Experimental Analysis of the Applicability of a Multi-Item Constrained EOQ Calculation Algorithm

S. Miranda, M. Fera, R. Iannone and S. Riemma
Department of Industrial Engineering
University of Salerno
Fisciano (SA) 84084, Italy

Abstract

The present paper proposes the results of an experimental campaign targeted to investigate the applicability in real contexts of a new approach for the calculation of the Economic Order Quantity (EOQ) when a multitude of items have to be stored in a limited space. The approach, already presented in the literature in its basic principles, has been modified in order to make it more suitable for real needs. Subsequently, the so obtained algorithm has been tested, in simulated scenarios, with the purpose of identifying the regulation parameters with significant influence on the final results, their best values and the average level of performances the approach is able to guarantee, in terms of total cost of stock and mean saturation of the warehouse. The outcomes of this study confirm the general validity of the approach even if some more deepen investigations are still necessary before a possible implementation in a real stock management application.

Keywords
Inventory Management, Constrained EOQ, Dynamic calculation

1. Introduction and Literary Review

The problem of the Economic Order Quantity (EOQ) calculation under constraint is one of the basic problems in the classical theory of the inventory management. Typical constraints are a limited availability of space in the warehouse or a maximum budget for the purchases. Among the possible approaches to the solution of the constrained EOQ calculation, the most known and widespread is surely given by the application of the Lagrange multipliers method. This because it can assure easiness of implementation with a stationary approach targeted to avoid that all products, when eventually reach a simultaneous peak of stock, do not exceed the space constraint (Hadley and Whitin 1963, Parsons 1966, Phillips et al. 1973).

Another classical approach is named “fixed-cycle method”. In this procedure a fixed cycle time for all items is assumed and the orders for the items have to be phased within the cycle. In this way situations where peaks of stock are reached simultaneously are avoided. The main problem of this approach is to decide on the joint ordering cycle and the phasing within the cycle, so as to minimize total cost while satisfying the capacity constraint throughout the cycle. (Parsons 1966, Goyal 1976, Zoller 1977). The two classical methods have been compared by Rosenblatt (1981), evidencing better results but a more difficult implementation of the fixed-cycle approach.

A third approach to the problem, called “basic cycle method”, has been proposed by Goyal and Belton (1979), Kaspi and Rosenblatt (1983) and Roundy(1985) and consists in the calculation of individual cycle times which have to be integer multipliers of a basic cycle time. Many solution procedures and applications were developed through this approach also for the unconstrained problem, with economic advantages due to the joint replenishment.

Rosenblatt (1985) provided a comparative study of the three classical methodologies. In this study the basic cycle approach has resulted computationally very efficient as compared with the fixed cycle approach but no general conclusions were furnished about the quality of the solutions, being them usually data dependent. For this reason the author suggested to develop some other more sophisticated heuristic algorithms to the basic cycle approach in order to conciliate best efficiency with best solutions to the problem.

The main limit of the three above described methodologies is that generate stationary ordering policies which are not very suitable to manage a strongly dynamic resource as the warehouse is. In fact, due to the strict connection of the
warehouse with the production and the replenishment activities, its availability cannot be considered static but an extremely dynamic constraint. For this reason some authors have considered the capacity of the warehouse as a decision variable and not a fixed data (Hall 1988, Rosenblatt and Rothblum 1990). More specifically, in these approaches extra space can be purchased or rented out if necessary in order to minimize the total cost of stock management. Of course, the higher flexibility of these models is balanced by a higher computational effort, with the risk of keeping them far from the possibility of an implementation in real contexts.

Although non stationary approaches could be able to produce good solutions in terms of reduced costs, very few paper have been published in the last years on the problem of the constrained EOQ calculation. The reason of that is the complexity of the optimizing algorithms which make them not very effective in real contexts (Guder et al. 1995). Among the most recent studies it is possible to cite Berretta and Rodrigues (2004) which propose a meta-heuristic evolutionary algorithm for the multistage lot-sizing problem with capacity constraint, showing good results in comparison to the Lagrangean relaxation. Mondal and Maiti (2003), Chang et al. (2005) and Minner (2007) propose alternative methods to the search for the optimal values of the lot sizes that could be successfully adapted to a constrained problem.

The present paper proposes an experimental analysis of the method proposed by Iannone et al. (2010), consisting of an iterative procedure which, after a first calculation of the constrained EOQs with the traditional Lagrangean method, corrects the lot sizes through the dynamic analysis of consumption rates, delivery times and management costs and determines the real overlapping of the stocks in a fixed period of time.

2. Algorithm description

The basic idea of the approach and the logic of the calculation algorithm have been already described in Iannone et al. (2010) and Iannone et al. (2012) and will be summed up briefly in the next subsection. Subsequently, some substantial modifications, recently implemented in the approach, are explained. Finally, the new calculation algorithm will be shown.

2.1 The dynamic approach to the constrained EOQ

The method is targeted to surmount the limits of the classical approach to the constrained lot calculation, typically based on the Lagrange multipliers method (Iannone et al. 2010). The proposed procedure calculates the lot sizes with a dynamic approach based on the simulation of the total stock behaviour in a fixed time horizon, with the aim of reducing the total management cost. The general formula for the EOQ calculation of the $i$th item is given by:

$$Q_i^* = \frac{d_i \cdot C_{iL} \cdot T}{\sqrt{\frac{k_i \cdot T}{2} + \lambda \cdot v_i + f \left( \frac{dC_i}{dQ_i}, \mu \right)}}$$  \hspace{1cm} (1)$$

where $d_i$ is the daily demand for the $i$th item, $C_{iL}$ is the cost of order launch for the $i$th item, $T$ is the time horizon, $v_i$ is the specific volume of the $i$th item, $k_i$ is the holding cost per unit of time of the $i$th item. The $\lambda$ parameter, similarly to what happens in the Lagrange multipliers method, is regulated with an iterative procedure in order to satisfy the space constraint, expressed, as a function of the time, by:

$$V(t) = \sum_{i=1}^{N} v_i q_i(t) \leq V_{TOT}, \forall t \leq T$$  \hspace{1cm} (2)$$

in which $N$ is the number of different items managed in the warehouse, $q_i(t)$ is the quantity in stock at the time $t$ of the $i$th item and $V_{TOT}$ is the total available space in the warehouse.

The function $f$ in the denominator of (1) has been introduced to correct the dynamic procedure, giving a more marked reduction of the lot-sizes relative to the items with a slighter slope of their cost function, while the items with steeper slope of the function sustain a more limited reduction of their initial lot-sizes. This with the aim of reducing further the total cost of the stock management. In order to obtain this correction, different alternative variations of the dynamic procedure have been tested (Iannone et al. 2012) and the best performances seem to be obtainable with a logarithmic variation, given by:
where \( \overline{\frac{dC}{dQ}} \) is the harmonic mean of the cost derivatives and \( \mu \) has the role of a regulation parameter to give the right weight to the corrective term. The best values of the two regulation parameters (\( \mu \) and \( \lambda \)) have to be found through a calculation algorithm in which they are progressively increased within fixed ranged until the cost function is minimized and the space constraint is satisfied.

2.2 Improvements of the approach

The above described approach has been modified in some aspects in order to make easier the experimentation phases and/or to respect with more accuracy the applicability of the model to real cases.

More specifically, the modifications are the following:

a) Simplification of the range of exploration for the \( \mu \) parameter.

In order to simplify the iterative algorithm for the search of the optimal solution, a modification of the formula (1) with variation (3) has been done. In particular the new formula for the lot sizes calculation is expressed by:

\[
Q_i = \sqrt{\frac{2\cdot C_{\text{I-D}}}{k_1 t + 2 \cdot \mu \cdot v_1 - 2 \cdot \mu \cdot \ln \left( \frac{dC_i}{dQ} \right)}}
\]  

(4)

In this way, although the range of admissibility for \( Q \) includes also negative values of \( \mu \), the range of convenience starts from \( \mu = 0 \) and can have also positive values. This because, due to the sign of the logarithm of the cost derivative ratio, the effect of \( \mu \) on the modification of the lot sizes leads to lower costs only with positive values of it, while, if \( \mu < 0 \), the result is an increase of the costs.

b) The cost derivatives, necessary to identify which lots have to be increased and which decreased, are clearly variable with the lot dimension. This variability cannot be included into the calculation algorithm because it becomes unstable and does not converge to any solution (Iannone et al. 2010). In the previous studies the cost derivatives were fixed at the values calculated with the lot sizes obtained after the dynamic application of the Lagrangean method. This approximation, necessary for the convergence of the algorithm, induces an error because overestimates the values of the derivative over the investigation range of the lot sizes. In order to reduce this error the cost derivatives have been substituted with the mean values given by the incremental ratios. The formula (4) is therefore modified as follows:

\[
Q_i = \sqrt{\frac{2\cdot C_{\text{I-D}}}{k_1 t + 2 \cdot \mu \cdot v_1 - 2 \cdot \mu \cdot \ln \left( \frac{dC_i}{dQ} \right)}}
\]  

(5)

in which the incremental ratio \( \Delta C_i / \Delta Q_i \) can be expressed in two different ways:

\[
\frac{\Delta C_i}{\Delta Q_i} = \frac{C_{\text{I-D}i - \text{EOQi}}}{Q_{\text{I-D}i - \text{EOQi}}}
\]  

(6)

or:

\[
\frac{\Delta C_i}{\Delta Q_i} = \frac{C_{\text{I-stat}i - \text{EOQi}}}{Q_{\text{I-stat}i - \text{EOQi}}}
\]  

(7)
where: $Q_{\text{din},i}$ and $C_{\text{din},i}$ are respectively the lot size of the ith item obtained with the application of the dynamic method and the relative management cost; $Q_{\text{stat},i}$ and $C_{\text{stat},i}$ are the lot size of the same item calculated with the classical Lagrangean method and its management cost. Figure 1 shows, for a generic item, the difference between the two incremental ratios. Since both the alternatives present advantages and weak points, the choice between them is made after an experimentation targeted to understand if it is significant for the algorithm and, in the affirmative case, which one is able to guarantee better results.

**c) The approximations of the $Q_i$ calculation.**

In this work two modifications of the lot sizes values are introduced. The first one concerns the meaning of the proposed approach. In fact, the above described logarithmic variation provide an increase of the lot sizes with a higher value of the cost derivative, balanced by a further reduction of the lots with lower derivative. Of course the increase of a lot has the target of reducing the total cost but this can happen until the lot dimension is lower than the relative EOQ; after that, an increase causes higher costs and more required space and for this reason, it is to be avoided. This simple consideration imposes another constraint to the lot calculation because its value cannot surmount the limit given by the relative EOQ value. The second modification is the approximation to the nearer integer of the values calculated through the (5) formula. In this way the dynamic of the stock consumptions is made more realistic. In fact, being the stock overlaps strongly affected by the values of the lot sizes, non integer values induce instability on its behavior with repercussions on the cost function and on the quality of the final solution. For these reasons the final formula for the calculation of the lot dimensions is given by:

$$Q_i = \text{round} \left( \min \left( \frac{2 \cdot D_i \cdot D_i}{k_i \cdot t + 2 \cdot \lambda \cdot v_i - 2 \cdot \mu \cdot \ln \left( \frac{DC_i}{EOQ_i} \right)}; \text{EOQ}_i \right) \right)$$

(8)

Figure 1: Incremental ratios of the cost function for a generic item

**2.3 The algorithm for EOQ calculation**

The algorithm for the lot sizes calculation presented in Iannone et al (2012) has been therefore modified in order to consider the improvements described in the previous section. Figure 2 shows the flow chart of the new algorithm, implemented in this work.

A first evident difference is relative to the initialization of $\mu$ which is always done at 0, while the first value of $\lambda$ is dependent by $\mu$ and given by the following condition:

$$\lambda = \max_i \left\{ 0; \frac{2 \cdot \mu \cdot \ln \left( \frac{DC_i}{EOQ_i} \cdot \frac{DC_i}{\Delta Q_i} - k_i \cdot t \right)}{2 \cdot v_i} \right\}$$

(9)
This condition is necessary to assure that all the solutions have real values. Also in this case the algorithm is structured in two concentric cycles. The external cycle is targeted to find the minimum cost with a progressive increase of $\mu$; at each value of $\mu$ the internal cycle increments $\lambda$, starting from (9), until the space constraint is satisfied. Subsequently the algorithm calculates the relative total cost and restarts the external cycle with an increased value of $\mu$. $\epsilon_\mu$ and $\epsilon_\lambda$ are the increments given to the two parameters at each iteration of the cycles; they can be fixed arbitrarily and, for this reason, will be object of experimentation in order to understand their significance and best values.

3. Experimental Analysis

Once the algorithm and its modifications have been defined and before testing the applicability of the approach in real contexts, the purpose of the experimentation is to characterize the influence of the main regulation parameters on the algorithm performances measured in terms of obtainable economic benefits. To perform the experimental campaign, the algorithm and the simulations have been implemented in MATLAB 7.5 with an appositely edited script. In order to analyze correctly the performance of the algorithm when the regulation and the input parameters vary, an opportune objective function is introduced. From a mathematical point of view, it is defined as the ratio between the cost saving with the application of the method and the maximum saving in case of ideal conditions, without any constraint. According to that, the objective function is defined as follows:

$$F_{ob} = \frac{\Delta C_{tot}}{\Delta C_{EOQ}} = \frac{C_{Lagrange} - C_{algorithm}}{C_{Lagrange} - C_{EOQ}}$$  (10)
where: \( C_{Lagrange} \) is the cost of stock management with the solution given by the Lagrange multipliers method, \( C_{algorithm} \) is the cost corresponding to the solution furnished by the proposed approach and \( C_{EOQ} \) is the benchmark, obtained with the calculation of the EOQ’s without constraints. It is evident that it will be always \( 0 \leq F_{ob} \leq 1 \) and the best results will be characterized by the higher values of \( F_{ob} \). The two limit cases, \( F_{ob} = 0 \) and \( F_{ob} = 1 \) correspond, respectively, to the case in which no improvement is obtained with the implementation of the algorithm and the case in which the result correspond to the ideal EOQ solution.

3.1 Experimental campaign definition and simulation execution

After the definition of the objective function, the algorithm regulation parameters and the respective levels have been defined. They are the following:

- The number of items (noise factor), with 3 levels (20 - 50 - 100);
- The derivative of the cost function (control factor), with 2 levels (static - dynamic);
- The coefficient \( \varepsilon_{\mu} \) (control factor) with 2 levels (0.01-0.001);
- The coefficient \( \varepsilon_{\lambda} \) (control factor) with 2 levels (0.01-0.001).

In particular, the number of items has been considered as a non-controllable parameter because it is not possible to regulate it in order to obtain better results of the algorithm but, at the same time, its effect can be significant and for this reason has to be investigated.

As concerns the derivative, it is not clear which one of the two alternatives is preferable and how this choice can affect the results. The two levels of the experimentation reflect these two alternatives, expressed by (6) and (7) and called, respectively, dynamic and static, in order to reflect the starting point of the incremental ratio.

Finally, for the \( \varepsilon_{\mu} \) and \( \varepsilon_{\lambda} \) coefficients, which determine the increment of \( \mu \) and \( \lambda \) in the iterative cycles of the calculation algorithm and, as a consequence its duration and the resolution level of the solution, the purpose is to understand how much these aspects are really significant for the quality of the result.

The so designed experimental plan consists of 24 combinations of factors (trials). Each trial has been repeated 5 times, obtaining 120 different simulation runs. For each run all the characteristics of the items have been randomly fixed within the ranges shown in Table 1.

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Measure Unit</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Daily Demand (d_i)</td>
<td>Units/day</td>
<td>1÷50</td>
</tr>
<tr>
<td>Ordering Cost (C_i)</td>
<td>€</td>
<td>20÷100</td>
</tr>
<tr>
<td>Holding cost (K_i)</td>
<td>€/(unit*day)</td>
<td>0,001÷0,05</td>
</tr>
<tr>
<td>Specific volume (v)</td>
<td>m³/unit</td>
<td>0,1÷10</td>
</tr>
<tr>
<td>Purchasing cost (P)</td>
<td>€/unit</td>
<td>1÷30</td>
</tr>
<tr>
<td>Available Space</td>
<td>%V_TOT</td>
<td>50%</td>
</tr>
<tr>
<td>Simulation period</td>
<td>days</td>
<td>300</td>
</tr>
</tbody>
</table>

3.2 Results analysis

The results of the 120 simulation runs have been analyzed through the ANOVA method in order to evidence which parameters, directly or in interaction with others, are significant for the final output. Table 2 resumes the results of the analysis.
Table 2: Results of the ANOVA on the selected parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>SS</th>
<th>Dof</th>
<th>F_{α=0.05}</th>
<th>F_{α=0.01}</th>
<th>Var</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Main Effects</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>No of Items</td>
<td>0.3613818</td>
<td>2</td>
<td>3.07</td>
<td>4.79</td>
<td>0.180691</td>
<td>240.2351</td>
</tr>
<tr>
<td>ε_μ</td>
<td>0.00066556</td>
<td>1</td>
<td>3.92</td>
<td>6.85</td>
<td>0.000666</td>
<td>0.88489</td>
</tr>
<tr>
<td>ε_λ</td>
<td>0.00319421</td>
<td>1</td>
<td>3.92</td>
<td>6.85</td>
<td>0.003194</td>
<td>4.246819</td>
</tr>
<tr>
<td>derivative</td>
<td>4.08864E-07</td>
<td>1</td>
<td>3.92</td>
<td>6.85</td>
<td>4.09E-07</td>
<td>0.00544</td>
</tr>
<tr>
<td><strong>Interactions</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Items x ε_μ</td>
<td>6.09831E-05</td>
<td>2</td>
<td>3.07</td>
<td>4.79</td>
<td>3.05E-05</td>
<td>0.04054</td>
</tr>
<tr>
<td>Items x ε_λ</td>
<td>9.60893E-05</td>
<td>2</td>
<td>3.07</td>
<td>4.79</td>
<td>4.8E-05</td>
<td>0.063877</td>
</tr>
<tr>
<td>Items x derivative</td>
<td>0.000111381</td>
<td>2</td>
<td>3.07</td>
<td>4.79</td>
<td>5.57E-05</td>
<td>0.074043</td>
</tr>
<tr>
<td>ε_μ x ε_λ</td>
<td>4.15642E-06</td>
<td>1</td>
<td>3.92</td>
<td>6.85</td>
<td>4.16E-06</td>
<td>0.005526</td>
</tr>
<tr>
<td>ε_μ x derivative</td>
<td>2.03763E-05</td>
<td>1</td>
<td>3.92</td>
<td>6.85</td>
<td>2.04E-05</td>
<td>0.027091</td>
</tr>
<tr>
<td>ε_λ x derivative</td>
<td>4.47552E-06</td>
<td>1</td>
<td>3.92</td>
<td>6.85</td>
<td>4.48E-06</td>
<td>0.00595</td>
</tr>
<tr>
<td>Error</td>
<td>0.078974901</td>
<td>105</td>
<td></td>
<td></td>
<td>0.000752</td>
<td>1</td>
</tr>
<tr>
<td>Total</td>
<td>0.444514345</td>
<td>119</td>
<td>0.003735</td>
<td>0.003735</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

It is evident that the number of items is strongly significant for the final output, with a level of confidence higher than 99.99%, while the ε_μ parameter has resulted significant with a level of confidence higher than 95%. The other parameters and all the interactions are not significant. Figure 3 and Figure 4 show respectively the main effects and the interactions of the parameters, evidencing also the best levels. In particular the algorithm presents best performances with higher number of items and higher resolution of the ε_λ parameter. No effect is given to the algorithm performances by modifying the resolution of the ε_μ parameter and, for this reason, is preferable a lower resolution in order to reduce the number of iterations of the algorithm.

4. Conclusions
The paper presents the results of an experimental campaign carried out on an innovative approach to the lot sizes calculation in presence of space constraints. The experimentation is targeted to identify which parameters affect significantly the performances of the calculation algorithm and to define the best setting configuration of these parameters. The tests have been conducted with a number of items which approximates with adequate accuracy a real context in order to verify its future applicability in industrial cases. Before the testing, the approach has been improved in order to make more effective the calculation algorithm, in terms of elaboration times and degree of repeatability of the solutions.

The results of the experimentation have been analyzed through the ANOVA technique which has led to interesting conclusions. In particular, the most significant parameter affecting the algorithm performances has resulted the number of items, which measures the dimensional scale and the complexity of the problem under analysis. The outcome is very encouraging because shows that the higher is the number of items and the better are the main performances of the approach. A little influence on the performances is also given by the increment resolution for λ (ε_λ) which has preferable lower values: This result is easily predictable because an higher resolution of the λ parameter makes more precise the approximation to the space constraint. On the contrary, no significant effect is evidenced for ε_μ. As a consequence of that, it can be set at the higher value, in order to speed up the calculation time without reducing the quality of the solutions.

Further studies will be targeted to enlarge the experimentation by analyzing the effect of other regulation and/or external parameters in order to deepen the investigation towards the capacity of producing satisfactory and robust results for an effective application in real cases. Finally, once the method has been tuned and verified, a comparison with methodologies different from the traditional Lagrangean approach will be carried out in order to obtain relative information about computational complexity and quality of the solutions in several external conditions.
Figure 3: Main effect of the parameters on the objective function

Figure 4: Interactions between parameters on the objective function
References

Biographies
Salvatore Miranda was born in 1972 in Salerno (Italy) and graduated magna cum laude in Mechanical Engineering in 1997. He obtained his PhD in Industrial Plants in February 2002 and in November 2002 he became Assistant Professor at the University of Salerno, in Italy. From December 2011 he has been working as Associate Professor in the same University. He currently has more than 50 publications in national journals, international journals, national and international conference proceedings. His research interests include Production Planning and Control, Supply Chain Simulation, Project Management, Logistics and Inventory Management. Currently he is referee in several journals in these fields.

Raffaele Iannone was born in 1974 and graduated magna cum laude in Mechanical Engineering in 2000. He became suitable for the role of university researcher in scientific-disciplinary sector Ing-Ind/17 (Mechanical Industrial Plant) at Faculty of Engineering of UNISA (2003), came to duty in the same university in 2005 and has had tenue since 2008. In 2005 he awarded a PhD in Advanced Manufacturing Systems Engineering with the thesis “Advanced techniques for supply chain and supply network management in manufacturing contexts”. In 2006 he
was tenured professor of Operations Management at the Faculty of Engineering of UNISA. His main research interests are in supply chain management, distributed simulation, healthcare management and ERP systems. He is author of more than 30 national and international publications.

**Marcello Fera** was Born on 1979 in Naples and graduated magna cum laude in Management Engineering on 2004 in Naples University "Federico II". He obtained the Ph. D. in Mechanical Systems Engineering on 2008 from the University of Naples "Federico II". Since the 2005 he is research fellow in the field of the operations management and the industrial plants optimization and design. He is author of more than 15 publications in the same field of research. His specific research interests are focused on assembly lines balancing problems, safety systems, obsolescence management and operations management in general. He is reviewer for some international journal in the same research fields.

**Stefano Riemma** was born in Naples (Italy) in 1964. M.Sc. (cum laude) in Mechanical Engineering (1988) at University of Naples; PhD in Industrial Plant Management at University of Naples (1997); associate professor in Operations Management, University of Salerno – Italy, (1998); full professor in Operations Management (2002). Since 2009 up to 2011 he is director of the PhD program on Innovation Engineering and Management at University of Salerno. He is chief of the Industrial Engineering Department. Prof. Riemma main research topics are Manufacturing Information Systems, Automation of Manufacturing Systems, Supply Chain Management, Production planning and control, Health Management; he is scientific coordinator of many research projects developed in partnership with some primary national firms; he is referee of the Italian Industry Ministry to evaluate projects of Innovation technology; he is member of APICS, POMS, ANIMP and INFORMS society.