Model of Integrated Production and Delivery Batch Scheduling Under JIT Environment to Minimize Inventory Cost

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Abstract

This paper considers a batch scheduling problem of a JIT supplier and integrates production and delivery decisions. Parts that have been sequenced are to be processed on a single machine, collected in a container, and sent to a customer in order to be received at a common due date. Scheduling involves the number of batches, batch sizes, scheduling of the resulting batches, and the number of vehicles delivering the batches. The objective is to minimize inventory costs wherein the holding costs of in-process parts and completed parts are distinguished. The holding cost is derived from the so-called actual flow time, defined as an interval time of parts spent in the shop since the starting time for processing a batch until its due date. The proposed problem is formulated as a non-linear programming solved by relaxing the variable of the number of batches to a parameter. An algorithm is proposed to solve the model and tested by numerical experiences. This research shows that there is a ratio between in-process parts and completed parts holding costs which affects the decision of the number of batches and their sizes. In addition, this research shows that the lower the in-process parts holding cost, the more advantageously to store parts as WIP than as finished product, and vice versa. The number of vehicles is determined from the number of the resulting batches.

Keywords
Integrated production and delivery, batch scheduling, actual flow time, distinguished holding costs of WIP and finished product.

1. Introduction

In the case of a company that produces parts on a production facility and then delivers the parts to customer on a delivery facility, production and delivery are two successive functions and interrelated. However, many companies manage these both functions separately. This separation could bring to a solution with a local optimum. In order to achieve the global optimum, these two functions need to be integrated (Chen, 2010b).

Integration of production and delivery decisions can be seen in the case of production of the parts that have been sequenced, so-called sequenced parts (junbiki), by a JIT supplier. In the JIT environment, the product must be received by the customer at the right time in the right quantities. This sequenced parts will be assembled to the customer's assembly line directly, and the product must be received at the time will be assembled in the right quantities. Lead time between the order received by the supplier and product received by the customer is too short to provide inventories (Iyer, et al., 2009). In order to provide 100 percent service level with minimum cost, the decisions of production and delivery must be integrated.

In this research, the sequenced parts processed on a single machine, collected in a container, and sent to a customer in order to be received by the customer at a common due date. For the next discussion, the term part will be used to refer to the sequenced part. Considering each container as one production batch, then many production batches that are delivered together in one vehicle are considered as a delivery batch, and the products are sent by a number of vehicles simultaneously at a common due date. The combined schedule involves determining the number of batches and their sizes, starting times of processing the resulting batches, and the number of vehicles.

Research on integrated production and delivery has been developed in recent decades. Chen (2010b) named it as IPODS (Integrated Production and Outbound Distribution Scheduling). Typical model of IPODS combines machine scheduling (to process the orders) and vehicle scheduling (to send the orders). The IPODS model is a problem of optimizing one or combination of time-based, cost-based and revenue-based performance (Chen, 2010b).
Many IPODS research with time-based performance has been conducted by Geismar et al. (2008) and Chen (2010a), while the cost-based performance has been addressed by Hall and Potts (2003 and 2005), Stecke and Zhao (2007), Zhong et al (2010), Chen and Pundoor (2009), Lee and Yoon (2010) and Armentano (2011), Yan et al. (2011). The IPODS research that combines cost-based and time-based performance has been carried out by Chen and Vairaktarakis (2005), and Gharehyakheh and Tavakkoli-Moghaddam (2012).

Much of cost-based performance of IPODS research does not distinguish the WIP holding cost and the finished product holding cost, but in practice these two holding costs are different. The research on integration of production and delivery and distinguishing between WIP holding cost and finished product holding cost was done by Lee and Yoon (2010). The holding costs are computed by multiplication of the holding cost per unit time and the flow time. Lee and Yoon (2010) assumed that all parts available in the shop at the beginning time of the scheduling period simultaneously. The resulting schedule is a combination of job scheduling in a production stage and batch scheduling in a delivery stage, without considering the due date.

This research develops Lee and Yoon (2010) model, and involves the decision of combining production and delivery batch scheduling under JIT environment. The performance used is the actual flow time proposed by Halim et al. (1994) instead of the traditional flow time (see Dobson et al., 1987, and Dobson et al., 1989). By this performance, the arrival of batches in the shop should not be at the beginning time of the scheduling period, but could be controlled to be at the starting times of processing the batch precisely. The objective is to minimize the total relevant costs.


The batch scheduling model discussed in this paper is developed from the batch scheduling problems that have been dealt in Halim et al. (1994). To ensure the due date is achieved, Halim et al. (1994) adopted backward scheduling by putting the first scheduled batch is closest to the due date and use the actual flow time as a performance measure. Halim et al. (1994) model did for the production stage only and considered the delivery time be zero.

2.1 The actual flow time of production batch scheduling

Halim et al. (1994) defines the actual flow time, $F^a$, as the time spent by material/job in the shop from starting time of processes part ($B$) until its due date ($d$). The actual flow time, $F^a$, of each part can be formulated as:

$$F^a_i = d - B_i, \quad i = 1, 2, ..., N$$  \hspace{1cm} (1)

In the batch scheduling, the batch processing time is calculated by multiplication of part processing time and the batch size. Figure 1 shows the actual flow time for each batch in a schedule. If $t$, $s$ and $Q$, respectively, stand for the part processing time, setup time per batch, and the batch size, then the actual flow time per batch, can be formulated as follow:

$$FL^a_i = \sum_{j=1}^{i} (s + tQ_j) - s, \quad i = 1, 2, ..., N$$  \hspace{1cm} (2)

The total actual flow time for $N$ batches can be written as follows:

$$F^a = \sum_{i=1}^{N} \left[ \sum_{j=1}^{i} (s + tQ_j) - s \right] Q_i$$  \hspace{1cm} (3)

Figure 1. The actual flow time
(Source: Halim et. al., 1994)
Batch scheduling models that distinguish in-process parts and completed parts have been developed in Halim and Ohta (1994). In-process parts are waiting in the batch until all parts of the batch have been processed, while completed parts are stored in the delivery stage until the starting time of delivery. The actual flow time of parts in both stages are shown in Figure 2, and can be formulated as below:

\[
F^a = \left\{ \sum_{i=1}^{N} tQ^2_i \right\} + \left\{ \sum_{i=2}^{N} \left\{ \sum_{j=1}^{i-1} (s + tQ_j) \right\} Q_i \right\} \tag{4}
\]

![Figure 2. The actual flow time of in-process parts and completed parts for the production batch](Source: Halim and Ohta, 1994)

### 2.2 Actual flow time of integrated production and delivery batch scheduling

Figure 3 shows the actual flow time of integrated production and delivery. The due date is defined as the time when the products arrive at the customer location. Let \( b \) is the departure time of vehicles, and \( v^0l \) is the travel time. The total actual flow time of parts in both stages is an addition of the actual flow time in production stage and in the delivery stage. The actual flow time in the production stage is equal to the Equation (3), while the actual flow time in the delivery stage is equal to the travel time. For a simultaneous delivery problem, the total actual flow time of \( N \) batches can be formulated as:

\[
F^a = \sum_{i=1}^{N} \left\{ \sum_{j=1}^{i} (s + tQ_j) \right\} Q_i + v^0l \sum_{i=1}^{N} Q_i \tag{5}
\]

![Figure 3. The actual flow time of integrated production and delivery](production)

The actual flow time of integrated production and delivery batch scheduling and distinguishing between in-process parts and completed parts is adopted from Equation (4), and shown in Figure 4. In a common due date problem, all vehicles departure simultaneously. The departure time is equal to the completed time of the first batch. Hence, all parts of the first batch available as in-process parts only. However, the availability of the other batches are differ from the first batch. Let take the second batch as an example (see Figure 4). The actual flow time of in-process parts in the second batch is an interval time between the arrival time of the batch until all parts of the batch have processed. After processing, all parts in the second batch is stored as completed parts and wait until the delivery
time. Thus, the actual flow time of completed parts of the second batch is defined as an interval time between completion time of the second batch and the delivery time, which is equal to the actual flow time of the first batch plus setup time.

The total actual flow time of \( N \) batches in the production stage and the delivery stage can be written as:

\[
F^a = \left\{ \sum_{i=1}^{N} tQ_i^2 \right\} + \left[ \sum_{i=2}^{N} \left\{ \sum_{j=1}^{i-1} (s + tQ_j) \right\} Q_i \right] + \nu^01 \sum_{i=1}^{N} Q_i
\]

(6)

The first term and the second term of Equation (6) shows the actual flow time of in-process part and completed parts, while the third term shows the actual flow time of delivered part that indicate the number of delivery.

![Figure 4. The actual flow time of in-process parts and completed parts of integrated production and delivery](image)

3. Problem definition

Let there be \( n \) parts of single item that are requested at a common due date, \( d \), and processed on a single machine. The total number of parts processed is equal to demand rate. The parts are moved in the container that called a batch. Consequently, the batch sizes are limited by container capacity, \( c \). Let assume that the setup activity does not require material, so that the arrival times of the batches in the shop are ensured at the same with starting times for processing the batch, \( B_i \). It is also assumed that the setup time of a batch, \( s_i \), is not affected by the batch sequence and the batch size, \( Q_i \).

All products must be received by the customer at the due date, simultaneously. Deliveries are done by a number of homogeneous vehicles, \( n \), which are able to carry \( k \) production batches. The availability of vehicles is not a constraint, because it is held by a third party. Travel time from suppliers to customers, \( \nu^0 \), is assumed constant.

The problems are how to batches all the requested parts so as minimize the total relevant cost, how to schedule the resulting batches, and how many vehicles are needed to deliver all batches. To ensure that the due date is not violated, a backward scheduling is chosen, i.e. scheduling by putting the first scheduled batch closest to the due date and move toward zero. The proposed problem is called the single machine-sufficient vehicle-common due date (SMSVCD). Figure 5 illustrates the SMSVCD problem.

![Figure 5. Illustration of SMSVCD problem](image)
The total relevant cost of SMSVCD problem is consist of the inventory cost in the production stage and in the delivery stage, and the delivery cost. The inventory cost is multiplication of the holding cost per unit time and the actual flow time. The delivery cost is proportional to the number of vehicles delivering the batches. There is a cost that is not effect by the actual flow time but must be accounted in the relevant cost, that is the cost for procuring containers.

Let $c_f$, $c_w$, $c_c$, $c_d$ and $n_c$ as the holding cost per unit time completed parts, the holding cost per unit time in-process parts, the procurement cost per unit containers, the delivery cost and the number of containers, respectively. Suppose that $F_{aw}^i$ and $F_{af}^i$ are the actual flow time of in-process part and the actual flow time of completed part, respectively. The total relevant cost, $TC$, for SMSVCD problem can be formulated as:

$$TC = c_w \sum_{i=1}^{N} F_{aw}^i + c_f \sum_{i=1}^{N} F_{af}^i + c_c n_c + c_v n_v$$

(7)

### 4. Model Formulation and Solution

#### 4.1 Model Formulation

The following assumptions are adopted in formulating SMSVCD problem:

- Parts moved in containers, and a container assumed as a batch
- Batch size is real positive value,
- Vehicle’s capacity is expressed in terms of containers,
- The number of containers in each shipment is always an integer,
- Completed time of the first batch is the same with the departure time of delivery,
- Travel time includes the activity of loading and unloading,
- Vehicles availability is sufficient.

Refering to Equation (6) and Equation (7), the SMSVCD problem can be presented as a non-linear programming as follows:

Minimize

$$TC = c_w \left( \sum_{i=1}^{N} tQ_i^2 \right) + c_f \left[ \sum_{i=2}^{N} \left( \sum_{j=1}^{i-1} (s + tQ_j) \right) Q_i \right] + \sum_{i=1}^{N} x_i c_c + c_v \left[ \frac{N}{k} \right]$$

(8)

Subject to:

$$(N - 1) s + \sum_{i=1}^{N} tQ_i + v^0 \leq d$$

(9)

$$\sum_{i=1}^{N} Q_i = n$$

(10)

$$Q_i \leq c, \quad i = 1, 2, ..., N$$

(11)

$$d - v^0 - B_i - (i - 1)s - \sum_{k=1}^{i} tQ_k = 0, \quad i = 1, 2, ..., N$$

(12)

$$x_i = \begin{cases} 
0 & \text{if } Q_i = 0, \quad i = 1, 2, ..., N \\
1 & \text{if } Q_i > 0, \quad i = 1, 2, ..., N \\
N \geq 1, \quad N = integer \\
Q_i, B_i \geq 0, \quad i = 1, 2, ..., N 
\end{cases}$$

(13)

(14)

(15)
Constraint (9) shows that all batches are processed and shipped from time zero to the due date. Constraint (10) states a material balance in the shop. Constraint (11) implies that the batch size does not exceed the container capacity. Constraint (12) ensures that all batches will be arrived at the customer location at the due date. Constraint (13) is a binary variable, that is 0 if the batch size is zero, and 1 otherwise. Constraint (14) shows that the number of batches must be an integer which greater than or equal to one. Constraint (15) shows the non-negative constraint.

4.2 Solution Method

The SMSVCD problem is solved by relaxing the decision variable N to be a parameter. Using Mathcad 14, we can find \( Q_i \) for certain value of N using Equation (8) to (15).

Proposition 1. In the batch scheduling problem with batch sizes is limited by the container capacity, \( c \), the batch sizes, \( Q_i \), are \( Q_i \leq c \), and the number of batches, \( N \), is \( \lceil n/c \rceil \leq N \leq n \), where \( n \) is the total processed parts.

Proof. The container can not store parts exceeded its capacity, \( c \). If the container is full of parts, the requirement of container will be minimum. The minimum requirement of container is computed by dividing the total number of parts processed (\( n \)) and the container capacity (\( c \)). However, the number of containers is an integer, so the minimum requirement of the containers is rounding up of \( (n/c) \). Conversely, if each container contains one part, the requirement of container will be maximum, so that the maximum of \( N \) occurs when \( N = n \). If one container is considered as one batch, then the maximum batch size (\( Q_i \)) is \( c \), the minimum number of batches is \( \lceil n/c \rceil \) and the maximum number of batches is \( n \). \( \square \)

Determining of the optimal solution of SMSVCD problem is started at \( N \) minimum according to Proposition 1 to find the total cost and the batch size as the initial solution. Computation followed by increasing \( N \) by 1 and its solution were compared with the previous solutions. This process is stopped when the solution of TC at the last \( N \) larger than the previous, or the value of \( N \) exceeded \( N \) maximum.

Proposition 2. In the integration of production and delivery problem and the vehicle capacity is expressed by \( k \) production batches, the minimum requirement of the vehicles delivering the batches for \( N \) production batches are \( [N/k] \).

Proof. The vehicle can not load the production batches exceeded its capacity, i.e. \( k \) batches. If the vehicle is loaded at the full capacity, the requirement of vehicle will be minimum. Thus, the minimum requirement of vehicles is computed by dividing the number of batches and the vehicle capacity, or \( (N/k) \). However, the number of vehicles is an integer, so the minimum requirement of vehicles is rounding up of \( (N/k) \), and stated as \( [N/k] \). \( \square \)

Refering to Proposition 1 and Proposition 2, an algorithm is proposed to solve the SMSVCD problem.

Algorithm SMSVCD

Step 1. Set predetermined value of parameters \( n, s, t, \nu^0, d, c_v, c_c, c_o, c, \) and \( k \).
Step 2. Using Proposition 1, compute N minimum,
Step 3. Solve Equation (8) to (15) with N minimum to determine TC and optimal batch sizes (\( Q_i \)). Set the current solution TC as the best solution (\( Z^* \)) and batch sizes as (\( Q^*_i \)).
Step 4. Increase the number of batch, \( N' = N + 1 \), then solve Equation (8) to (15) with \( N' \) to find TC and batch sizes (\( Q_i \)). Set the value of the total cost as \( Z' \) and the batch sizes as \( Q'_i \).\nStep 5. If \( Z' < Z^* \), then set \( Z^* = Z' \) and \( Q^*_i = Q'_i \), go to Step 6.
Otherwise, set \( N \text{ optimum} = N' \) go to Step 7.
Step 6. If \( N' > N_{\text{maximum}} \), then \( N \text{ optimum} = N' \) go to Step 7.
Otherwise, set \( N \text{ optimum} = N' \) go to Step 4.
Step 7. Using Equation (12), compute \( B_i \), and according to Proposition 2, compute the number of vehicles, \( n_v \).
Step 8. Stop
5. Numerical Experiences and Analysis

In order to show the behavior of the solutions, several numerical problems are presented here. The first numerical experience is done to demonstrate how the proposed algorithm solves the problem in one value of the parameter. The result of the experience shown in Table 1.

Table 1 shows that the proposed algorithm able to solve the SMSVCD problem. By taking one value for all parameter shown in Table 1, it is found that the minimum total cost occurs when the number of batches is 5, or \( N \) optimum is 5, the number of vehicles is 2 with simultaneous departure at 90. Figure 6 shows the Gantt Chart of SMSVCD problem.

<table>
<thead>
<tr>
<th>( N )</th>
<th>( Q_i )</th>
<th>TC</th>
<th>( B_i )</th>
<th>( b )</th>
<th>( n_p )</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>{20; 20; 20; 19; 11}</td>
<td>51,470</td>
<td>{80; 68; 56; 44.5; 37}</td>
<td>{90}</td>
<td>2</td>
</tr>
<tr>
<td>6</td>
<td>{20; 20; 20; 18; 10; 2}</td>
<td>51,480</td>
<td>{80; 68; 56; 45; 38; 35}</td>
<td>{90}</td>
<td>2</td>
</tr>
</tbody>
</table>

![Figure 6. Gant Chart of SMSVCD problem](image)

The second numerical experience is intended to find how the changes of in-process parts holding cost will affect the decisions of the number of batches and their sizes. The changes of in-process parts holding cost is expressed by the ratio of \( \frac{c_w}{c_f} \), whereas the other parameters were not change. Computations are conducted for several values of \( N \), and the result of the experiences are shown in Table 2.

<table>
<thead>
<tr>
<th>( N )</th>
<th>( Q_i )</th>
<th>TC</th>
<th>( Q_i )</th>
<th>TC</th>
<th>( N=7 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.6</td>
<td>{20; 20; 20; 20; 10}</td>
<td>48,930</td>
<td>{20; 20; 20; 20; 10}</td>
<td>48,930</td>
<td>53,140</td>
</tr>
<tr>
<td>0.65</td>
<td>{20; 20; 20; 20; 10}</td>
<td>49,780</td>
<td>{20; 20; 20; 20; 10}</td>
<td>49,780</td>
<td>50,620</td>
</tr>
<tr>
<td>0.7</td>
<td>{20; 20; 20; 20; 10}</td>
<td>50,620</td>
<td>{20; 20; 20; 20; 10}</td>
<td>50,620</td>
<td>53,140</td>
</tr>
<tr>
<td>0.75</td>
<td>{20; 20; 20; 19; 11}</td>
<td>51,470</td>
<td>{20; 20; 20; 18; 10; 2}</td>
<td>51,480</td>
<td>51,580</td>
</tr>
<tr>
<td>0.8</td>
<td>{20; 20; 20; 18.3; 11.7}</td>
<td>52,310</td>
<td>{20; 20; 20; 16.7; 10; 3.3}</td>
<td>52,280</td>
<td>52,380</td>
</tr>
<tr>
<td>0.85</td>
<td>{20; 20; 20; 17.9; 12.1}</td>
<td>53,140</td>
<td>{20; 20; 20; 15.7; 10; 4.3}</td>
<td>53,070</td>
<td>53,170</td>
</tr>
</tbody>
</table>

Table 2 shows that for \( N = 5 \) and \( N = 6 \) and the value of \( \frac{c_w}{c_f} \) less than or equal to 0.7, the batch sizes for the batch number \( i \) until number \( (N-i) \) are equal with the container capacity, while the last batch is equal with the remaining parts. However, if \( \frac{c_w}{c_f} \) greater than or equal to 0.75, the minimum total cost will be achieved when the last several batches will share the remaining parts. It means that in the in-process parts holding cost much lower than
the completed parts holding cost, there is a tendency to store parts as in-process parts than as completed parts, and vice versa.

The reasoning can be described as follows: the bigger the batch size, the longer the actual flow time of in-process parts in its batch. Referring to the objective function as shown by Equation (8), the longer of actual flow time of in-process parts will increase the inventory cost of in-process parts.

This numerical experience shows that there is a ratio of \( \frac{c_w}{c_f} \) affecting the decisions of the number of batches and their sizes. This value is called a changing point. If \( c_w/c_f \) less than the changing point, the batch sizes of the first batch until the batch number \((N-1)\) will be maximum, and the batch size of the last batch is equal to the remaining part. However, if \( c_w/c_f \) is greater than the changing point, the decision of batch sizes will tend to avoid fulfillment in the last several batches.

6. Conclusion
The integrated production and delivery batch scheduling problem distinguishing the holding costs of the in-process parts and the completed parts is affected by several variable decisions, e.g. the number of batches \((N)\), the batch sizes \((Q)\), scheduling of the resulting batches and the number of vehicles delivering the batches \((n_p)\). The problem can be formulated as a non-linear programming model. Referring to the numerical experiences, it can be concluded that there is a ratio between the holding costs of in-process parts and completed parts which affects the decisions of the number of batches and their sizes. In addition, the lower the in-process parts holding cost, the more advantageously to store part as a WIP than as a finished product, and vice versa. The number of vehicles is determined from the number of resulting batches.

References


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