

# **Facility Location in a Three-Echelon Supply Chain with Stochastic Demand: A Discrete-Event Simulation Approach**

**Hasan Hosseini Nasab and Morteza Elahi**

**Faculty of Industrial Engineering**

**Yazd University**

**Yazd, Yazd 85287, Iran**

**Hamed Elahi**

**Faculty of Industrial Engineering**

**Sadjad Institute**

**Mashhad, Khorasan Razavi 9188148848, Iran**

## **Abstract**

The facility location problem under numerous conditions and restrictions has been discussed extensively in past studies. The location problem of supply chain facilities and centers may be considered as an example. According to numerous studies conducted in the area of centers' location in the supply chain, the examination process has moved closer to more realistic assumptions. This paper discusses the problem of locating facilities in a three echelon Supply chain with the consideration of responding to multi products in a stochastic demand environment. As the proposed model requires a long computational time, an innovative problem-solving method based on discrete-event simulation is utilized and presented with a numerical example.

## **Keywords**

Facility location problem, Facility location in supply chain, Discrete-event simulation

## **1. Introduction**

The problem of finding the optimal location of facilities has been prepared an open space for extending of many studies so far. Generally, the Facility location problem has been following a procedure to find the best location for the facility layout with multiple objectives and variables that is divided into many types under all constraints and objective functions (Arabani and Farahani 2012). Koopmans and Beckmann (1957) provide one of the first studies that were introduced a problem as a facility layout. Facility layout problems are classified by using various viewpoints. Categorizing based on type of facilities location and time horizon of planning is common method that Arabani and Farahani (2012) had been studying and classifying it. Based upon the approach of this category, the whole problems in the first level divided into two separate groups called static and dynamic that is sorted by the length of planning horizon. In the next level of this classification, problems have been organized based on discreteness or continuousness of candidate places.

Facility location has an important role in supply chain design. Supply chain management is about planning and control of operations in an efficient way all over the supply chain, and a part of this planning procedure is about supply chain configuration. Since the activities associated with the transportation of materials is composed of about 20 to 50 percent of the total operating budget of manufacturers and in other words 15 to 70 percent of the manufacturing cost of a single product (Tompkins et al. 1996), optimal layout of facilities and distribution centers have an important effect on supply chain design. Before introducing the facilities layout in the supply chain, most studies had only been focused on the design of distribution systems (Klose and Drexl 2005) and thanks to introducing layout subjects in the supply chain, location studies in distribution centers have been gone to a point in which the supply chain can be seen in very details.

Current research deals with distribution centers location among candidate points in a three-echelon supply chain, and in this research demand value is not a definite one and obviously, it has a probability distribution. The goal is to reduce the whole transportation cost that is composed of transportation from factory to distribution centers and from distribution centers to demand places (see Figure 1).

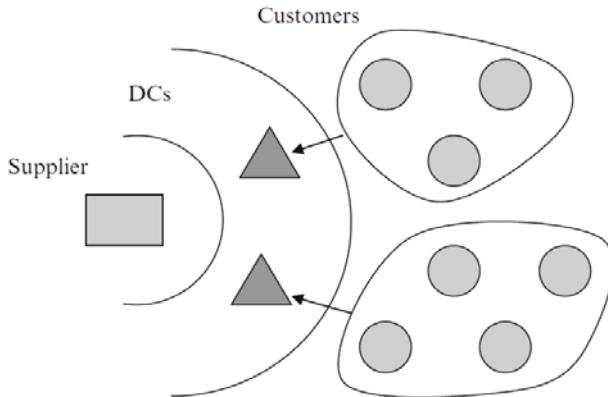


Figure 1: Graphical display of the three-echelon supply chain in the current problem

Innovative standpoint to find a solution for this model, designing scenarios and making a good comparison between these scenarios with the help of discrete event simulation and ultimately, resulting in choosing the best scenario that has an appropriate place for the layout. In the rest of this paper most of the conducted research about the supply chain will be discussed in the section 2 of this article. In the third section, the mathematical model of problem will be described and in the fourth section, the solution procedure in this paper will be illustrated. In the fifth section, the used approach will be described with a numerical example and its modeling in Arena that is a simulation software.

## 2. Literature Review

So far, any conducted studies in the facility layout have been classified with different viewpoints and as an example, the classification which is carried out by Arabani and Farahani (2012) can be cited. In the highest group of this classification problems were examined based on time horizon. After separating the problems into two categories called dynamic and static, in the second group they were sorted based on discreteness or continuousness of candidate places. In another classification facility layout problems is classified based on the type of problem that can be solved in that area. A mentionable example is various location problems in the supply chain (Farahani and Hekmatfar 2009). In a research carried out by Melo et al. (2009), the appropriate classification of facility location articles in the supply chain was provided. Based upon one of the classifications, in the first level classification was done based on the number of supply chain levels and in the next level any studies which had located facilities in one level of the supply chain were separated from those facilities that are located in the two-echelon supply chain. Finally, articles were grouped by several criteria such as number of product types, probability or definiteness of parameters, time horizon of planning. Among the available articles about supply chain location some of them had been assumed some of the parameters as indefinite ones, and they were very close to the real circumstances in order to that assumption (Farahani and Hekmatfar 2009). A research conducted by Chan et al. (2001) is one of those articles. This research is explained about supply chain location in a single-echelon and single-commodity supply chain with these following circumstances, probability of demand, and by using innovative algorithm.

Among the two-echelon and single-commodity supply chain location articles in the stochastic environment, an article carried out by Daskin et al. (2002) can be cited. This study is about the single-commodity circumstances and has been following a precise procedure to find the number of distribution points in the chain, and in the next level that wants to determine inventory levels policies in each of the distribution centers. Wang et al. (2011) was to minimize the costs of allocation and distribution, and by using genetic algorithm, has been seeking the best place in a condition with stochastic demand. Among the articles in the field of supply chain location, many research have been examined the single-commodity condition, but fewer number of studies have been working on multi-commodity conditions (Melo et al. 2009). A research conducted by Guillen et al. (2005) can be cited here, that is about a two-echelon supply chain location in a way that several commodities were in indefinite condition.

Conducted studies in the field of the supply chain location with more levels are far less than single-echelon supply chain or two-echelon one, and this issue will also be noted in future studies of Melo et al. (2009) & Wang et al. (2011). In most three-echelon supply chain locations have been made references to single-echelon facility location in the supply chain (Melo et al. 2009). Dogan and Goetschalckx (1999) is among a few papers in the field of three-echelon supply chain location which is done in multi-commodity condition and all parameters of model have been considered

as definite ones. Current research deals with distribution centers location in a three-echelon supply chain. The goal is to reduce transportation cost when there are several types of goods in the chain, and demand points were determined but demand is indefinite and has a probability distribution.

### 3. Problem Modeling

The topic in this study is to determine the location of distribution centers in a three-echelon supply chain. In this section, model, variables and parameters in the general form of problem will be introduced. The problem is working on the distribution centers location with several types of goods when manufacturers and the locations of demand (consumers) are determined. Each center can send any order to each of the demand points and can receive products from every one of factories, and distribution policy specifies each of the distribution centers only capable of storing and distributing one type of goods. Number of all distribution centers for each of the goods types has already been set as a policy. In this model demand is probable, and in each period demand is stated by probability distribution function. The aim is to find the location of distribution centers in a way that the sum of transportation costs will be minimized by using stochastic demand.

#### 3.1 Model parameters

$c_{ij}$  Cost of shipping a unit load per unit distance from plant i to distribution center j

$c_{jk}$  Cost of shipping a unit load per unit distance from distribution center j to demand point k

$d_{ij}$  Distance from plant i to distribution center j

$d_{jk}$  Distance from distribution center j to demand point k

$s_l$  Number of distribution centers for commodity type l

$u_l$  Capacity of distribution center for commodity type l

$w_{kl}$  Stochastic demand during a certain period of time for commodity type L ordered from demand point k

Relating to indexes: (i=1...M), (j=1...N), (k=1...K) and (l=1...L)

#### 3.2 Decision variable

$y_{jl}$  is a decision variable and can get zero or one figures indicating L type of distribution center to be placed in j site. As Banks (1998) has suggested, mathematical models cannot be presented for the problems which will be solved with simulation approach, although they can be applied to other methods, but owing to the explanation of the topic, general model of operation research (OR) is determined for the general form of the problem.

$$\text{Min} \quad \sum_i^M \sum_j^N c_{ij} \cdot d_{ij} \cdot y_{jl} + \sum_i^M \sum_k^K c_{jk} \cdot d_{jk} \cdot y_{jl} \quad (1)$$

St.

$$\sum_i^M y_{jl} = 1 \quad \forall j = 1 \dots N \quad (2)$$

$$\sum_j^N y_{jl} = s_l \quad \forall l = 1 \dots L \quad (3)$$

$$\sum_{k=1}^K w_{kl} \leq s_l \cdot u_l \quad \forall l = 1 \dots L \quad (4)$$

1<sup>st</sup> equation shows that objective function must be minimized. As an explanation, if the total number of distribution centers (SL) has already been determined in a multi-echelon supply chain with stochastic demand, some points among the candidate ones will be chosen and by using those points the sum of distribution costs must be minimized. In this function, distribution costs included transportation cost from factory to distribution centers, and transportation cost from distribution centers to the locations of demand. Constraint (2) ensures that in each of the candidate points must only be constructed one distribution center.

Constraint (3) states that the number of distribution centers for L type of goods must not exceed the number that is determined by the management policies (SL).

Finally, constraint (4) can make the total capacity of all centers more than the total number of stochastic demands of L type of goods in each period and due to that there will be no shortage of capacities.

As already mentioned, in problems that were solved by simulation approach, mathematical model of the problem is only represented for more explanation on the subject and it's the general form of the model, ultimately, problems will be solved with a different method that will be examined in the next section.

#### **4. Computer Simulation**

Solving approach to this location problem is using discrete event simulation. In addition to dealing with a three-echelon supply chain, some other circumstances will be examined in this paper, including the probability of demand and being multi-commodity. As there is a spread of using discrete event simulation models in different areas, the current paper is one of the few studies that in order to solve facility layout problem will use discrete event simulation models. Due to the fact that in all simulation problems, a problem must be modeled by using the system, and must avoid dealing with the whole system (Evn et al. 2001), according to general condition that have been stated, a problem gives a figure and it will be modeled and at the end, an optimal solution must be found. So, those steps will be followed that are in section 5 and all are equivalent to the steps of the most simulation studies. Thus, after presenting the problem, all important logics that is used in the model and setup time of the model will be determined. After confirmation of the created model, the optimal answer will be sought by designing an appropriate experiment. Since a few studies have been carried out by using stochastic parameters in location problems, discrete event simulation and the innovative approach that is presented for the facility layout, will be provide suitable circumstances in order to choose most of the model parameters as stochastic ones to make the real conditions more available. Different layout conditions will be applied to the model as various scenarios, and after multiple runs of each scenario and optimizing of each scenario, all results of various scenarios will be compared with each other in the ANOVA table. Afterwards, the best layout scenario will be selected by using stated objective function and examining paired comparisons analyses.

#### **5. Numerical Example**

In this section, a problem will be created; after modeling different scenarios, the optimal scenario must be selected and obviously, it would be the optimal layout.

##### **5.1 Problem Description**

In a three-echelon supply chain, a factory with two products is in the first echelon and for each type of the products must be established a distribution center. ( $SL_1 & SL_2 = 1$ )

There are three demand points for the first type of product and four demand points for the second type of product. The locations of demand points are definite, but demand value in different points is stochastic. Table 1 shows the time distribution function for consumption of each shipment in each center.

Table 1: The time probability function for consumption of each product in the demand points

Demand Point	Probability Function
DPA1	Uniform(5,10)
DPA2	Uniform(3,7)
DPA3	Uniform(6,9)
DPB1	Uniform(5,8)
DPB2	Uniform(4,6)
DPB3	Uniform(5,10)
DPB4	Uniform(3,6)

There are three candidate points for distribution centers layout and first or second type of distribution center can be placed in each of the candidate locations; distribution centers are equal in capacity and there are two types of transportation companies for carrying all products. Due to the fact that distribution functions have been stated by applying the duration for consumption of each shipment and all demands will be ordered by using the policy of shipping unit, there's no necessity to debate about the capacity of the container. One of transportation companies is used to carry commodities from the factory to two distribution centers and another company ships commodities form

the distribution centers to demand points. There're 14 carriers for the first company and 8 carriers for the second one. Table 2 shows the distances between all required routes per unit distance.

Table 2: Distance matrix in the supply chain network

	DPA1	DPA2	DPA3	DPB1	DPB2	DPB3	DPB4	Factory	DC 1	DC 2	DC 3
DC 1	2	5	6	3	4	5	2	30	—	6	9
DC 2	2	3	4	4	6	5	4	20	6	—	3
DC 3	4	1	5	5	2	2	1	35	9	3	—

All the figures in the preceding table are distances that carriers must be passed through in the three echelons of the supply chain. There's the goal of finding the best layout for the distribution center and it has been defined to minimize sum of the transportation costs along the chain. Transportation cost of each shipment is 5 units of money per unit distance.

## 5.2 Simulation Model

Computer model of the program was created in Arena 13.5 simulation software. Following the majority of discrete event simulation studies, in this section some of the most important and frequently used logics within the model will be quoted. Each consignment is intended for an entity as a unit and moves in the model. As shown in the picture below, this model depicts three levels of the supply chain. All distances between the stations came from the distance matrix and are given to the carriers, and the nearest carriers have always been called by each of the senders for sending shipments. In order to make the model so similar to this policy in all "Request" modules, the carrier selection policy (Smallest distance) has been set. So that all rules related to distribution in two levels can be applied to the model in the best possible way, two carrier groups have been produced. A group called FTDC is responsible for moving commodities from the factory to two distribution centers that has been allocated to "Request" module of the factory, another group called DCTDP transfers commodities from those distribution centers to demand centers and this group has been assigned to both "Request" modules that are for the two distribution centers.

There are many decision modules in the middle level of the model (distribution centers) to guarantee that each demand will be sent to which one of the applicants for commodities, and distribution policy has always been this following way and due to the fact that there are many demand points to order commodities, one demand point must receive its demand sooner than others and it's the one that ordered it sooner than others. Naturally, this policy must be applied to other demands as well. Inasmuch as any particular unit of commodities is defined as a shipment or entity and stochastic duration is shown within the model, in the form of the time distribution function for consumption of each shipment in each demand point, minimum/maximum inventory policy in any demand point or any distribution center was set in the model utilizing the technique of number of entities in the queue for each center.

Figure 2 shows the software model.

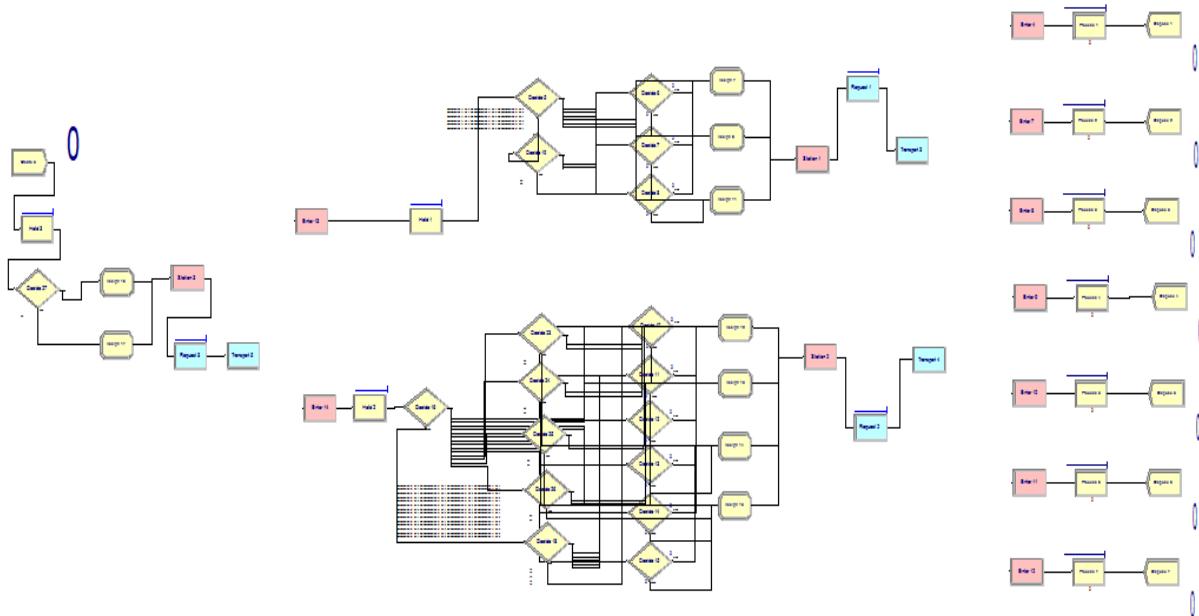


Figure 2: Three-echelon supply chain model simulated in Arena 13.5

After implementing the model as mentioned above, there is an inevitable confirmation of the simulation model to perform. In the confirmation step of the model, it will be checked whether the created computer model is exactly the same as the logical model of the system or not. In order to confirm the model, Christopher Chung (2003) was introduced a method that is used here. In this method, the model is confirmed after utilizing an approach for dividing model into tiny parts, the observation of moving entity in the model and the examination of logicality of the model. This is accomplished using dissimilar animations in various segments for entities, examining verification of steps for the model and utilizing animation in other items such as resources, etc. Eventually, results of examining entity movement in different steps demonstrate that the software model tallies with the logical model.

By examining trend changes in primary variables of the system, warm-up period of the model can be estimated. This point can be in a place that drastic changes of variables dropped and model condition is very close to normal circumstances of the system. Indeed, variables converge more and more after this point. According to the debates raised by Evon et al. (2001), for examining the trend, one of the most important variables of the system will be analyzed. The variable for consumption rate of total commodities within the supply chain in every moment can be an appropriate parameter for analyzing the reliability trend of the system. Figure 3 is a trend chart for this variable.

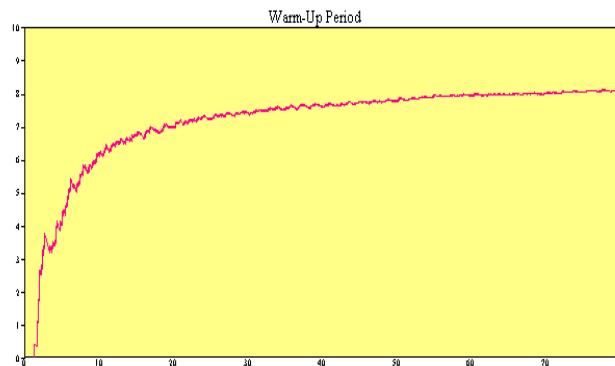


Figure 3: Chart of consumption rate for shipments per unit time.

According to the chart of warm-up period which was drawn for the consumption rate of commodities, the first 10 hours is chosen as warm-up period of the model and all information about the first 10 hours must be omitted. All results in the following sections are extracted from the model, when the warm-up period is set in the model as was stated.

### 5.3 Design of Experiments

At this stage several scenarios will be designed so that after examining results and model outputs and analyzing them in the next steps, the best circumstances will be delineated for the system. Since three points have been nominated for two centers and these centers have got different characteristics and demands to each other, there are a choice of six different scenarios for the centers layout, consequently, six layout scenarios must be examined. In order to test various layout designs, the technique for design of experiments was used and it has been done utilizing the stated method proposed by Montgomery (2004). Total transportation cost in each scenario was specified as a response variable; the establishment of the two distribution centers has been determined as influential factors and layout choices for each center have also been designated as levels of the factors. In order to be able to inactivate random terms of the probability distribution in the results, three replications is used in each scenario. Any replication represents the best result that is obtained using software-generated random figures in that replication. Thus an Arena module called "Opt Quest" is utilized. Table 3 shows the result of 6 layout scenarios. Figures inside the table shows the result of objective function or total transportation cost after running the model for 200 hours.

Table 3: Results of objective function for 6 layout scenarios of distribution centers in a three-echelon supply chain

	Locating the distribution center for "A" commodity in DC1		Locating the distribution center for "A" commodity in DC2		Locating the distribution center for "A" commodity in DC3	
	Locating the distribution center for "B" commodity in DC2	Locating the distribution center for "B" commodity in DC3	Locating the distribution center for "B" commodity in DC1	Locating the distribution center for "B" commodity in DC3	Locating the distribution center for "B" commodity in DC1	Locating the distribution center for "B" commodity in DC2
Rep 1	9083	8060	8349	7778	8061	7510
Rep 2	9063	8022	8401	7788	8005	7495
Rep 3	9061	8001	8367	7802	8035	7519

### 5.4 Analysis of Results

In order to analyze the preceding result, ANOVA and SPSS 19 were used here, and software results will be analyzed in two parts. The first section is an ANOVA table to give an answer to three stated hypothesis testing. In the second section, after utilizing the tables for paired comparisons analysis, it will be diagnosed in which level the influence of each factor is more than others, in other words, by proving there are differences among the various levels of any factor, it will be examined which level is different from the rest.

Table 4: ANOVA results

**Tests of Between-Subjects Effects**

Dependent Variable:Cost

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	4389209.333	5	877841.867	1869.517	.000
Intercept	1.191E9	1	1.191E9	2535844.770	.000
A_Product	3428200.333	2	1714100.167	3650.474	.000
B_Product	2066756.333	2	1033378.167	2200.758	.000
A_Product * B_Product	484128.000	1	484128.000	1031.035	.000
Error	5634.667	12	469.556		
Total	1.195E9	18			
Corrected Total	4394844.000	17			

According to the results of Table 4 with a 0.95 confidence interval for all tests, rejection or acceptance of the tests is examined. P-Value for all the three tests related to influence of the place of A/B product and impact of interference effect on the determination of layout for sites of the two distribution centers is obtained 0.000, This means that all three tests can be rejected without any doubt and examination of changing the layout of both centers should be examined, because their interference effect is known as a significant effect. Here the best scenario in terms of result will be found using paired comparisons analyses. Figure 4 is obtained using SPSS.

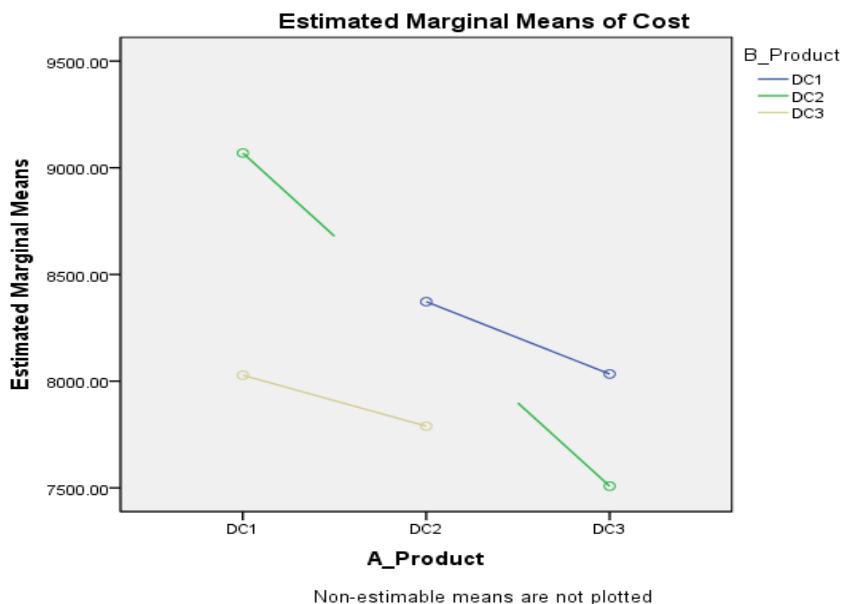


Figure 4: Chart for comparing 6 scenarios of distribution centers

In the diagram above, for various layout scenarios of distribution center for A product is drawn different points, and for various layout scenarios of distribution center for B product is drawn different lines. Since the objective function wants to minimize transportation cost, then the best scenario is lower than others. The diagram shows that in the defined three-echelon supply chain, the minimum cost will be achieved when distribution center for a product is in DC3 and distribution center for B product is in DC2. Regarding to stochastic demand and the confidence interval for this statistical analysis, there is a %95 confidence to rely on the obtained results and these results all are high accuracy ones. Finally, it can be said that in the stated circumstances, the sixth scenario which shows the distribution center layout for A product in DC3 and the distribution center layout for B product in DC2 is determined as the best layout scenario, and after that the fourth scenario is determined as the second appropriate scenario for establishment.

## 6. Conclusions

Current study have been examined distribution centers layout in a three-echelon supply chain with the approach to discrete event simulation, and its aim is to find the best layout design among the candidate pointes with the lowest transportation cost. According to the three candidate points and two types of distribution centers, six possible scenarios for layout were made. Simulation results were examined with statistical analysis and it was shown that the sixth scenario is the least cost of layout designs. This paper demonstrated that discrete event simulation can be utilized for layout design among the candidate pointes. Though in the present model, only demand for final costumers were applied to the model in a probabilistic manner, but according to the created model can be understood that advantage of simulation approach in contrast with other approaches used in the layout design is an ability to easily use probability distributions in many of definite model parameters. Future studies in this field can cover probability distribution for velocity of the carriers, probability for type of ordered commodity in any distribution center, probability for production rate in factories of the supply chain and many other items.

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## Biography

**Hasan Hosseini-Nasab** is an associate professor at the University of Yazd, Iran. He received his Ph.D at the University Of Bath, UK, 2001. He has worked on facility location and cost management for 20 years.

**Morteza Elahi** is received his B.S. and M.S. degrees in Industrial Engineering, from IUST University and Yazd University respectively. His interest areas are simulation models, system dynamics methodology, modeling/ analysis of socio-economic problems. He has performed diverse consulting projects in human resources planning, strategic marketing and supply chain using System Dynamics.

**Hamed Elahi** is currently a freelance software developer, and as he has sufficient experience in computer systems analysis and MATLAB programming, is working on a program for aviculture management. He holds a Bachelor of Science degree in Industrial Engineering from Sadjad Institute, Mashhad, Iran. He has published a conference paper. His research interests include optimization, simulation, operation research and systems analysis.