that proper decision making at design stage significantly influences life cycle cost (LCC). Lida et al., (2007) have proposed a decision support system for system design under concurrent engineering environment, using fuzzy decision making mathematical approach. In this method, the evaluation of a system design is carried out on the basis of two concurrent subsystems, i.e., external concurrent subsystem and internal concurrent subsystem. These include market investigation, material, and external components, functional design, assembly design, manufacturing design and environment design.
Hodgson and Harper (2004) proposed a holistic approach of material selection for the system life cycle design. In this research work, the authors have demonstrated the effective use of material for various aspects of life

cycle of the system. The authors have also identified various attributes of material selection for life cycle design of a system. These are performance, impression, manufacturability, environment, legislative compliance, reliability, maintainability and usability. Suh and Huppes (2005) have carried out the review of various proposed life cycle inventory methods for the system design. In this review, the authors have compared six life cycle inventory methods with one another and have also compared these methods with ISO standards. Spitzlay et al.,(1997) carried out the life cycle cost (LCC) analysis of a fuel tank for two different design alternatives on the basis of design for end of system, design for manufacturing, design for use and design for environment. In this research methodology, multilayer high density polyethylene (HDPE) tank and steel tank (ST) were used for comparison. It was revealed that design alternative (i.e., HDPE tank), has lower LCC, as compared to ST.

Kato and Kimura (2003) proposed a new system life cycle technology using the Quality Function Deployment (QFD). In this research work, the researchers have proposed that future technologies need to consider life cycle aspects, such as, system judgment for reuse or supply and demand matching, by carrying out an analysis about the relationship between requirements and technologies in the field of automobile. Tomiyama et al., **(**1997) have proposed a holistic approach to life cycle design, marketing, material acquisition, manufacturing, serviceability and disposal / recycling. Kato and Kimura (2004) have developed a system life cycle design method using a strategic analysis. In this methodology, the authors have modeled the relationship of price and cost in each process of part exchange, transportation, remanufacture and selling. Kainuma (2004) proposed multi-criteria decision analysis for life cycle assessment (LCA) on the basis of five attributes. These are air pollution, water pollution, energy consumption, waste generated and customer satisfaction. Hundal (1997) and Hundal (1998) have carried out a detailed design analysis on environment and other aspects of life cycle need to be considered during system development process. In this research study, it was revealed that consideration of eco-design, waste prevention, innovation, etc. at design stage are useful for successful development of system.

In addition, these researchers have also considered that system sustainability is a multi-criteria decision making process. The various design concepts are considered as the various alternatives for system design. Sustainability parameters such as, design, manufacture, safety etc., have been considered as the design criteria for evaluation and comparison of different concepts of system design. However, these researchers have not taken into consideration all aspects of sustainability of a system and also their interrelationship / interdependence with each other. These models can be analyzed using appropriate matrix to develop sustainability expression of a system [17-23]. The recent standards of ISO -14040 (2006) has made it necessary for manufacturers and end-user of the system to carry out LCA to reduce environmental hazards, created due to solid waste, disposal and recycling methods of systems. In addition, it also suggests ways and means to be adopted at design stage for designing sustainable systems. The results thus obtained from these LCA studies are useful for a decision making process at system design stage. Wani and Anand (2010) carried out the sustainability modelling and sustainability evaluation of triboelements. In this research study, the researchers have considered PTFE (Polytetraflouroethylene) as a material for life cycle analysis using a multi criteria decision making approach. It is evident from these research studies, that various attempts have been made for design and evaluation of sustainability of a system separately or in combination. In addition, various sustainability features/attributes of a system in general have been identified.

Literature review, presented in above section, indicated the identification of all the attributes of system sustainability. These have been thoroughly investigated previously by Anand and Wani, (2010). These are discussed in following subsections.

1.1 Functionality/Performance based Evaluation

A design process is initiated by identifying the customer requirements through need analysis Ullman, (1997). These customer requirements are transformed into design parameters or functional requirements, Pahl and Beitz (1984). The main function is decomposed into various sub-functions depending upon the complexity and are then transformed into design objects. The design object is a physical object, i.e., component or assembly and is also called physical domain/structure of the system , Suh (1990). Desired output of the system is obtained through functional interaction. The performance of a system during its entire life cycle depends upon how best the functions are performed by the physical objects i.e., components or assemblies. Any cognitive weakness at the initial design stage of the system gets accumulated over its entire life cycle which not only culminates into poor performance of the system, but also leads to higher LCC. Complexity increases with the increase in the number of functions, which it has to perform during its life cycle. Functional analysis also helps in identifying critical components at system conceptual design stage, which helps a designer to enhance the reliability of system, Wani and Gandhi (2008). Based on the above discussion, the performance/ functional characteristics of the system during the life cycle depends upon how best, the initial design is carried out. Therefore, the first criterion for design and development of a system in a Sustainability process is identified as design for performance and is abbreviated as F/P. This criterion is termed as an attribute of system sustainability. This attribute is shown in Table 1.1. Second and third column of the table show requirements and facilitating factors and then, the Sustainability attribute of a system is derived and is shown in the fourth column. The information given in column second and third will help designers in identifying the attribute of system life cycle in a systematic and easier way.

1.1.2 Manufacture based Evaluation

Design features which facilitates the ease of manufacturing, reducing cost of manufacturing, less emission of toxic gases during processing and machining, minimum material wastage, material and energy conservation, reuse of the material at the end of life etc., potentially contribute towards designing the system from manufacturing point of view. Use of standard components and appropriate fastening techniques, provision for hinges and captive fasteners facilitate accessibility, assembly and disassembly of systems. Appropriate selection of manufacturing technologies and their processes need to be well ascertained at system conceptual design stage by the designer. All these aspects contribute towards second attribute for design and development of system life cycle. This attribute is called design for manufacture and is abbreviated as M. This attribute is shown in second row of Table 1.1.

1.1.3 Marketing based Evaluation

In the present day global market competition, the success of every system during sales and marketing is determined on the basis of various factors. These factors are shelf life of a system, aesthetics, ease of installation and commissioning, ease of transport, operation / handling, safety, weight, durability, operational cost, energy requirements, etc. The success of a system at this phase of the system life cycle also depends on environment friendly design of the system. All these aspects contribute towards the third attribute for design and development of system life cycle. This attribute is called design for marketing and is abbreviated as M_{KTG}. This attribute is shown in third row of Table 1.1.

1.1.4 Maintainence based Evaluation

LCC of a system is determined to a large extent by the cost of maintenance incurred during operational stage. It is relevant to mention here that often the cost of ownership is less than the cost of maintenance. This problem is further exaggerated by incorporating electrical, electronic, hydraulic and pneumatic components/assemblies in the system. Therefore, for maintaining these systems high skilled men are required. Moreover, labor cost and losses due to failures are of serious concern for manufacturers, as well as for domestic users of these systems. Therefore, designers need to include system design features/characteristics which facilitate maintenance of system at operational stage, with minimum cost of labor and spares. Design features, such as accessibility, ease of assembly and disassembly of components/assemblies, diagnosability, standardization of parts, ergonomic aspects, etc., help in reducing maintenance cost of the system at operational stage. A designer need to in-built these features / characteristics in the system at design stage.

Therefore, designer has to consider the maintainability of the system for improving the sustainability of the system. This is possible if the designer includes design for maintainability as one of the attributes of the sustainability of the system. This is shown in 4th row (3rd column) in the table.

1.1.5 Safety based Evaluation

Failures at operational stage are inevitable. These failures not only culminate into losses and wastage of resources but also lead to human losses at times during the operational stage of systems. Critical components and assemblies of the system need to be identified at system design stage, preferably at system conceptual design stage. This will enable the system manufacturers and engineers to built-in quality control during this phase of system life cycle. Reliability of a system helps in reducing failures at operational stage and also ensures safe use of the system by the end user. In the present day global market competition, designers need to incorporate reliability of various components and assemblies of the system, at system conceptual design stage. This is achieved by the principle of fail-safe, failure–free systems and also by increasing the redundancy of the system. Moreover, methods such as, FTA, FMEA, FMCEA etc., have also been used for increasing the safety, reliability and for identification of critical components and assemblies of the system. All these aspects contribute towards the fifth attribute for design and development of system life cycle. This attribute is called design for safety and is abbreviated as S. This attribute is shown in fifth row in the table.

1.1.6 Environment based Evaluation

The increasing problem of environmental degradation is a serious threat to humanity. Many steps have been taken previously in this regard and efforts are still on to maintain the ecological balance. The industries are one of the major sources of pollutants which adversely affect the environment. Environmental consciousness has become a major thrust area for all designers in the recent years. Designers have to take into consideration, conservation of energy, conservation of materials, minimum waste systems, reduction in emission of toxic substances, eco-friendly technology, reduction in air and noise pollution, use of longevity materials and components, etc. for the design of system at system conceptual design stage. All these aspects contribute towards the sixth attribute for design and development of a system in a sustainability process. This attribute is called design for environment and is abbreviated as E. This attribute is shown in sixth row of Table 1.1.

1.1.7 Disposal / Recycle based Evaluation

At the final phase of system life cycle, its disposal becomes equally important for the end user. In the recent years, it has been observed that Municipal Solid Waste (MSW) is of serious concern for designers, manufacturer and end user of the system. Moreover, increasing concern for material and energy conservation, demand that designers need to design the system and its components in such a way that there is further scope for recycling or reuse of the material and retention of quality. Designer has to consider various factors for recycling of systems at system conceptual design stage. These are ease of disassembly, simple to identify, and reconditioning. In addition, the factors facilitating disposal of system assemblies and components, such as conservation of energy and materials, appropriate disposal methodologies, use of technology compatible with ISO standards, emphasis on sustainable or green design, etc., need to be critically analyzed at design stage. These factors are considered for design and development of system from sustainability point of view under Design for Disposal /Recycle. This is called Design for Disposal attribute of sustainability and is abbreviated as D/R. This attribute is shown in 7^{th} row in Table 1.1.

Sustainability attributes for a system, in general are identified in this section. For this, the attributes like Performance based Evaluation (P/F), Manufacture based Evaluation (M), Marketing based Evaluation (M_{KTG}), Maintenance based Evaluation (M_{AINT}), Safety based Evaluation (S), Environment based Evaluation (E), Disposal/ Recycle based Evaluation (D/R) are identified. These are defined as sustainability attributes of system and are represented by A^d_i. These are shown in Table 1.1.

The sustainability attributes identified above and given in Table 1 are represented mathematically as :

$$A^d = (A^d_1, A^d_2, \ldots\ldots\ldots\ldots A^d_n) \qquad (1.1)$$

where A^d represents the Sustainability attribute and A^d_n represents n^{th} attribute.

These sustainability attributes are used in developing system sustainability model and it leads to the evaluation of sustainability of a given system. Using this, a designer can evaluate various design alternatives of a system at system conceptual design stage. These are discussed in the following sections.

Table 1.1: Sustainability attributes of a system

S. No.	Sustainability Requirement	Sustainability Facilitating Factor	Sustainability attribute
1.	Identify the customer requirements, Analysis of actual need, Concept hunt, transformation of customer requirements into design specifications, Identification of critical components and functions, Development of design concepts, Feasibility of design concepts.	Need analysis, Brainstorming, Development of a functional and structural tree of the system, Quality function deployment (QFD), Failure Mode Effect Critical Analysis (FMECA),	***performance/Function*** (F/P)
2.	Ease to manufacture, less emission of toxic gases, Ease of assembly / disassembly, Use of simple and standard components and assemblies, Minimum labor/ machine costs, Minimum material wastage, Ease of Disposal/Recycling.	Selection of appropriate manufacturing technologies, use of eco-friendly / bio compatible materials, accessibility, assembly, use of standard joining techniques like Fastening, welding, soldering, wrapping, Latching, connections and edge connectors, Hinges, captive fasteners, use of less number of machines, simple design, systematic sequence of manufacturing processes.	***Manufacture*** (M)
3.	Long shelf life, ease of installation and commissioning, ease of transport, operation/ handling, safety, durability , low energy requirements, Less number of failures.	Longevity of material, Failure free design, Less (weight, size and volume), number. of components, minimum number of corners/ sharp edges, etc., protection from heat leakage, insulation, attractive looks/ display, low operational cost, material selection.	***Marketing*** (M_{KTG})
4.	Easy opening/ fastening of parts and components of various assemblies and sub assemblies, reduced number of components and	Fastening, welding, soldering joints, wrapped connections and edge connectors, weight size and volume of assembly, accessibility, assembly,	***Maintenance*** ($M_{AINT.}$)

	assemblies, Compatibility between mating components when replacing a faulty item/component, high proficiency in carrying out maintenance work and diagnosis, hazard free environment for maintenance work.	use of standard components e.g., bearings, fasteners with dimensional and functional tolerances, Skills of maintenance personnel, weight size and shape of equipments from ergonomics point of view, Insulation, Leak proof system.	
5.	Operational techniques, Minimum number of failures, higher availability, Low breakdowns, perform desired functions, Higher safety Minimum emission of toxic wastes, Increased Functional life.	Hazard and operatibility study (Ergonomics),Failure free design, part, materials and processes, proper testing and inspection processes, FMEA, FMECA, BIT, Quality maintenance, control/review, use of standard components and materials, Insulation and safety guards, use biodegradable/ biocompatible materials, chemicals, etc., providing corrosion resistance, methods of lubrication,	***Safety*** (S)
6.	Minimum emission of toxic substances to environment, Ease of disposal/recycling, minimum waste of materials and energy, minimum energy consumption, extended life of lubricants, materials and components, Ease of assembly /disassembly Surface coatings.	Systematic use of machinery (tools and equipments), Use of biodegradable materials, innovative machines and strategies (vibration isolation), proper selection of components, and processes. Long time lubrication, biodegradable materials. Minimum number of components, use of suitable coating materials.	***Environment*** (E)
7.	Ease to assemble and disassemble, eco-friendly, minimum cost, easy to machine, conservation of energy(retention of quality/ strength), material conservation, recycling of materials.	Minimum no. of sub assemblies or parts, use of technology compatible with ISO standards, selection of appropriate disposal methods, standard materials (biodegradable), Dust collection system, Equipment to neutralize and separate out harmful components or sorting mechanisms for waste separation.	***Disposal/ Recycle*** (D/R)

1.2 Sustainability Modelling

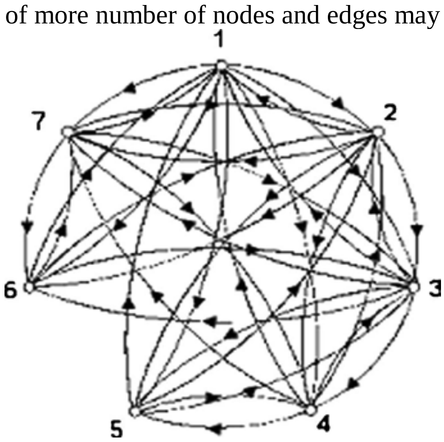
Sustainability attributes of a system have been identified in the previous section. Each attribute facilitates the system sustainability through its contributing factors/features. Each Attribute possesses distinctive characteristics which helps to develop relationship among these attributes, i.e., how one attribute is related to the other in determining the performance of a system during its entire life cycle. The relationship among the attributes is called degree of relationship. The relationship between the attributes varies. It may be taken as, strong, to none as two extremes of degree of relationship. In between, this is assumed as medium and weak relationship. These are developed and are also represented in Table 1.2. The table shows clearly that the degree of relationship varies among attributes. The degree of relationship is represented in last column of the Table 1.2. The Seven attributes of sustainability of a system in general are shown with their serial numbers. The degree of relationship for a sustainability attribute among the other attributes is shown against the attribute in its row entry as their serial number.

It is a fact that any cognitive weakness at the initial phases of system sustainability, gets accumulated over its entire life cycle which not only culminates into poor performance of the system, but also leads to higher LCC. Therefore, sustainability features /characteristics which facilitates all aspects of life cycle of a system are in-built in the system at first phase of system design, i.e., at Design for Performance, in terms of sustainability requirements/facilitating factors. This shows that a strong relationship exists between F/P and other attributes of sustainability of a system. This is shown in Ist row, 3rd column in Table 1.2. The M and M_{aint}, are facilitated by design characteristics accessibility, assembly, standardization, etc. These Sustainability facilitating factors ascertain that there also exists a strong relationship between these two attributes. This is shown in 2^{nd} row, 3^{rd} column in the table. Similarly, the relationship among the design attribute M_{ktg} and other attributes (F/P, M, M_{aint}, S, E, and D/R) is weak and is shown in 3^{rd} row, 3^{rd} column in the table. However, the relationship of design attribute S is none with attributes E and D/R and is shown in 5^{th} row, 3^{rd} column in the table. In the same way, the relationship among attributes is given in the Table 1.2. It is however, mentioned again that the relationship among these attributes need to be derived by a concurrent engineering team comprising of designers and experts from other fields of system life cycle. Sustainability modelling of system requires consideration of Sustainability attributes and their relationship. This can be conveniently represented using graph–theoretical concepts [21].

Sustainability attributes digraph G^d (N^d, E^d) for a system is defined, where N^d= ($N^d{}_1$, $N^d{}_2$, ……….., $N^d{}_N$) is a set of nodes representing Sustainability attributes. Sustainability attributes digraph is abbreviated as Sustainability ^{g}. An edge E^d={$e^d{}_{12}$, $e^d{}_{13}$,…., $e^d{}_{ij}$ } is a set of edges among nodes representing degree of interrelationship e.g., the edge $e^d{}_{12}$ connects node $N^d{}_1$ to $N^d{}_2$. The direction of edge $e^d{}_{12}$ from node $N^d{}_1$ to node $N^d{}_2$ indicates the relationship of $A^d{}_2$ and $A^d{}_1$, i.e., $A^d{}_1$ is related to $A^d{}_2$. It is possible that any two attributes ($A^d{}_i$ and $A^d{}_j$) of an attribute digraph are related to one another. This is represented by two directed edges ($e^d{}_{ij}$ and $e^d{}_{ji}$) in the opposite directions forming a closed loop. Sustainability ^{g} of a system in general, is developed considering the seven attributes identified in above sections, and their relationship-strong, medium or weak, to none (Table 1.2). This is shown in Fig. 1. In Fig. 1, the direction of edge $e^d{}_{12}$ from node $N^d{}_1$ to node $N^d{}_2$ indicates the relationship of F/P and M, i.e., F/P is related to M. The direction of edge $e^d{}_{21}$ from node $N^d{}_2$ to $N^d{}_1$ indicates relationship of M is related to F/P. In this way, the edge $e^d{}_{12}$ and $e^d{}_{21}$ from node $N^d{}_1$ to node $N^d{}_2$ and node $N^d{}_2$ to $N^d{}_1$ respectively form a closed loop. In similar, way edges between the nodes in Fig.3.1 represent the relationship among the attributes.

Sustainability^g is a graphical representation of Sustainability attributes and their interrelationship. The graphical representation augments further understanding the Sustainability of systems, which needs to be exploited at design stage. This shows a clear visual picture of system Sustainability.

However, for handling the digraph conveniently it is represented by matrix due to the fact that presence of more number of nodes and edges may lead to a complex figure.



1.F/P 2. M
3.M_{ktg} 4. M_{aint}
5. S 6. E
7. D/R

Figure 1: Sustainability Attributes Relationship Digraph of a System

1.3 Matrix Representation of A Sustainability Attributes Digraph

Matrix representation of the Sust^g for a system consisting of all the important sustianbility attributes e.g., F/P, M, M_{KTG}, M_{AINT}, S, E, and D/R. In this case, the facilitation among all these attributes is considered to develop the sustianbility expression of the system. The Sust^g of these seven attributes is developed based on the discussion in above section and is shown in Fig. 1.

Let the Sust^g in general, with N nodes be represented by N^{th} order binary matrix [r_{ij}], where r_{ij} represents relationship of i^{th} attribute with j^{th} attribute with r_{ij}=1, if i^{th} attribute is related to j^{th} attribute, otherwise r_{ij}= 0, as an attribute cannot have relationship with itself. Sustainability matrix for the Sust^g is shown in of Fig. 1 and is written as:

$$R^d = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 0 \end{bmatrix} \qquad (1.2)$$

The diagonal elements in the matrix have value 0 and off diagonal elements have value 0 or 1. This implies that in this matrix only relationship among attributes is considered and value of the attributes is not taken into account. To incorporate this, a new matrix called Sustainability Attributes Relationship Permanent Matrix is defined. An Sustainability Attributes Relationship Permanent Matrix representing the sustainability attributes digraph shown in Fig. 1 is written as:

$$Q^d = [AI^d + R^d] = \begin{bmatrix} D & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & D & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & D & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & D & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & D & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & D & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & D \end{bmatrix} \qquad (1.3)$$

Where I^d is identity matrix and D is value of the attributes. It is noted from the matrix expression (1.3), that all the diagonal elements have been assigned the value of D i.e., each attribute has equal value in the system. However, this is not true in actual practice. Also, the relationship of one attribute with the other attribute (i.e., r_{ij}) may take any value rather than extreme value 0 and 1.Thus there is a need for considering general attribute value (i.e., diagonal elements D_i) as well as degree of relationship (i.e., off diagonal elements r_{ij}) to develop matrix expression characteristic of the Sustainability of a system.

These are taken into consideration through a new matrix called Variable Sustainability Attributes Relationship Permanent Matrix ($VSust^{per}$). A permanent is a standard matrix operation used in combinatorial mathematics. It is an analog of a determinant where all the signs in the expansion by minors are taken as positive [20]**.** Let the off diagonal elements of matrix Q^d is represented as r_{ij} instead of 1, where i^{th} attribute is related to j^{th} attribute. Let us also define a diagonal matrix, H^d, with diagonal elements D_i representing variable value of i^{th} attribute. If an attribute is excellent in a system, it is a maximum value. If an attribute is weak it is assigned a minimum value. In general, most of the attributes are assigned intermediate values of the interval scale, as attribute may have medium contribution in sustainable design. The attribute value may be assigned qualitatively or quantitatively.

Variable Sustainability Attributes Relationship Permanent Matrix for the digraph shown in Fig. (1) is given as:

$$Q^d = [H^d + F^d] = \begin{bmatrix} D_1 & r_{12} & r_{13} & r_{14} & r_{15} & r_{16} & r_{17} \\ r_{21} & D_2 & r_{23} & r_{24} & r_{25} & r_{26} & r_{27} \\ r_{31} & r_{32} & D_3 & r_{34} & r_{35} & r_{36} & r_{37} \\ r_{41} & r_{42} & r_{43} & D_4 & r_{45} & 0 & r_{47} \\ r_{51} & r_{52} & r_{53} & r_{54} & D_5 & 0 & 0 \\ r_{61} & r_{62} & r_{63} & 0 & 0 & D_6 & r_{67} \\ r_{71} & r_{72} & r_{73} & r_{74} & 0 & r_{76} & D_7 \end{bmatrix} \qquad (1.4)$$

It may be noted that any matrix expression (1.4), represents value of attributes (D_i's) and their relationship (r_{ij}'s) for the given system. Permanent of this matrix (or Per (D)) i.e., $VSust^{per}$, is called variable sustainability attributes relationship permanent function, abbreviated as VPF-d. VPF-d is characteristic of the system sustainability as it contains number of terms, which are its variant.

As stated above, permanent is a standard matrix function and is used in combinatorial mathematics [120]. Use of this concept in sustainability modeling will help in representing structural information from combinatorial consideration. This is desirable to associate proper physical meaning to the structural components and their combinations. Moreover, using this no negative sign will appear in the expression and hence no information is lost.

VPF-d of matrix, expression (1.4) is written in sigma form as:

$$
\begin{aligned}
Per(D) = {} & \sum_{i=1}^{7} D_i + \sum_i \sum_j \sum_k \sum_l \sum_m \sum_n \sum_o (r_{ij}.r_{ji}) D_k D_l D_m D_n D_o + \\
& \sum_i \sum_j \sum_k \sum_l \sum_m \sum_n \sum_o (r_{ij}.r_{jk}.r_{kl} + r_{lk}.r_{kj}.r_{ji}) D_l D_m D_n D_o + \\
& \sum_i \sum_j \sum_k \sum_l \sum_m \sum_n \sum_o (r_{ij}.r_{jk}.r_{kl} r_{lm} + r_{ml} r_{lk}.r_{kj}.r_{ji}) D_m D_n D_o + \\
& \sum_i \sum_j \sum_k \sum_l \sum_m \sum_n \sum_o (r_{ij}.r_{jk}.r_{kl} r_{lm} r_{mn} + r_{nm} r_{ml} r_{lk}.r_{kj}.r_{ji}) D_n D_o + \\
& \sum_i \sum_j \sum_k \sum_l \sum_m \sum_n \sum_o (r_{ij}.r_{ji})(r_{kl}.r_{lk})(r_{mn}.r_{nm}) D_o + \\
& \sum_i \sum_j \sum_k \sum_l \sum_m \sum_n \sum_o (r_{ij}.r_{jk}.r_{kl} r_{lm} r_{mn} r_{ni} + r_{in} r_{nm} r_{ml} r_{lk}.r_{kj}.r_{ji}) D_o + \\
& \sum_i \sum_j \sum_k \sum_l \sum_m \sum_n \sum_o (r_{ij}.r_{ji})(r_{kl} r_{lm} r_{mn} r_{no}) + (r_{on} r_{nm} r_{ml} r_{lk}) + \\
& \sum_i \sum_j \sum_k \sum_l \sum_m \sum_n \sum_o (r_{ij}.r_{jk}.r_{kl} r_{lm} r_{mn} r_{no}.r_{oi} + r_{io} r_{on} r_{nm} r_{ml} r_{lk}.r_{kj}.r_{ji})
\end{aligned}
\qquad \text{...............} \ (1.5)
$$

The equation consists of number of terms. These are arranged in eight (i.e., N+1 = 8, with N=7 in this case) groupings in descending order of number of attributes value. The first grouping contains only one term and is a system of seven attribute values, (i.e., $D_i D_j D_k D_l D_m D_n D_o$). The second grouping is absent as there are no self loops present in the digraph. The third grouping contains number of terms and each term is a multiple of five attribute values, (i.e., $D_k D_l D_m D_n D_o$) and 2-attribute relationship loop ($r_{ij} r_{ji}$). Similarly, Terms of 4th, 5th and 6th groupings are a multiple of attribute values and their relationship loops. The 7th and 8th groupings contain two subgroupings. The terms of first subgrouping consist of one attribute value (D_o) and 3-attribute relationship loops (i.e., ($r_{ij} r_{ji}$) ($r_{kl} r_{lm}$) ($r_{mn} r_{nm}$)). The terms of second subgrouping is a multiple of 1-attribute value (i.e., D_o) and 6-attribute relationship loops (i.e., ($r_{ij} r_{jk} r_{kl} r_{lm} r_{mn} r_{ni}$) or its pair ($r_{in} r_{nm} r_{ml} r_{lk} r_{kj} r_{ji}$). The terms of 8th subgrouping are also arranged in two subgroupings. The terms of first subgrouping are a multiple of 2- attribute relationship loops (i.e., $r_{ij} r_{ji}$) and 4- attribute relationship loops (i.e., $r_{kl} r_{lm} r_{mn} r_{no}$) or its pair (i.e., $r_{on} r_{nm} r_{lm} r_{lk}$). Each term of second subgrouping consist of 7-attribute relationship loop (i.e., $r_{ij} r_{jk} r_{kl} r_{lm} r_{mn} r_{no} r_{oi}$) or its pair (i.e., $r_{io} r_{no} r_{nm} r_{lm} r_{lk} r_{kj} r_{ji}$). It is once again stated that VPF-d when expanded takes into consideration all terms of the matrix expression and thus no information is lost. This shows that VPF-d is a powerful expression for analyzing sustainability of a system.

Table 1.3: Scoring Criteria for Sustainability attribute – F/P

S. No.	**Description of Scoring Criterion**	**Score (D_i)**
1.	Customer requirements are well established, functional interaction efficiently provide desired output, quality and reliability analysis reduces failures and increases safety	4
2.	Fulfils a few of the above requirements	2
3.	None of the above.	0

Table 1.4: Scoring Criteria for Sustainability attribute - M

S. No.	**Description of Scoring Criterion**	**Score (D_i)**
1.	Ease to machine with minimum emission of gases to the environment, Ease of assembly/ disassembly, and Environmental hazards are less.	4
2.	Fulfils a few of the above requirements	2
3.	None of the above.	0

Table 1.5: Scoring Criteria for Sustainability attribute –D/R

S. No.	**Description of Scoring Criterion**	**Score (D_i)**
1.	Bio compatible materials, innovative disposal and recycling strategies, Minimum emission of toxic gases during recycling/disposal, Minimum energy requirements.	4
2.	Fulfils a few of the above requirements	2
3.	None of the above.	0

However, the suggested score values as per Tables 1.3 and 1.4 will be useful for the user to start with. This assessment becomes easier for the team comprising of designers and other experts of life cycle engineers, rather than an individual. It is expected that a team or individual fully conversant with system will have no difficulty in assessment.

Numerical value of permanent, i.e., VPF-d becomes a powerful means for Sustainability and evaluation of a system as it contains various structure invariant of Sustainability. An index called

Sustainability index is defined as the numerical value of the permanent ($VSUST^{per}$). This index is a measure of the ease of Sustainability of a system. Based on the index value, the design alternatives are evaluated. The best alternative is the one having the highest index value and is selected . The ideal value I^d_{ideal} of Sustainability index is calculated to be $14.3*10^6$. Comparison of Sustainability value of system (I^d_i) can be relatively made with ideal value I^d_{ideal} .This comparison show to what level Sustainability of the system is achieved of the ideal value. This is obtained as:

$$I^d_r = \frac{I^d_i}{I^d_{ideal}} \times 100 \ \% \qquad (1.6)$$

where, I^d_r is the relative Sustainability index, which represents Sustainability value of the system as % of the ideal value of index. This relative index provides designer qualitative information for improving Sustainability of the system. A scale 0-4 is proposed for assigning value to attributes and their degree of relationship. The user may select an appropriate scale e.g., 0-5, 0-10 or 0-100. However, it is desirable to select lower scale value to obtain manageable value of index and also to reduce subjectivity. The lower scale is adopted so that there is a limited range of score values available to the designers for uniform evaluation of design alternative from Sustainability point of view. The final result will not be changed if the user chooses a different scale. The relative Sustainability index value (I^d_r) i.e., Eq. (1.6) will be useful in this regard.

1.5 Steps - Sustainability Analysis and Index Evaluation

The procedure proposed previously for Sustainability and evaluation of index for a system is given now. Consider the given system and its various conceptual design alternatives (q= 1, 2,……). Study functions and structure details, and design features from life cycle design point of view.

1. Consider the first alternative of the system (i.e., q=1). Identify Sustainability attributes (A^d_i, $i=1,2,3……,N^d$) of the system (Refer section 3.1) and also assign values to attributes i.e., D_i, $i= 1,2,…..,N^d$ (Refer section 1.3 and also use Table 1.3 and 1.4) .
2. Identify the relationship among attributes i.e., in terms of degree of relationship (r_{ij}). Assign value to r_{ij} using Table 1.2.
3. Develop $SUST^g$ for the system alternative.
4. Write $VSUST^{per}$ (refer sections 1.3 and 1.4). This will be a N x N matrix with diagonal elements D_i's and off diagonal elements r_{ij}'s.
5. Derive Sustainability expression (VPF-d) or permanent function i.e., permanent ($VSUST^{per}$) on the line of expression (1.5). Refer section 1.4 for details.
6. Evaluate the ideal value of Sustainability index I^d_{ideal} from VPF-d obtained in step 6 by substituting D_i= 4 and r_{ij} as obtained in step 3.
7. Use VPF-d and substitute the value of D_i and r_{ij} obtained in step 2nd and 3rd to evaluate Sustainability index I^d_i. Determine also the value of I^d_r using equation (1.6).
8. Consider the 2nd alternative (i.e., q=2) and repeat step 2 to 5 and 7.
9. Compare the Sustainability of all alternatives based on step 6 to 8 and identify the best alternative from life cycle point of view.

1.6 Example

The proposed methodology can be used for both new and existing designs for design and evaluation of life cycle of a system. An example of a mechanical brake system is considered here in this section. The example considered here is of a mechanical brake system with two design alternatives, and is meant for illustrating the proposed procedure.

1.6.1 Example - Mechanical Brake System

An example of Sustainability of a mechanical brake system is considered here for illustrating the proposed procedure. The designer has developed two design concepts/alternatives for this brake system. These are Drum brake and, Disc brake system. The two design alternatives are to be evaluated from Sustainability point of view and then compared for selecting best design alternative. In both the brake systems available, the objective is to perform efficient braking action.

First of all, it is necessary to study the system concept details of first design alternative i.e., Disc brake. These Sustainability attributes are F/P, M, M_{KTG}, M_{AINT}, S, E and, D/R. The value to various attributes is assigned on the basis score values using Tables above. For example, the value of attribute F/P is obtained from Table 1.3

Therefore, the values assigned to various life cycle attributes in case of first design alternative i.e., Disc brake system are V_1=4, V_2=4 V_3**=4,** V_4**=4,** V_5**=4,** V_6=2**,** V_7=2**.** This completes step 2 of the design procedure.

The relationship among these attributes, i.e., the degree of interrelationship r_{ij} are also identified.

The ideal value of index, $I^d_{ideal} = 1.19*10^6$ is obtained from matrix expression as discussed above and this completes step 6 of the design procedure.

Therefore, $I^d_1 = 9.86552*10^5$. The value of I^d_r in this case is 82.85 %. This completes step 7.

Now, the second design alternative, i.e., Drum brake system is taken into consideration for carrying out the life cycle design analysis.

So, $I^d_2 = 4.56844*10^5$ .The value of I^d_r in this case is 38.31 %. This completes step 8.

This completes the step 9 of the procedure. It is observed that the sustainability index decreases from $9.86*10^5$ to $4.56*10^5$ for the two design alternatives considered here in this example. The relative index value also decreases from 82.85 % to 38.31%.

However, the alternative with highest value of life cycle design index, i.e., $I^d_1 = 9.86*10^5$ is considered as the best design alternative from Sustainability point of view. Therefore, in this case, alternative-I i.e., Disc Brake System the best design alternative among both the design alternatives.

This simple example has been elaborated for the benefit of readers. Although the procedure may appear troublesome and time consuming if performed manually, however it is not so when using a computer and more so using an expert system.

Table 1.7: Mechanical Brake System – Sustainability Comparison

S. No.	**Design Alternative**	**Index Value (I^d_i)** ($*10^5$)	**I^d_r Value** (%)
1.	Disc Brake System	9.86	82.64
2.	Drum Brake System	4.56	38.31

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