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A Mathematical Model for Profit Maximization General Lot-sizing and Scheduling Problem (PGLSP)

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Abstract

This paper extends the general lot-sizing and scheduling problem (GLSP), to include demand choice flexibility. In Profit maximization general lot-sizing and scheduling problem with demand choice flexibility which is named PGLSP in this paper, the accepted demand in each period, lot-sizing and scheduling are three problems which are considered simultaneously. The proposed problem will result in better decisions especially when the mix of products is decided in mid-term planning and short-term decision is about scheduling of different machines. A mathematical model is developed for the PGLSP.

Keywords
Lot-sizing, scheduling, profit maximization, demand choice flexibility

1. Introduction
Lot-sizing and scheduling are important problems in the field of production planning. Most of the time, decisions for these two problems are made hierarchically. In this method, the lot-sizing problem is solved at first and then the result is used for sequencing and scheduling problem [1]. GLSP considers lot-sizing and scheduling problems simultaneously due to their dependency.

In most of the models for production planning and especially in lot-sizing and scheduling problems, the objective function is minimizing costs. In these models, the assumption is that all the customers' demands should be met. However, in a business with a goal of maximizing profit satisfying all potential demands may not always be an optimal solution [2]. In this situation, appropriate selection of demand (to be met) is an effective step for demand management.

In this paper, the profit maximization general lot-sizing and scheduling problem with demand choice flexibility is studied. The accepted demand in each period can vary between its upper and lower bounds. The upper bound could be the forecasted demand and the lower bound could be the organization commitments towards customers or minimum production level according to production policy. In other words, the amounts of accepted demands, lot-sizing and scheduling are problems which are considered simultaneously. According to these assumptions the objective function of the problem is maximizing the revenue of sales minus the production, inventory holding, and setups costs. The result of this model is the amount of accepted demand for each product in each period, the size of production lots, and the sequence of them. In this problem, the mix product problem which pertains to mid-term planning and lot-sizing and scheduling problems which relate to short-term scheduling are considered in one problem.

The proposed problem will result in better decisions especially when mix of products is decided in mid-term planning while short term decision is about scheduling of different machines. This model is presented based on the characteristic of industries like moquette weaving industry. In this type of industry setup times and costs are considerable so it is wise to use the GLSP model. Besides that selecting the demand without considering back order costs is also possible and predicting the exact amount of demand is impossible and it is more reliable to define upper and lower bounds for it. Wholesalers accept this policy, so sometimes the amount of products they delivered is less than their orders. For preserving the percentage of products and enhancing the service level, the lower bounds are greater than zero.

We have a few products in this company in different models and colors and the planning period of two weeks and the planning horizon of 8 weeks are suitable for the company.

2. Literature Review

Because PGLSP, which is a new problem, is a combination of GLSP and demand choice flexibility, we discuss the related literature of these two problems.

GLSP was first presented by Fleischmann and Meyr [3]. For modelling GLSP, the authors use two kinds of time buckets, small buckets, and large ones. Small buckets or positions are within the large buckets or periods. The positions are used for sequencing. Meyr [4] extended GLSP to deal with sequence-dependent setup times. Kocklar [5] represented a new model for GLSP with sequence-dependent setups by using the combination of Meyr [4] approach for modelling and transportation problem. The basic idea in this model is to disaggregate the production variables by relating each production quantity to the period at which it will be required.

Gupta and Magnusson [6] represented a new model based on the capacitated lot-sizing problem (CLSP) with sequence-dependent setup times. This problem is related to travelling salesman problem (TSP) or the vehicle routing problem (VRP) problem. Setup costs in GLSP are like distance cost in TSP. Because TSP is an NP-hard problem, CLSP with sequence-dependent setup is also NP-hard [6]. Almada-lobo et al [7] have demonstrated that the Gupta's model does not avoid disconnected sub tours . Almada-lobo et al [8] presented a more efficient model especially in comparison with the models which use the Meyr [4] approach for scheduling.

Simultaneous lot-sizing and scheduling is noticed in many practical situations. Pattloch et al [9] represented a heuristic algorithm for production lot scheduling in tobacco industry. Quadt [1] discussed the lot-sizing and production scheduling for flexible flow-line and represented a case study in a semiconductor factory. This problem was modelled and solved for soft drink production [10,11], steel casting factory [12] and animal food plant [13]. These studies emphasized the importance of considering lot-sizing and scheduling at the same time due to their interactions.

Companies often produce many products, but one of the most important problems is selecting which products to produce. This problem is known as the product mix problem [14]. The problem is to find a production plan that can maximize the net profit of the company by considering the resource constraint and each product demand. This problem only calculates the amount of production and doesn't take into account lot-sizing and scheduling problem. Amount of production should be within its upper bound and lower bound. PGLSP is a combination of the mix product problem and GLSP which are pertaining to mid-term planning and short-term scheduling.

For most production planning problems with the assumption of responding to all demands, because the net profit from selling all products is fixed, the objective is to minimize different costs. Flexibility in demand choice results in changing this objective to maximizing the sales revenues minus different costs.

Haugen et al [15] studied profit maximization capacitated lot-sizing problem (PCLSP) and represented a mathematical model for it. In this model, demand is a function of price and price is a variable in the model. In this problem the demand is determined indirectly by the model. The objective function is maximizing sales revenue minus different costs. In another paper, Haugen et al [16] studied the large scale of the same problem.

Geunes et al [17] represented a model that implicitly decides, through pricing decisions, the demand levels of the firm which should be satisfied in order to maximize the profit. Merzifonluoglo and Geunes [18] studied a planning problem which determined the optimal level of demand, production, and inventory for every planning period when flexibility exists in selecting demands and their delivery timing. They use a variable in their model which determines the proportion of accepted demand with the objective of maximizing the profit. Shen [2] studied the demand choice flexibility in a supply chain and with the objective of profit maximization.

3. PGLSP Definition and Formulation

3.1 PGLSP definition

PGLSP is described as follows:

Having P products and T planning periods, decision maker wants to define: (1) the accepted demand of each product in each period which is between upper and lower bounds, (2) the quantity of lots for each product, and (3) the sequence of lots. The objective function is maximizing the revenue of sales minus production, holding, and setup costs. Assumptions of the model are as follows:

- Backlog is not allowed.
- Setup times and costs are sequence-dependent. The triangular inequality is existed between setup times.
- The model has the characteristic of setup preservation. It means that if we have an idle time, set up state would not change after it.
- The breakdown of setup time between two periods is not allowed and the setup is finished in the same period in which it begins.

In this part we propose a model for PGLSP. Parameters for the model are represented in table 1

Table 1: Model parameters

Parameters		
C_t	Available capacity in each period	$t = 1, \dots, T$
Ld_{jt}	Demand lower bound for product *j* in period *t*	$j = 1, \dots, P, t = 1, \dots, T$
Ud_{jt}	Demand upper bound for product *j* in period *t*	$j = 1, \dots, P, t = 1, \dots, T$
h_j	Holding cost for one unit of product *j*	$j = 1, \dots, P$
r_{jt}	Sales revenue for one unit of product *j* in period *t*	$j = 1, \dots, P, t = 1, \dots, T$
Cp_j	Production cost for one unit of product *j*	$j = 1, \dots, P$
p_j	Processing time for one unit of product *j*	$j = 1, \dots, P$
S_{ij}	Setup cost for transition from product *i* to product *j*	$i = j = 1, \dots, P$
St_{ij}	Setup time for transition from product *i* to product *j*	$i = j = 1, \dots, P$
I_{j0}	Initial inventory level for product *j*	$j = 1, \dots, P$

3.2 The model of PGLSP

In this model lot-sizing variable is disaggregated according to period which is used and scheduling method uses the concept of TSP. Scheduling method is based on Almada-lobo et al [8] model and uses the concept of TSP. This model and its decision variables are proposed as follows:

Q_{jtk}: Production quantity of Product j in period t to fulfil the demand of period k
D_{jt}: The accepted demand of product j in period t
X_{ijt}: Binary variable which is 1 when setup occur from product i to product j in period t
A_{jt}: Positive variable which is 1 when machine is setup for product j at the beginning of period t
Y_{jt}: Auxiliary variable that assign product j to period t
Q_{j0t}: Quantity of initial inventory which is used in period t
R_{j0}: Unused portion of initial inventory at the end of the planning horizon

(1)

M

Subject to:

j (2)

j (3)

j (4)

j (5)

t (6)

t (7)

j (8)

i (9)

t

i (10)

t

j (11)

t

j (12)

The objective function of the model (1) is to maximize total revenue minus production and inventory cost of production quantities in different periods, setup costs, initial inventory holding cost until the period which is used, and the unused initial inventory holding cost. CQ_{jt} is a parameter which combines production cost and inventory cost of product j which will be used t periods after production period. This parameter is calculated by statement (13).

i (13)

Constraints (2) show that the accepted demand in period t is fulfilled by productions for this period or initial inventory. Constraints (3) guarantee that the accepted demand in each period is between its lower and upper bounds. Constraints (4) ensure that the initial inventory is equal to initial inventory which used in different periods and the unused portion of it. Constraints (5) guarantee that the machine is setup for production, and the production upper bound in these constraints is shown in statement (14).

j (14)
t

Constraint (6) shows the capacity limit for production. Constraints (7) to (9) determine the sequence of products, changing the setup states and transition of setup states between different periods. Constraint (7) shows that at the beginning of each period there is only one setup state. Constraint (8) guarantees that the setup states network is a connected network and constraint (9) avoids disconnected sub tours. Constraints (10) to (12) show the different kinds of variables in the model.

We can omit variable D_{jt} and use the statement $\sum_{t=0}^{k} Q_{jtk}$instead, consequently, constraint (2) is omitted from the model, and constraint (3) changed to constraint (15). With this change, a set of variables and constraints are omitted from the model, but because the model is MIP, this change should be well studied.

$$Ld_{jt} \leq \sum_{t=0}^{k} Q_{jtk} \leq Ud_{jt} \qquad j = 1, \dots, P, t = 1, \dots, T \qquad (15)$$

Number of binary variables and continuous non-negative variables for each model are represented in table 2.

Table 2: Number of variables

Continuous non-negative variables	Binary variables
2	*P*

In the models, because production lots are disaggregated according to the periods in which they will be used, the number of production variables and constraints is increased intensively by the increase in number of periods.

4. Numerical Experiments

4.1 Test data

In this part, we discuss experimental results. We use the method of Hass and Kimms [19] for generating data sets for GLSP with some modifications when needed. The method of producing these data is represented as follows.
Demand upper bounds, Ud_{jt}, have uniform distribution between [50, 70] and its lower bounds, Ld_{jt}, have the same distribution between [30, 49].

Production capacity in each period is calculated by statement (16).

(16)

In statement (16) capacity in each period is calculated based on the demands upper and lower bounds in the period. “util” is a parameter which adjusts the capacity and if it is increased, the capacity will be tighter. In different scenarios this multiplier is equal to 0.4, 0.6, or 0.8.

Setup times, St_{ij}, have uniform distribution with parameters [2,10] and setup costs are generated by statement (17).

(17)

Production cost of one unit of a product is generated based on its average setup costs and is calculated by statement (18). In this statement “Co” is a multiplier for adjusting production cost. This parameter in different scenarios is equal to 8 or 12.

(18)

Sale price for each product is calculated by relation (19). Sales prices have relation with production costs which is adjusted by "Pr" as a multiplier. This multiplier in a scenario with high profit is equal to 1.25 and in a scenario with low profit is equal to 1.1.

(19)

Without losing generality, we assume that initial inventory is zero and machine is setup for product 1 at the beginning of planning horizon.

According to different multipliers, 12 problem sets are proposed. These sets are shown in table 3.

Table 3: Data sets and their multipliers

NO.	1	2	3	4	5	6	7	8	9	10	11	12
Util	0.4				0.6				0.8			
Co	8		12		8		12		8		12	
Pr	1.1	1.25	1.1	1.25	1.1	1.25	1.1	1.25	1.1	1.25	1.1	1.25

Computational tests were executed on a computer with a 2.8 GHZ CPU and 3.25 GB RAM. For solving mathematical models we use GAMS 22.1 software and CPLEX solver. According to large amount of tests, for organizing input parameters and output results, we link C++ and GAMS.

4.2 Numerical tests

In this part the performance of the model and the effect of omitting variable D_{jt} is studied. As mentioned before, we can omit this variable from models and substitute the equal statement. To study this change, problems with different sizes have been generated. Number of products and periods for these problems are presented in table 4.

Table 4: Number of product and periods for Numerical tests

No.	$P\times T$	No.	$P\times T$	No.	$P\times T$	No.	$P\times T$
1	3×5	5	3×25	9	5×20	13	10×5
2	3×10	6	5×5	10	7×5	14	10×10
3	3×15	7	5×10	11	7×10	15	15×5
4	3×20	8	5×15	12	7×15	16	20×5

Two states of the Model were run in a time limit of 3600 seconds. For comparison, the percentage difference between computation time of an state and the best computation time is calculated. The best computation time is the minimum of the two states computation times for each problem. The average of this percentage (AvgDif%) for all problems in a set is represented as an evaluator for the model in that set. The higher this evaluator is, the worse is the model performance. The standard deviations of these percentages (DevDif) are also calculated. According to time limit, number of problems in each set in which the best solution is found (nBest) is also represented. Comparative results are shown in table 5.

Table 5: The results of models comparisons

No.	Util	Co	Pr	model with variable D_{jt}			model without variable D_{jt}		
				nBest	AvgDif%	DevDif	nBest	AvgDif%	DevDif
1	0.4	8	1.1	12	14.1	28.6	11	15.7	28.9
2			1.25	12	17.2	35.5	13	19.7	47.9
3		12	1.1	13	93.5	260.1	12	26.7	63.1
4			1.25	10	7.2	14.4	16	0	0
5	0.6	8	1.1	10	19.3	35.9	14	1.1	35.4
6			1.25	14	17.6	53.8	12	9.4	26.2
7		12	1.1	14	12.9	35.4	12	19.1	51.0
8			1.25	12	19.3	42.6	12	11.5	25.0
9	0.8	8	1.1	3	676.5	1589.1	16	0	0
10			1.25	12	17.2	35.5	13	19.7	49.0
11		12	1.1	5	1350.7	4337.1	16	0	0
12			1.25	6	283.3	675.2	16	0	0

As we can see in table 5 the performance of the model states are different for various sets. In other words, although the second state has a better performance in comparison with the first we cannot introduce one state as the best one. For example the model without D_{jt} variable in the set 1 has the worst performance; on the other hand, the model has the best performance in the 12th set.

To study the effect of different parameters on performance of the models, previous evaluators are calculated for different input parameters. To do this, problems in different data sets with similar parameters are put in to one set and evaluators are calculated for each new set. Table 6 shows the effect of multipliers “util”, “Co” and “Pr” on model performance.

Table 6: Multipliers effect on models performance

NO.	Multiplier	model with variable D_{jt}			model without variable D_{jt}		
		nBest	AvgDif%	DevDif	nBest	AvgDif%	DevDif
Util							
1	0.4	47	33.0	157.80	52	15.5	263.3
2	0.6	50	17.3	185.88	50	10.3	72.3
3	0.8	26	581.9	5190.62	61	4.9	77.1
Co							
1	8	63	127.0	886.00	79	10.5	200.0
2	12	60	294.5	2021.55	84	9.5	454.6
Pr							
1	1.1	57	361.2	4246.29	81	10.4	217.6
2	1.25	66	70.9	410.77	82	10.1	81.7
Total							
1		123	210.7	3021.43	163	10.2	51.9

Omitting D_{jt} variables from the model has been investigated and we can say when these variables can be easily defined, their existence results in better performance. When the capacity is not tight these variables are more easily determined. For example when "util" is equal to 0.4, the capacity is high so inventory costs are low and the D_{jt} can be defined easily by the upper bounds of demands because higher production means higher revenue. When the "util" is equal to 0.8 and capacity is tighter, the D_{jt} variables cannot be defined easily and their existence increases the number of variables and constraints of the models.

In the second part of table 6, the effect of multiplier "Co" is shown. In both scenarios model without variable D_{jt} has much better performance; this means that the effect of this multiplier is not the same as "util". When "Co" is equal to 12, the production cost is higher. In this case, performance of both model states become worse in comparison with the case that "Co" is equal to 8. Because by increase in costs, the profit of each product is decreased, the distinction between products is decreased and selecting products becomes harder.

The third part shows the effect of multiplier "Pr" on the model performance. When this multiplier becomes higher, the price of each product increases and products become more distinctive and Production decisions easier. From table 6, the performance of both states becomes better when this parameter is increased from 1.1 to 1.25. When "Pr" is equal to 1.25 the performance of model with D_{jt} variables is better than the model without D_{jt}, because in this scenario high profit result in more distinctive products so it is easier to define D_{jt}.

Models' performances according to all solved problems are shown in table 6 in "Total" section. The model performance without D_{jt} variables is better than the model with D_{jt}. Highest number of problems with best solutions also pertains to the first state. So in general, the model without D_{jt} is better in these samples.

4.3 Solved problems' size

Since the model without D_{jt} variables has the better performance in general, to study the dimensions of problems which are solved optimally, we use this model state with a time limit of 7200 seconds. In table 7, the maximum number of products with a definite number of periods in different sets is represented. The dimensions of solved problem in sets 9, 11, and 12 are low in comparison with other sets. In these sets, according to low distinction between products, the low dimensions are acceptable. In a number of problems due to low memory space we couldn't find any optimal solutions.

Table 7: solved problems' size

NO.	Util	Co	Pr		Number of periods					
					5	10	15	20	25	30
1	0.4	8	1.1	Maximum number of products	25	20	7	7	7	3
2			1.25		25	10	7	7	5	5
3		12	1.1		30	20	10	7	5	5
4			1.25		30	15	15	7	7	5
5	0.6	8	1.1		30	10	7	3	5	5
6			1.25		20	20	10	10	5	5
7		12	1.1		30	20	10	7	7	5
8			1.25		30	10	10	7	3	3
9	0.8	8	1.1		20	7	7	5	3	3
10			1.25		25	15	10	5	5	5
11		12	1.1		15	7	7	5	5	3
12			1.25		20	7	5	3	3	*

*Optimal solution is not found in 7200 second

5. Conclusion and Future Research Directions

In this paper, profit maximization general lot-sizing and scheduling problem with demand choice flexibility is studied. In this problem, decisions about lot-sizing, scheduling and demand selection are made in order to maximize the total revenue minus production, holding, and setup costs. This new problem which is an extension of GLSP by adding the assumption of flexibility in choosing demands, integrates two different levels of production planning, short-term planning and mid-term scheduling. The result of this model is the accepted demand in each period for each product, the quantity of production lots, and the sequence of them. In this paper a mathematical model represented for PGLSP. Representing exact methods which try to find an optimal solution based on the model structure like branch and bound algorithm is a good aspect for future research. Efficient heuristic and meta-heuristic algorithms are also appropriate methods to solve PGLSP. Considering the effect of transportation costs in choosing demands, obtaining the delivery time of each product as one of the output of the model and extension of the problem environment to more complex one like flow shop and job shop are suggested for future researches.

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