The Operating Decisions of Corporate Business Based on the Brusselator Model

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Abstract

There are a variety of modes of operating business in modern enterprise. Different modes will directly affect the operational effectiveness and efficiency, and they are essential for the survival and development of enterprises in the highly competitive and complex environment. All the modes can be divided into self-operation and outsourcing. According to the theory of dissipative structures, decision-makers can choose the appropriate way to operate business with the method of building the Brusselator model. The steady state solution of the system can be solved from the classical model and its expansion of the Brusselator model. Then applying perturbation for the solution, and the decision-makers will know whether the system is stable or not, that is whether the system can be a dissipative structure, which determines which modes is suitable for the corporate business between proprietary trading and outsourcing. So it’s the innovation of the article that the Brusselator was improved by expanding the primeval model and the dissipative theory and then the results from the analysis of models can provide effectively theoretical reference for the corporate operations.

Keywords
Brusselator, Dissipative Structures, Business Operation, Self-operation, Outsourcing

1. Introduction

According to the enterprise life cycle theory, adjusting the direction of the development of enterprises timely plays a role in maintaining the vitality of enterprises (Ichak Adize 1997). Since the development of enterprises involves in their business operations, the different ways of business operations will directly affect the effectiveness of corporate management and its expanded reproduction.

The modern enterprises are operated in various ways that can be divided into different categories based on different standards. From the geographical scope of business activities, they can be divided into domestic and transnational business operation; from the size of business, they can be divided into single and multiple business management; from the operation period length, they can be divided into short-term and long-term business management and so on (Wangchen Lu 2002). In the light of different operating right of enterprises, business operations include self-operation and outsourcing in the article. Especially operational cooperation among the enterprises belongs to outsourcing. It’s no doubt that business self-operation can enable enterprises to master the self-control, prevent the disclosure of trade secrets, revitalize existing business and enhance their visibility, but meanwhile it can cause complex management problems and poor coordination and other issues; business outsourcing focus on their core business competitiveness and reduce operating costs, but it’s inevitable for the risks of joint management and customer relationship management. So both business self-operation and outsourcing have their pros and cons which the article focuses on the study of reasonable selection for them.

2. Literature review

As to the research about business operations, the related literatures domestically aren’t rare because of various perspectives for the business operations. For example, Zhou Jizhi (1987) elaborated the advantages and disadvantages of contracting operation, leasing operation, stock operation from the size of enterprises, operating conditions, operating mechanism and the macroeconomic. According to transaction costs theory, Zhang Jincheng (2011) researched how companies choose different business strategies to reduce transaction costs. From Lei Lin and Liu Qian’s book (1999), game theory was applied to make decisions of an enterprise about business operations. However the researches that determine the choice between self-operation and outsourcing mostly are related to logistics. According to the main factors affecting the number of different decision-making, logistics operations can
be divided into the single dimensional determinism (Wang Shuyun 2004) pointed out that enterprises should apply scarce resources on core competencies, and then the business of non-core ability should be contracted to an outside organization), binary determinism (Qinghong Xie 2010) and his research term agreed with the point so that they provided a general reference with the two-dimensional decision model they designed) and the multi-dimensional determinism (Tian Yu and Gong Guohua 2000) built three-dimensional model with cost factors based on the Ballow model).

Generally the researches abroad for business operations have qualitative and quantitative criteria. So the qualitative researches usually confirm the ways of a business operation on the basis of the analysis of enterprise tangible or intangible resources. For example, Ronan McIvor (2010) analyzed the functions and limitations of business operations in which the resource theory involved and concluded the market conditions play a vital role in outsourcing decisions. Kumar Sameer (2007) and his team used the process approach to analyze the outsourcing decisions in services and provided references for the enterprise management and cost saving. It’s commonly found in the quantitative researches that mathematical methods are applied on business operations. Nayarit Shirkouhi (2011) and his team made use of fuzzy multiple criteria group decision to determine whether the enterprise's information systems adopt outsourcing. Kumar Sameer (2010) decided the way of production enterprise business operations based on the closed-loop of production, distribution capabilities and associated costs. Jabbour Liza (2013) built dynamic probabilistic model and explained outsourcing produced a large of sunk costs and market thickness affected the stability of outsourcing decisions.

These studies point out determinants that impact organizational forms (such as transaction frequency, asset specificity, corporate strategy, transaction costs, market maturity, etc.) and then make use of some theory to make decision between outsourcing and proprietary. Nevertheless an enterprise is composed of various elements which the role of the different elements is constantly changing in different stages of business development. Only from a systems perspective consider the impact of the various forces for business operations, and decision makers will choose a suitable way for enterprise development. The article studied business operation in the terms of dissipative structures theory and provided the theoretical basis for determining the choice between outsourcing and self-operation.

3. The Characteristics of Dissipative Structure in Business Operations

The dissipative structures theory that belongs to complex systems theory can avoid the issues above. From the study for the input and output of the whole system, the reasonable business decisions of enterprises depend on the orderly corporate structure of business operations. The corporate business operations have the characteristics of dissipative structures which need to constantly exchange matter or energy with the surroundings in order to maintain a stable macro-ordered structure system (Xiaofeng Shen and Gang Hu 1987).

Firstly, the enterprise is an open system that constantly exchange matter, energy and information with the outside. It is because of operating various businesses that the enterprise can finish the exchange. In this case, the total entropy of the system decreases because negative entropy outside input the system continuously. Thereby the enterprise improves its self-organization.

Secondly, the various business operations cause the system to produce different order parameters that describe the degree of order inside the system and the phase change process. Furthermore, these parameters aren’t identical in different periods of corporate life cycles. Hence the faster order parameters change, the more transient new stabilization remains. In other words, the system always stands in the phase transition that is far from equilibrium state and fluctuations happen all the time under the circumstances. The instability of system promotes the change of the way of business operations and makes enterprises avoid the fatal "short board" in businesses.

Moreover, both outsourcing and self-operation obtain negative entropy from the external environment constantly. With the self-catalyst the choice of different ways of business operations has periodicity and produces the oscillations in time and the diffusion in space. It is also because of this alternation that the balance in the original system is constantly broke and forms non-linear transition that amplifies the fluctuations mentioned in the previous paragraph.

In addition, the choice for the mode of business operations can be considered as a self-organized phenomenon within the social systems. It means that subjective initiative, which promotes the innovation of enterprise
management and improves the dynamic adaptability for the surroundings in the business decisions, plays a key role in the self-organization of the system.

The formation mechanism of dissipative structure can be summarized by the Brusselator model and the basic model is as follows:

\[
\begin{align*}
    A & \xrightarrow{k_1} X \\
    B + X & \xrightarrow{k_2} Y + D \\
    2X + Y & \xrightarrow{k_3} 3X \\
    X & \xrightarrow{k_4} E 
\end{align*}
\]

(1)

Here \(A\) and \(B\) are the initial reflection. \(D\) and \(E\) is reflected products. The intermediate component \(X\) and \(Y\) can change over time. \(2X + Y \rightarrow 3X\) is the most important reflection because it causes the non-linear diversification that is essential to the special behavior of the system. \(K_i (i=1, 2, 3, 4)\) are the different catalysts in the processes and their amounts affect the rate of reaction. This reaction path can be represented by Figure 1 (Niclis G and Prigosine I 1977).

![Fig.1 the reaction path of the classical Brusselator](image)

4. The Brusselator Model for Business Decisions

The feasible way of business operations is related to the corporate ability to generate maximum profit. So it’s the most important task after confirming the business that the enterprise makes the choice of the model of the operations. The issue will be analyzed by establishing the Brusselator model below.

4.1 Problem assumptions and definitions

Now we convert the factors of business operations to the Brusselator model’s variables \(A, B, C, D, E\) and \(F\) as follow.

The model assumes \(A\) as the main businesses of the enterprise and their demands in the market and \(B\) as the non-core businesses and their demands. Both \(A\) and \(B\) represent not only the elements above but also their concentrations that mean the intensity, the importance and the urgency of the issues.

\(X\) can be seen as the corporate structure that operates the main businesses, and \(Y\) is the non-core business structure. The structures include all kinds of factors of production (labor, land, capital and entrepreneurship), the organizational structure and culture, institutional policies and technical level etc.

\(D\) and \(E\) are the productions from the business operations. The non-core businesses output \(D\) and the main businesses output \(E\). The outputs include the services or goods that the enterprises provide for the market, customers’ satisfaction, the share of the market, the firm’s size and so on.
$K_i$ ($i=1, 2, 3$ and $4$) indicate the conditions and the environment in the different periods of the business development and catalyze the manner and intensity of the operations, such as policymakers’ anticipation, the policies in the market, the level of informatization, the consciousness of innovation and the awareness of risks and consumption habits etc.

Accordingly the corporate business decisions can be described as the Brusselator model as follows (the model I):

\[
\begin{align*}
A \xrightarrow{K_1} X \\
B \xrightarrow{K_2} + X \xrightarrow{K_3} Y \\
Y + 2X \xrightarrow{K_4} 3X \\
X \xrightarrow{K_5} E
\end{align*}
\]

As the main source of corporate profits and reflection of the core competitiveness, the core businesses $A$ stimulate or catalytic the main operational structure under certain conditions $K_1$. In order to make more profits, the enterprise invests additional resources in core business continuously so that the structure of the main business continues to be improved, that is $X \rightarrow 2X$. With the development of the businesses, the enterprise continues to expand, but the new businesses can’t become the company's main business immediately so that non-core businesses $B$ generates in the system. At this time, $X$ model can’t meet the development of $B$ and the non-core business structure $Y$ produces in the enterprise under the conditions $K_2$. With the competition and cooperation between $X$ and $Y$, the core businesses $X$ aren’t replaced by $B$ but upgrade obviously. In other words, the system achieves the transition from junior to senior order. In this case, $X$ and $Y$ produce cross-catalytic under the effect of $K_3$, that is $Y+2X \xrightarrow{K_3} 3X$. According to the cross-catalytic and the self-catalytic, $X$ will generate the new development under the new role of $K_4$.

In the period of the development of whole business operations, if the impact of $B$ in the model is not enough to make total system entropy change within thresholds, the existing model of business operations is still available. If the impact reaches or exceeds the threshold, the existing structure of the core business operations can’t meet the demand in the market so that the structure of the non-core business operations comes into being and then obtains greater development.

### 4.2 Dynamics equation’s solution in model I

According to the theorem of mass action and the reaction equations, the model I can be solved by chemical kinetic equations (Deyi Li and Songtao Liu 1997) below,

\[
\begin{align*}
\frac{dX}{dT} &= K_1A - K_2BX + K_3X^2Y - K_4X + \rho_1\Delta X \\
\frac{dY}{dT} &= K_2BX - K_3X^2Y + \rho_2\Delta Y
\end{align*}
\]

(3)

Here $\rho_1$ and $\rho_2$ are the diffusion coefficients of $X$ and $Y$. There are hypothesizes for the equations as follows:

\[
\tau = K_4T, \quad x = \sqrt{\frac{K_1}{K_4}}, \quad y = \sqrt{\frac{K_4}{K_4}}
\]

\[
a = \sqrt{\frac{K_2K_3}{K_4}}, \quad b = \frac{K_2}{K_4}
\]

If we assume that it does not consider diffusion formula ($\rho_1=\rho_2=0$), equation (3) can be converted to the following equation:

\[
\begin{align*}
\frac{dx}{d\tau} &= a - bx + x^2 - x \\
\frac{dy}{d\tau} &= bx + x^2y
\end{align*}
\]

(4)

Making $dx/d\tau = 0$ and $dy/d\tau = 0$, and getting steady state solution by solving the equation:
In order to check the model conveniently, the catalytic $K$ can be assumed as 1. So the steady state solution is: $X_0 = A$ ; $Y_0 = B/A$. From the significance of the steady state solution, the prerequisites that an enterprise meets the demands $A$ are the formation of the main business structure $X$ which can solve the problems in the $A$. So $X_0 = A$. Because of the mutual promotion and coordinated development between $X$ and $Y$, the new business operational structure forms in the enterprise which can also solve the problems in both $A$ and $B$. So $Y_0 = B/A$.

4.3 The analysis for the stability of model I

Based on the above analysis, a uniform steady state exists in the system and the stability of enterprise business system can be analyzed with the method of applying perturbation to the steady state. If the result will stabilize gradually, the dissipative structure can form in the system. Therefore, the self-operation of the non-core business isn’t suitable for the corporate development. Conversely if the minor disturbances bring about large fluctuations in the model’s solution, it proves that the enterprise business operations can appear the possibility of producing dissipation branch and in this case the model of self-operation will contribute to the formation of dissipative structures.

This article will solve the multi-dimensional linear system with the method of the linear-stability analysis. So $x = x_0 + \mu_1$, $y = y_0 + \mu_2$ (\(\mu_1\) and \(\mu_2\) are minimal variables). The Taylor series of multivariate function can be expanded near the point \((X_0, Y_0)\) and get the formula:

\[
\begin{align*}
\frac{d\mu_1}{d\tau} &= (b-1)\mu_1 + a^2 \mu_2 \\
\frac{d\mu_2}{d\tau} &= -b\mu_1 - a^2 \mu_2
\end{align*}
\]

(6)

Its characteristic equation is

\[
\lambda^2 - \omega\lambda + a^2 = 0, \quad \omega = b - 1 - a^2
\]

(7)

The solution of the equation is:

\[
\lambda_{1,2} = \frac{1}{2} (\omega \pm \sqrt{\omega^2 - 4a^2})
\]

(8)

Based on the analysis of normal mode, we can determine the stability of the system by the value of $\omega$:

(1) When $\omega > 0$, $\mu_1$ and $\mu_2$ will change over time. And the disturbance for the steady solution will make $x$, $y$ away from the solution and form an unstable focus. Thus the system will form a branch of dissipative structures and then a new ordered structure. It illustrates self-operation of non-core business has the potential to promote enterprise form dissipative structures and the ancillary business has the possibility to become the main business. Therefore the decisions of business self-operations are reasonable and effective.

(2) When $\omega < 0$, the disturbance for the steady solution will make the $x$, $y$ still approach the steady state by degrees and form a stable node so that the system won’t have a branch of dissipative structures. At this time, the outputs from the business self-operations will be more harm than good for the enterprise, and then the non-core business operations should try to choose the model of outsourcing or unitization.

5. Improving the Model

In regard to the business that can’t be operated by the enterprise itself, they also keep doing the businesses by the outsourcing or the unitization. Then the evolution of the system will be the "dual-core" and even the "multi-core" development. Hence, the model I is improved to accommodate the model of the outsourcing and the unitization so that the business outsourcing can be illustrated whether suitable for their own development or not.

5.1 definitions for the new model

Assuming that only one contractor is responsible for the outsourcing or the unitization of the corporate business in the same period, then the Brusselator model was extended as follows (Model II):
The meaning of each variable in the model is converted as follows:

The contents of $A$, $B$ and $X$ in the model II have the same meaning as what in the model I. $Y$ is the supplier’s structure of the business operations. So the formation and development of $Y$ can be influenced by not only $B$ and the core business structure $X$ but also the business autocatalytic and the cross-catalytic between $Z$ (the structure of the non-contracting businesses operations from the agents) and $Y$ from the agents. In addition, the disturbance for the agents from the external environment $C$ can also reflect to $Y$. It means that the relationship between $X$ and $Y$ grow from the original auxiliary relations like the "Single core" to the unbeatable combination like the "dual core". In other words, if there are industrial barriers or monopolies’ crowding and other issues when the enterprise operates the new business, the alliance with the advantages of enterprise in the business is a better choice for the new development than the corporate self-operations. $D$, $E$ and $F$ are the outputs of the core businesses, the contracting businesses and the non-contracting businesses from the agents respectively.

\[ T_i (i = 1, 2 \ldots 6) \] represent the environment and the conditions in different periods, which stimulate and catalyze the development of the business structure, such as the competition from the similar products or services, the game between the enterprise and its contractors, the trade policy, the degree of sharing information among the enterprises, the integration of resources, the distribution of benefit etc. the paths of the reaction in model II can be represented by Figure 2.

\[ \tau \]

Figure 2 the reaction path of the expanding model

Like formula (3), dynamic equations that solves the model II can be created below,

\[
\begin{align*}
\frac{dx}{d\tau} &= a - bx + x^2 - x \\
\frac{dy}{d\tau} &= bx + x^2 y + zy^2 - cy \\
\frac{dz}{d\tau} &= cy - zy^2
\end{align*}
\]

(10)

$T_i$ are assumed as 1, so that

\[
\begin{align*}
\frac{dx}{d\tau} &= \frac{dy}{d\tau} = \frac{dz}{d\tau} = 0
\end{align*}
\]

(11)

The model’s solution in the steady state is $X_0=A, \ Y_0=B/A, \ Z_0=CA/B$. 

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5.2 The analysis for the stability of model II

Some disturbance are applied into the solution in the steady state above, and then multi-dimensional linear stability of the system can be solved with the method of the analysis for a linear system stability. Making $X = x_0 + \mu_1$, $Y = y_0 + \mu_2$, $Z = z + \mu_3 (\mu_1, \mu_2$ and $\mu_3$ are minimal variables), and Taylor series of multivariate function will be expanded near the point($X_0,Y_0,Z_0$), so that

$$\begin{align*}
\frac{d\mu_1}{dt} &= (b-1)a_2 \mu_1 + b_2 \\
\frac{d\mu_2}{dt} &= -b\mu_1 + (c-a^2)\mu_2 + \left(\frac{b}{a}\right)^2 \mu_3 \\
\frac{d\mu_3}{dt} &= -(c+b/a)\mu_2 - \left(\frac{b}{a}\right)^2 \mu_3
\end{align*}$$

(12)

Therefore, the characteristics of the differential equation is

$$\begin{vmatrix}
b-1 \lambda & a^2 & 0 \\
-b & c-a^2 \lambda & \frac{b^2}{a^2} \\
0 & -c-b/a & \frac{b^2}{a^2} \lambda
\end{vmatrix} = 0$$

(13)

Its eigenvalues is $\lambda_k = p_k + q_k i$, $k = 1,2,3$. In turn, the solution of model is like

$$\mu_s = e^{\tau \lambda_k} \cdot (a_k cosq_k \tau + \beta_k sinq_k \tau) \cdot m_s(\tau), \ s=1, \ 2, \ 3.$$  

(14)

When $\lambda_k$ have n-fold roots, $m_s(\tau)$ is n polynomial.

According to the analysis of differential equations motion, we can find whether the system is stable through the eigenvalues $\lambda_k$ and the general solution (14),

(1) When the real components of $\lambda_k$ are negative, the solutions of equations gradually are flattening out with the growth of time $\tau$. It shows that the system is asymptotically stable. At this point, the new business of enterprise should strengthen the control and supervision of business operations and maintain moderate elasticity of outsourcing. At the same time, the enterprise should strengthen the ability of self-operation for the new businesses, and it’s the final purpose that the enterprise gradually takes over the operation of the new business.

(2) When $\lambda_k$ doesn’t have a positive real part but a zero real part, if the root of zero real part is a single root, the system is stable. In other words, the fluctuations within the system don’t reach the threshold so that the mutations branch can’t appear in the system and it will not become dissipative structure through self-organization. At this time, outsourcing isn’t suitable for the overall development and self-operation can be considered as the model of developing the new businesses. If the root of zero real part is multiple roots, the stability of the system is uncertain because the stability of equations is in a critical state. It means that the outsourcing can’t always been conducive to the development of the enterprises and the operations of new businesses should make a short-term decision and adjust immediately according to the actual problem.

(3) When there’s a positive real part in the real part of $\lambda_k$, the system is unstable. The fluctuations in the system will be amplified at this time, and the unstable situation and the mutations branch appears in the system. Then the system forms the dissipative structures which show that the outsourcing has the ability to promote the healthy development of the new businesses. In the situation, the enterprise should help the agents realize their goals and tasks within the cooperation, adjust the policy of outsourcing actively and integrate the resources for production so that the mutation can develop towards the positive direction.
6. Conclusion
The Brusselator models for the business operations above indicate a transition of the business operations and describe the process that the business operations optimize and upgrade constantly. From the analysis for the models, we find the business operations depend on the material, energy and information within the enterprise and are limited by the external environment. The choice about the model of the new business operations is affected by the "density" of not only A and B but also C which works through impacting the agents indirectly.

Different from the original model, the model II reflects parallel reactions (see figure.2) of Brusselator model, which is theoretical innovation in the field of dissipative structures. The innovation improves the complexity of the original model and promotes the development of the theory towards practical application. In the light of the entire analysis from above sections, we can get the decision flowcharts (see Figure.3) about the business operations in the enterprises based on the Brusselator model which can describe the procedure enterprises make decisions by above models. In addition, the detail about making decisions to the model of the business operations in the different periods of the corporate development can be concluded in Table 1. From table1, those policymakers can find the scientific modes and the theoretical reference in their business operations.

Figure.3: The decision-operating flow chart of a corporate business based on the Brusselator model
Table 1: the decision table for the operation of new business based on the Brusselator model

<table>
<thead>
<tr>
<th>$\lambda_k = p_k + q_k \dot{i}$</th>
<th>Model I</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k = 1, 2, 3$</td>
<td>$B &gt; 1 + A^2$</td>
</tr>
<tr>
<td>$p_k &lt; 0$</td>
<td>Self-operations</td>
</tr>
<tr>
<td>$p_k \leq 0$ and there's $\lambda_k$ have single root</td>
<td>Self-operations</td>
</tr>
<tr>
<td>$p_k = 0$ and there's $\lambda_k$ have Multiple root</td>
<td>Self-operations</td>
</tr>
<tr>
<td>if there's $p_k &gt; 0$</td>
<td>Short-term outsourcing Long-term self-operations</td>
</tr>
</tbody>
</table>

In the meanwhile, the system of the business self-operations reaches a steady state easier than the outsourcing from the solution of the models. It can be explained that self-operations which facilitate the management and the supervisory control for the businesses let the enterprises closer to the market. By contrast, the outsourcing is influenced more easily by the disturbances from the external environment which affects both the enterprises and their agents. Furthermore, the changes of the business in the agents also impact the business operations in the enterprise. So the changes among the factors of the disturbance for the outsourcing business are more significant and the system is more prone to instability. In addition, it’s also proved that the vast majority of businesses in the enterprises are usually operated by themselves from the perspective of self-organization theory.

From the analysis above, the model I can provide the theoretical support for the business self-operations in the enterprises and the model II is focused on the evolution and development of the “dual-core” or even the “multi-core” system which is composed by the outsourcing business. By determining the order parameter of the business operations and the conditions of the dissipative structures, theoretically decision-makers can find the suitable approaches for the corporate business operations in different periods and conditions so that a theoretical reference and important basis for the business decisions can be provided for the enterprises.

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