Mathematical Programming based Synthesis of Rice Drying Processes

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Abstract

Various simplified (empirical) and rigorous (theoretical) drying models have been developed in the extent which they are available for the analysis of drying processes in a variety of practical drying systems. However, most were focused only on a single unit operation; mainly the dryer. Nevertheless other unit operations such as cooling and tempering units are also employed in industrial drying systems. Therefore, a synthesis problem of rice drying systems which takes into account all the interactions between the units that appear in a drying process was proposed. The aim is to select a process out of the large number of alternatives and operating conditions to maximize the rice quality using both empirical and theoretical models. Solving the synthesis problem, mathematical programming will be used as a tool. For the synthesis problem using empirical models, the problem was formulated as an MINLP model which was transformed from generalized disjunctive programming (GDP) model to exploit a disjunction part of a GDP model for integrating alternative choices of empirical drying models which are valid only in a small range of operating conditions. The synthesis problem using theoretical models arising from the simultaneous heat and mass transfer balances gave rise to a mixed-integer nonlinear programming (MIDO) model. A hybrid method which combines genetic algorithms (GAs) and control vector parameterization (CVP) approach was proposed to solve this problem. Results show that empirical models are easier to use for the synthesis problem but they are valid only within their developed ranges. Also, there is a need for developing a model for each particular unit employed in rice drying processes. For the synthesis problem with theoretical models, this problem gives rise to the most difficult class of optimization problems; however, a theoretical model provides a better understanding of the drying kinetics happening in rice grain. Moreover, theoretical models alleviate the need to develop models for each particular unit employed in rice drying systems.

Keywords
Synthesis Problem of Rice Drying, MINLP, MIDO

1. Introduction

Rice is the world's second most-widely grown cereal after wheat. However, unlike wheat, rice is eaten as a whole, and thus the presence of broken grains lowers the market value of rice (the price of head rice is approximately twice that of broken rice). Since rice is usually harvested at high moisture content, it must be dried within 24 hours for safe storage. However, improper drying operations can cause fissuring in the rice grain, reducing the yield of head rice. Therefore, rice drying is a critical process that needs to be performed carefully to maximize the proportion of head rice. Various drying models both theoretical and empirical model have been developed in the extent which they are available for the analysis of drying processes in a variety of practical drying systems. However, most were focused only on a single unit operation; mainly the dryer. Nevertheless other unit operations such as cooling and tempering units are also employed in industrial drying systems. Therefore, there is an important need for an integrated analysis of rice drying systems which takes into account all the interactions between the units that appear in a drying process.

The overall objective of this research is to thoroughly investigate the use of various types of drying models both simplified (empirical) and rigorous (theoretical) models to the synthesis problem of rice drying process. In particular, the issues related to the application of a mathematical programming approach to solve different class of optimization problems arising from different types of process models applied to the problem are to be investigated. The solution of the synthesis problem sought is an optimal configuration and operating conditions of a rice drying
system which dry rice from initial moisture content to desired final moisture content while maximizing the rice quality.

2. Background

2.1 Drying Methods

Rough rice is a hygroscopic, living and respiring biological material. It is often harvested at high moisture content ranging from 25 to 40% dry basis, which renders the grains very susceptible to attack by micro-organisms, insects and pests. Newly harvested grain with high moisture content must therefore be dried within 24 hours to about 14 percent for safe storage. However, heat and mass transfer that occur during the drying process differ in each layer of rice grain. The hull, see Figure 1, starts losing its moisture before the inner kernel loses its own moisture to the hull; thus, a fast dried grain develops a significant amount moisture gradient between both layers.

![Figure 1: Grain structure of rice kernel.](image)

Multi-pass drying systems are used in industry to gradually bring moisture content to desired levels (Brooker et al. 1992). Three main kinds of operations can be involved in a pass: drying, cooling and tempering. Drying units remove the moisture content within a rice grain; cooling or sometimes known as air ventilation units lower the temperature of the grain to prevent moisture accumulation on the grain surface and remove some amount of moisture content at low temperature (Prachayawarakorn et al. 2005), and finally tempering units equalize the moisture gradient that has developed during the drying processes. Tempering is accomplished by keeping grains at rest in an incubator at a constant temperature. Therefore, we assume that no heat or mass transfer occurs between the grain and surrounding air in the tempering unit.

The operating conditions of each drying system can vary considerably. Different configurations and designs of rice drying systems have been employed around the world, such as the cross-flow dryer, re-circulating rice dryer shown, two-stage concurrent-flow dryer, and the multi-pass drying system of drying and tempering units. In the traditional tempering-drying procedure, grains are heated in a drying section for 8 to 10 minutes then sent to a tempering section for 2 to 3 hours in which moisture in the inner layer of grains is allowed sufficient time to transfer to the outer layer of grains (Chen and Wu 2000). In Foster’s dryeration process, grains are dried at 60°C to within 2% of the desired final moisture content and transferred to a separate tempering bin where they are held for 6 to 8 hours without aeration. Finally, they are cooled slowly by ambient air for 8 to 12 hours (Gunasekaran 1986).

In the intermittent drying, grains undergo tempering for 40 to 120 minutes following a 3 to 15 minute drying period. The drying-tempering cycle is repeated until the grain reaches the desired final moisture content (Shei and Chen, 1998). In the cross-flow and mixed-flow drying systems, the amount of moisture removed from rice grain per pass is limited to 2.5-3 % (w.b.) in the first pass and 1.0-2.0% (w.b.) in the remaining passes, with the retention time of the rough rice not exceeding 20-30 minutes per pass. However, long tempering time (6-24 hours) has to be used since moisture content within rice grains is not uniform after the drying process (Brooker et al. 1992). In a three-stage concurrent-flow dryer, the moisture evaporated in one drying stage does not exceed 1.5-2.0% (w.b.), with time period in which rice is subjected to the hot air limited to 15-20 seconds. The rice temperature in the tempering zones is not allowed to exceed 43°C. The air temperatures are limited to 150-175°C, 100-150°C, and 75-125°C, respectively, in the first, second, and third stages, and the grain velocity is maintained at 5-7 m/hr. Under these conditions, the temperature and moisture content of the rice kernels entering the tempering zone are so uniform that tempering periods between drying stages can be reduced to one hour each (Brooker et al. 1992).
2.2 Modeling of Drying Process

Models that describe the entire drying process in a system fall within two main groups: empirical models and theoretical models. Empirical models are developed by fitting experimental data with a simple mathematical function, often in the form of an exponential function (Agrawal and Singh 1977; Wang and Singh 1978; Sharma et al. 1982; Noomhorm and Verma 1986; Basunia and Abe 1998; Shei and Chen 1998; Chen and Wu 2000). They are easy to implement due to the simplicity of the used mathematical functions; however, they are valid only for the particular range of operating conditions for which they are developed. Theoretical models are based on the principles of heat and mass balances (Abud-Archila et al. 2000a; Rumsey and Rovedo 2001; Wu et al. 2004). As such, they are not restricted to certain operating conditions; their validity is limited only by assumptions and simplifications up on which they are based. However, they are often more difficult to implement than empirical models.

2.2.1 Empirical Model

No single drying model can represent the drying process of rice grain over a wide range of drying operation. Here, we select drying models from the literature to cover various ranges of drying operation. These particular models were selected because they were formulated in terms of the same operating variables (drying temperature, relative humidity of drying air and drying time), and because they were derived from Page’s model, which has the general form

\[ MR = M_t - M_e = \exp(-kt^N) \]

where \( MR \) is moisture ratio, \( M_t \) is moisture content of grain at anytime (%d.b.), \( M_i \) is initial moisture content of grain (%d.b.), \( M_e \) is equilibrium moisture content of grain (%d.b.), \( t \) is drying time, and \( k \) and \( N \) are drying parameters.

**Wang and Singh’s model**

Wang and Singh (1978) developed empirical formulas for the parameters \( k \) and \( N \) in terms of drying temperature \( T(\degree C) \) and relative humidity of drying air \( RH(\%) \) in the ranges 30-55 \( \degree C \) and 15-85% respectively:

\[ k = 0.01579 + 0.0001746T - 0.01413RH \]  
\[ N = 0.6545 + 0.002425T + 0.07867RH \]  

**Phongpipatpong and Douglas’s model**

Phongpipatpong and Douglas (2003) developed drying models that are valid over a wide range of drying operation and simple to use in the synthesis problem. Thus they have first simplified the basic Page’s model by setting \( N = 1 \) and \( M_e = 0 \) to

\[ MR = \frac{M_t}{M_i} = \exp(-kt) \]

They then formulated specific formulas for drying in the ranges of 30-150 \( \degree C \) and 0-65% RH, cooling in the ranges of 15-30 \( \degree C \) and 40-60% RH and the tempering time model following Equation (5) to (7), respectively.

\[ \text{Drying model: } k = 0.023962T_D + 0.219931RH_D - 0.037472T_P RH_D \]  
\[ \text{Cooling model: } k = 0.004927T_C - 0.037351RH_C \]  
\[ \text{Tempering model: } t_p = 10.91926 - 0.22236T_{D/C} - 0.00034T^2_{D/C} + 0.00241T^2_{D/C} + 0.09664T_{D/C} mPin \]

where \( t \) is time (hr), \( T \) is air temperature (\( \degree C \)), \( RH \) is relative humidity (%), \( mPin \) is moisture content of rice entering a tempering unit (% d.b.). Subscript \( C, D, \) and \( P \) are cooling, drying, and tempering, respectively.

2.2.2 Theoretical Model

In this work we adopt the compartmental model developed by Abud-Archila et al. (2000a) to predict the drying behavior of rice grain. The actual rice grain in Figure 1 is assumed to consist of two homogeneous compartments as shown in Figure 2.
This model assumes that mass transfer occurs by diffusion between the two compartments and by vaporization at the surface of the rice grain, and that heat transfer happens only at the grain surface. The grain temperature is considered uniform and equal for both compartments. The compartmental model is described by the following equations:

\[
\frac{dx_1}{dt} = \frac{\beta_1}{\rho_g \cdot \tau_1} (x_2 - x_1)
\]

\[
\frac{dx_2}{dt} = \beta_2 \cdot S_{sg} \cdot (p_a - p_g) - \frac{\beta_1}{\rho_g \cdot \tau_2} (x_2 - x_1)
\]

\[
\frac{dT_g}{dt} = \frac{\alpha \cdot S_{sg} \cdot (T_a - T_g) + \beta_2 \cdot S_{sg} \cdot (p_a - p_g) \cdot L_v}{\rho_g \cdot (C_{pg} - C_{pw} \cdot \bar{x})}
\]

where \(x_1\) and \(x_2\) are the grain moisture contents of compartment 1 and 2 (%d.b.); \(T_g\) and \(T_a\) are the grain and air temperature (°C); and \(p_g\) and \(p_a\) are the partial vapor pressures at the grain surface and in the drying air; \(\beta_1\) is mass transfer coefficient between the two compartments (kg water. m\(^{-2}\).Pa\(^{-1}\).s\(^{-1}\)); \(\beta_2\) is mass transfer coefficient between the outer compartment and the air (kg water. m\(^{-2}\).Pa\(^{-1}\).s\(^{-1}\)); \(\tau_1\) is volume fraction of the inner grain compartment (m\(^3\).m\(^{-3}\)); \(\tau_2\) is volume fraction of the outer grain compartment (m\(^3\).m\(^{-3}\)); \(\rho_g\) is dry rice density (kg.m\(^{-3}\)); \(S_{sg}\) is specific dry grain surface (m\(^2\).m\(^{-3}\)); \(L_v\) is specific heat of vaporization (J.kg\(^{-1}\)); \(C_{pg}\) is specific heat of dry grain (J.kg dry matter.°C\(^{-1}\)); and \(C_{pw}\) is specific heat of water (J.kg dry matter.°C\(^{-1}\)). Since the compartmental model predicts the drying rate of a single grain kernel, further assumptions have to be made so that it can be used for the synthesis problem:

- Drying, cooling and tempering units are considered homogeneous systems; that is, drying behavior and characteristics of rice grains in the units are the same.
- Rice grains in any unit operation receive the same quality of air (i.e., same air temperature and relative humidity).
- The dynamic behavior of state variables of the grain (moisture content and grain temperature) is taken into account, while properties of air (air temperature and relative humidity) are assumed constant.
- In drying and cooling units, coupled heat and mass transfer between grain and air phases is considered; but in tempering units, only mass transfer between the two compartments is considered.

### 3. Mathematical Programming of the Synthesis Problem

In this work, mathematical programming is used as a tool to solve the synthesis problem. It involves three major steps: (1) the development of a superstructure to represent all alternatives from which the optimum solutions can be selected, (2) the formulation of a mathematical program which transforms the qualitative information from a
superstructure into a quantitative one, and (3) the development of a solution strategy for the optimization model from which the optimal solution is obtained (Yeomans and Grossmann 1999).

The synthesis problem of rice drying processes aims to select a process out of a large number of alternatives and operating conditions to maximize the rice quality (head rice yield). Due to this work employed both empirical and theoretical models for the synthesis problem, both empirical and theoretical rice quality models from the literatures will also be employed. One is a model developed by Phongpipatpong and Douglas (2003) as shown in Equation (11)

\[ HRY = 1 - k_q t \]  

where \( k_q \) is a constant that depends on the processing unit used (drying or cooling), and \( t \) is the processing time (hours). Another one is the theoretical quality model developed by Abud-Archila et al. (2000b) is used for the synthesis problems as follows

\[ \frac{d(HRY)}{dt} = -k_0 (x_1 - x_2)^3 \exp\left(\frac{-E_a}{R(T_g + 273.1)}\right) HRY^2 \]  

(12)

Where \( k_0 = 1.56 \times 10^{27} \) is the quality degradation rate coefficient (s\(^{-1}\)); and \( E_a = 1.657 \times 10^5 \) is the equivalent activation energy for quality degradation kinetic (J mol\(^{-1}\)).

### 3.1 Superstructure Representation

A superstructure is a graphical representation that shows all possible design alternatives of process equipments and their connectivity (Grossmann et al. 1999). The developed superstructure shown in Figure 3 allows each pass to take one of five alternative configurations: (1) drying-cooling, (2) drying-tempering, (3) cooling-tempering, (4) drying-cooling-tempering, and (5) drying-tempering-cooling. These alternatives represent all possible configurations of practical uses of intermittent rice drying systems. In the \( j \)th pass, rice grains having moisture content \( M_{in_j} \) will pass through drying unit \( D_j \), cooling unit \( C_j \), and/or tempering unit \( P_j \), and finally end the current pass with \( M_{out_j} \) moisture content. The backward path in this figure represents the possibility of starting a subsequent pass (\( j+1 \)). Note that the splitting nodes \( S_{1j} \) to \( S_{4j} \) and the mixing nodes \( M_{1j} \) to \( M_{4j} \) are dummy nodes used to enable the use of a compact form for the superstructure. However, for an empirical models, only three alternatives which do not include the dashed line between node \( S_{3j} \) and \( M_{4j} \) and between node \( S_{4j} \) and \( M_{2j} \) are considered: (1) drying-tempering, (2) cooling-tempering, (3) drying-cooling-tempering. The reason is that the synthesis problem with empirical models use the developed empirical model of head rice yield from the work of Phongpipatpong and Douglas (2003) and this model did not take into account the effect of required time rice spend in a tempering unit as a factor which affects the yield of head rice. In other words, the developed models consider only drying and/or cooling conditions as a cause of reduction of head rice yield. Therefore, using the empirical models in the synthesis problem is not able to provide the information of the importance of having a tempering unit in a rice drying system. As a result, the synthesis problem with empirical models cannot explain the different effects between having a configuration of drying-cooling-tempering and drying-tempering-cooling in a drying system on head rice yield.

![Figure 3: A superstructure of rice drying processes.](image)
3.2 Problem Formulation

3.2.1 Empirical Model

One of the objectives of our proposed synthesis problem using empirical models is to integrate alternative choices of drying models which are valid only in a small range of their experimental conditions. Therefore, a generalized disjunctive programming (GDP) framework proposed by Raman and Grossmann (1994) will be employed for the problem formulation of the synthesis problem with empirical models due to the fact that this framework provides distinctive constraints which can facilitate the formulation of the problem containing the choices of drying models.

The synthesis problem using empirical models as a GDP model is shown below.

\[ \text{Max} \prod_{j=1}^{8} HRY_j \]

Subject to

\[ g_j(x_j) \leq 0 ; \forall j \in J \]

\[ \forall i \in I_{kj}, h_{ij}(x_j) \leq 0 \]

\[ \Omega(Y) = \text{True} ; \forall i \in I_k, \forall j \in J, \forall k \in K_j \]

\[ x_j \in \mathbb{R}^n, Y_{ij} \in \{\text{True, False}\} \]

where \( HRY_j \) is the yield of head rice by pass \( j \in J \); \( x_j \) is the vector of continuous variables in pass \( j \); and \( g_j(x_j) \) are common constraints that hold regardless of the discrete decisions. Each pass \( j \in J \) is associated with \( K_j \) disjunctions. Each disjunction \( k \in K_j \) is composed of a number of terms \( I_k \) that are connected by the OR operator (\( \lor \)). In each term, a set of constraints \( h_{ij}(x_j) \leq 0 \) are enforced if the corresponding Boolean variable \( Y_{ij} \) is true, and are ignored otherwise. Finally, the set \( \Omega(Y) = \text{True} \) are logic propositions for the Boolean variables.

Note that the common constraints represent empirical process models in Section 2.2.1 which describe the operations in the processing units involved in the superstructure and this constraints hold irrespectively of the discrete decisions. The disjunctive constraints which represent the discrete decisions in continuous space are included in the model for: (1) the decision of an existence of \( j+1 \) to dry rice from initial moisture content to desired moisture content; (2) the relation of the existence of inlet MC at each process unit to its connectivity in the superstructure; (3) the expression of an alternative choices of drying models with different ranges of their own drying operations and finally, the proposition constraints are used to represent the logical relationships in the superstructure. The readers are recommended to find the full detailed of the problem formulation as the GDP model in the work of Younes et al. (2010).

3.2.2 Theoretical Model

Using empirical models for the synthesis problem, there is a need for developing an empirical model for each particular unit operation representing in a rice drying processes. However, due to the fact that a drying process happening in any unit operations existing in a drying system can be theoretically described by the same coupled heat and mass transfer process, therefore, using theoretical models for the synthesis problem becomes an interest. This problem involves a nonlinear set of differential-algebraic equations (DAEs) and a discrete set of process alternatives which leads to the problem type called mixed-integer dynamic optimization (MIDO) problem which can be written as follows.

\[ \text{max} \quad HRY(t_f) \]

Subject to

\[ f(\dot{x}(t), x(t), u(t), y(t)) = 0 \]

\[ x(t_0) = x_0 \]

\[ x(t_f) \leq M \]

\[ u_L \leq u \leq u_U \]
where $HRY(t_f)$ is the yield of head rice at the end of drying process (%), $x(t)$ is the vector of state variables, $\dot{x}(t)$ is the time derivative of $x$, $u(t)$ is the vector of control variables (operating conditions), and $\bar{x}(t_f)$ is average moisture content at the final time. Note that Equation (19) is the set of DAEs describing state variables of moisture content and grain temperature in each unit operation as described earlier in Section 2.2.2.

### 3.3 Solution Strategy

Different kind of drying models applied to the synthesis problem give rise to a different class of optimization problems and this requires also different optimization techniques needed to solve the problems.

#### 3.3.1 GDP Model

Due to the extensive development of algorithms and codes which are available for solving many practical MINLP problem, most GDP models are transformed and found to be solved in algebraic forms. The GDP model is transformed into MINLP model with the help of BIG-M constraints technique for the disjunctive part and the proposed systematic method of Raman and Grossmann(1991) to linear constraints for proposition part and solved in GAMS.

#### 3.3.2 MIDO Model

The combined genetic algorithm (Gas) and control vector parameterization (CVP) is proposed to solve the MIDO problem. Our strategy is to decompose the MIDO problem into an outer integer programming problem and an inner dynamic optimization (DO) problem. A GA will be used to tackle the discrete part of the problem (i.e., to search among alternative configurations) while a CVP approach will be used to solve the continuous, dynamic optimization problem. The basic idea of a GA is to start from initially generated set of random solutions called population from the solution space. Each candidate solution in the population called chromosome will undergo the evolutionary mechanism of GA through selection, crossover and mutation process to explore and exploit the existing solution in a current generation in hope that the better one will be generated in a next generation. A review of GAs, their implementation issues and limitations can be found in Gen and Cheng (2000) and Younes et al. (2009).

For a CVP approach, it is a deterministic optimization method widely used for solving optimization problems involving systems of differential equations or transient processes. The basic idea of the CVP method is to transform an infinite-dimensional optimization problem into finite-dimensional NLP through approximation of control profiles by piecewise polynomial elements varying from simple piecewise constant to complicated polynomial one. Then, the properties of these elements become the decision variables of optimization problem (NLP). Using CVP approach, two subproblems are generated. One is master (outer) NLP and another one is (inner) initial value problem (IVP). The IVP is decoupled from the optimization stage and is integrated using existing DAE solvers in order to evaluate the objective function and the constraints. Then, the outer NLP, which is in the term of parameters defining the piecewise elements, is solved using well-known NLP techniques. In each iteration, the NLP algorithm adjusts the control parameters on the basis of gradient information obtained from sensitivity equations of objective function and constraints. This approach is also sometimes called sequential direct strategy (Banga et al. 2005). Note that the detailed strategy of this approach can be found from Wongrat et al. (2011) and the proposed algorithm was implemented in MATLAB.

### 4. Test Problems

The synthesis problems with both empirical and theoretical models of rice drying processes can be stated as follows: "Harvested rice grains with a head rice yield ($HRY$) of 70% are to be dried from the initial moisture content ($M_i$) of 34% dry basis (d.b.) to a final moisture content ($M_f$) that does not exceed 14% dry basis (d.b.), what is the optimum configuration of units and their operating conditions that maximize the rice quality?" The solutions sought for the synthesis problem consist of the following: (1) total number of passes required to dry rice from initial moisture content to final moisture content; (2) flowsheet configuration that indicates the existence of unit operations in each pass: a drying unit, a cooling unit, and/or a tempering unit; and (3) operating conditions of each unit in the
flowsheet in each pass. For drying and cooling units, they are air temperature, air relative humidity, and operation time. For tempering units, only operation time is sought.

4.1 Synthesis Problem with Empirical Model
Two drying models are considered in the GDP model to enable the analysis of drying units over a wider range of operating conditions. The first model is called low temperature drying model, developed by Wang and Singh (1978) for a temperature range of 35-55 °C (Equation (1) to (3)). The second one is called high temperature drying model, developed by Phongpipatpong and Douglas (2003) for a temperature range of 55-150 °C (Equation (4) to (5)). The cooling and the tempering units use the models shown in Section 2.2.1. The bounds on operating conditions of each model are summarized in Table 5.

<table>
<thead>
<tr>
<th>Model</th>
<th>Property</th>
<th>Lower bound</th>
<th>Upper bound</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low temperature drying</td>
<td>Air temperature (°C)</td>
<td>35</td>
<td>55</td>
</tr>
<tr>
<td></td>
<td>Air relative humidity</td>
<td>0.15</td>
<td>0.85</td>
</tr>
<tr>
<td></td>
<td>Time (hours)</td>
<td>0</td>
<td>6</td>
</tr>
<tr>
<td>High temperature drying</td>
<td>Air temperature (°C)</td>
<td>55</td>
<td>150</td>
</tr>
<tr>
<td></td>
<td>Air relative humidity</td>
<td>0.05</td>
<td>0.65</td>
</tr>
<tr>
<td></td>
<td>Time (hours)</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>Cooling</td>
<td>Air temperature (°C)</td>
<td>15</td>
<td>30</td>
</tr>
<tr>
<td></td>
<td>Air relative humidity</td>
<td>0.4</td>
<td>0.6</td>
</tr>
<tr>
<td></td>
<td>Time (hours)</td>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>Tempering</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

After solving the problem, the best found flowsheet with maximum head rice yield of 69.87% is shown in Figure 4. Note that because of the nonlinearity of the used models, the reported solutions are not the global optima; they are the best found solutions using different initial guesses. In this solution, eight drying-tempering passes were recommended to dry rice from initial MC to required MC. The operating conditions of drying units in all passes are the same and these belong to the low temperature drying model. The selected drying condition is the best one (highest air temperature and lowest relative humidity) from the two drying models which maximize the dryer efficiency while preserving the rice quality. The percentage moisture reductions in passes 1 to 8 are 4.1, 3.5, 3.0, 2.5, 2.2, 1.8, 1.6, and 1.3 respectively.

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**Figure 4:** The best found flowsheet of synthesis problem using empirical models.

Min=34%  Mt=29.9%  Mt=26.4%  Mt=23.4%  Mt=20.9%  Mt=18.7%  Mt=16.9%  Mt=15.3%  Mout=14%
4.2 Synthesis problem with theoretical model

The compartmental model as shown in Equation (8) to (10) was employed to this synthesis problem to predict state variables (moisture content in each compartment and grain temperature) of rice grain happening along each unit operation involved in drying processes. The bounds on operating conditions employed for each unit are summarized in Table 6.

<table>
<thead>
<tr>
<th>Unit</th>
<th>Description</th>
<th>Lower bound</th>
<th>Upper bound</th>
</tr>
</thead>
<tbody>
<tr>
<td>Drying</td>
<td>Drying air temperature</td>
<td>35</td>
<td>80</td>
</tr>
<tr>
<td></td>
<td>Relative humidity of drying air</td>
<td>5</td>
<td>80</td>
</tr>
<tr>
<td></td>
<td>Drying time</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>Cooling</td>
<td>Cooling air temperature</td>
<td>20</td>
<td>30</td>
</tr>
<tr>
<td></td>
<td>Relative humidity of cooling air</td>
<td>5</td>
<td>80</td>
</tr>
<tr>
<td></td>
<td>Cooling time</td>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>Tempering</td>
<td>Tempering time</td>
<td>0</td>
<td>6</td>
</tr>
</tbody>
</table>

After solving the problems, different configurations were obtained with very small difference of their quality (the differences are in the order of 10^-3) as shown in Table 7. These results suggest that the key factors in keeping the rice quality are operating conditions not the choice of configurations. The best found solution in all runs and the moisture content profile in each compartment are shown in Figure 5.

<table>
<thead>
<tr>
<th>Run</th>
<th>Best configuration</th>
<th>Best quality (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3 4 3 4 3 3 2 0</td>
<td>69.99891</td>
</tr>
<tr>
<td>2</td>
<td>3 5 3 3 3 3 2 0</td>
<td>69.99925</td>
</tr>
<tr>
<td>3</td>
<td>5 5 3 5 3 5 0 0</td>
<td>69.99917</td>
</tr>
<tr>
<td>4</td>
<td>3 2 3 4 3 1 5 0</td>
<td>69.99878</td>
</tr>
<tr>
<td>5</td>
<td>2 2 3 4 4 4 2 5</td>
<td>69.99921</td>
</tr>
<tr>
<td>6</td>
<td>1 5 3 2 1 2 2 0</td>
<td>69.99897</td>
</tr>
<tr>
<td>7</td>
<td>5 3 5 4 3 3 4 0</td>
<td>69.99932</td>
</tr>
<tr>
<td>8</td>
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5. Conclusions

The synthesis problems of rice drying processes with various types of drying models both empirical and theoretical were thoroughly investigated in this work. Due to the nonlinear characteristic of both problems, various drying configurations which can maintain decent quality of head rice have been found and these provide a broader vision on the operation of drying systems. Also, the results imply that the key factors in keeping the rice quality are operating conditions not the choice of configurations. However, one should keep in mind that the solutions found depends on the models used in the synthesis problem. With the use of different models, optimal solutions might differ significantly. For the synthesis problem with empirical models, there is a need of different empirical models for each particular unit represented in a rice drying process while the problem with theoretical model only one drying model describing coupled heat and mass transfer of rice grain is required. In term of the problem formulation, the empirical models leads to a simpler MINLP model which is able to solved in GAMS while the theoretical model give rise to the more complicated problem type called MIDO model which a hybrid method combining GA and CVP is proposed to solve the problem. Nevertheless, the results from the synthesis problem using the theoretical model provide a more detailed analysis and better insight of rice drying processes when compare to the empirical model.
Figure 5: (a) The best found flowsheet of synthesis problem using empirical models; (b) Progression of moisture contents in the best found configuration.

References


**Biography**

**W. Wongrat** holds a Bachelor degree in Food Engineering and a Master degree in Chemical Engineering from Kasetsart University and a Ph.D. degree in Chemical Engineering from the University of Waterloo. At Waterloo, she conducted research on the mathematical programming based synthesis of rice drying processes. She is currently a lecturer and Assistant Dean for Administration at Faculty of Engineering at Kamphaeng Sean, Kasetsart University, Kamphaeng Sean Campus, Thailand. Her research interests are in process systems engineering and optimization with applications to waste and energy minimization in the food processing industry and also logistic management for agricultural product.

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