Multi-manned assembly line balancing problem: Resource constrained case

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Abstract

In this paper, a new approach on multi-manned assembly line balancing problem is presented. The goal of proposed approach is to establish balance of multi-manned assembly line with minimum number of worker, workstation and resources. For this purpose, mathematical model is developed for the problem. Finally, this model is solved for a numerical example using Gurobi to indicate the applicability of the proposed mathematical model.

Keywords  
Assembly line, Assembly line balancing problem, Multi-manned assembly line balancing, Mathematical modelling, Resource constraint.

1. Introduction

Assembly lines are the most common method for high volume production. Assembly line balancing (ALB) is the problem of assigning tasks to workstations while optimizing one or more objective, subject to the constraints occurring on the line (Gökçen et al., 2006, Alavidoost et al., 2014). A considerable amount of literature is devoted to solving conventional ALB problems (Baykasoğlu and Dereli, 2008). In addition, conventional ALB problems are known as the NP-hard problems (Ege et al., 2009, Yeh and Kao, 2009, Triki et al., 2014). Detailed reviews of these studies are given by Scholl and Becker (2006) and Becker and Scholl (2006).

Most researchers have studied on assembly line balancing problems for more than 50 years and the different surveys on the line balancing problem have been published in literature (Toksari et al., 2010). Especially in recent times, due to advances in technology, it has become possible to produce more complex goods. If such goods are produced in conventional simple assembly lines, hundreds of stations will be needed. (Kellegöz and Toklu, 2012). Much research has addressed the advantages of MMAL each station benefits from the presence of more than one worker and thereby results in shorter production times. Furthermore, duration of throughput is shorter and it is handled with less material. It also keeps work in process low. (Cevikcan et al., 2009, Buckhin and Masin, 2004).

Line designers enable stations to accommodate several workers, thus balancing out the line and avoiding the disadvantages of conventional assembly lines. Multi-manned assembly lines (MMAL) differ from conventional assembly lines due to multi-manned work stations, where worker groups assemble the tasks on the same workstation, as depicted in Figure 1 (Ege et al., 2009).
In MMALB problems, tasks are assigned to an order sequence of stations according to precedence relations. While assigning tasks to the stations, specific objectives must be optimized, such as minimizing the cycle time for a given number of the stations or minimizing the number of the stations for a given cycle time \((C_t)\). Additively, in literature efficient usage of resources that carry out assembly line operations has been targeted. In this paper, multi-manned assembly line balancing problem is studied with resource constraints.

The paper is organized as follows: the second section will address the resource constrained multi-manned assembly line balancing and developed integer programming model for proposed problem. In section 3, solution and comparison of multi-manned assembly line and resource constrained case on a numerical example is given. Finally in Section 4, conclusion and future studies are provided.

### 2. Resource constrained multi-manned assembly line balancing (RCMMALB) problem

Despite the fact that MMAL has been frequently applied in industry, very few studies about this approach have been reported. The first study that applied a teamwork approach to assembly line design was performed by Johnson (1991) who provided a new branch-and-bound algorithm and also mentioned the importance of teams. Bartholdi (1993) addressed an assembly line configuration that consists of two serial lines in parallel where two workers could be present in one multi-manned station. McMullen and Tarasewich (2006) and McMullen and Frazier (1998) addressed the assembly line balancing problems in which more than one worker performs the same tasks on individual product moving through a workstation. Bukchin et al. (1997) designed a new method for team-based assembly systems. Problem solving approach is based on assigning bill of material elements to teams and a hierarchical design approach is presented. Bukchin and Masin (2004) designed a team-oriented assembly line process by allocating tasks to teams according to a bill of material structure, while determining the required number of teams and number of workers in each team. Additionally, they introduced a branch and bound algorithm to solve the multi-objective design problem and a heuristic algorithm to address large-scale problems. Dimitriadis (2006) modified a heuristic algorithm to solve assembly line problems which include multi-manned workstations. Proposed heuristic algorithm is applied to well-known benchmarking problems and a real-life automobile assembly line problem. Çevikcan et al. (2008) devised a mathematical model for multi-manned stations in mixed model assembly lines and developed a scheduling based heuristic algorithm which include both model sequencing and worker transfer systems. Fattahi et al. (2011) presents a mixed integer programming model and a heuristic algorithm, based on ant colony optimization approach, developed to solve the medium and large-scale balancing problems of multi-manned assembly lines. In this model, the first objective is to minimize the total number of workers and the second objective, to minimize the number of open multi-manned workstations. Kellegöz and Toklu (2012) considered an assembly line balancing problem with parallel multi-manned stations, and following the problem definition, a mixed model multi-manned assembly line balancing problem with the aim of minimizing total cost per production unit. They develop a mathematical formulation of the problem, and propose a genetic algorithm (GA) to solve the problem. Yılmaz and Yılmaz (2015) considered an assembly line balancing problem with multi-manned workstations in order to minimize the total number of workers on the line, the number of opened multi-manned workstations and the differences of the workers total task load. Kellegöz and Toklu (2015) presented an efficient branch and bound algorithm for assembly line balancing problems with parallel multi-manned workstations. In their study, VWSolver algorithm is compared with the proposed algorithm and the results illustrates that the proposed algorithm outperforms the VW Solver in terms of both quality of feasible solutions and CPU times. Roshani et al. (2013) proposed a simulated annealing heuristic to solve assembly line balancing problems with multi-manned workstations and as the performance criteria the line length, efficiency and the smoothness index are considered. The performance of algorithm is tested on problems in the literature. Kazemi and Sedighi (2013) consider mixed model multi-manned assembly line balancing problem with the aim of minimizing total cost per production unit. They develop a mathematical formulation of the problem, and propose a genetic algorithm (GA) to solve the problem. Yılmaz and Yılmaz (2015) presented a mathematical model for assembly line balancing problem with multi-manned workstations in order to minimize the total number of workers on the line, the number of opened multi-manned workstations and the differences of the workers total task load. Kellegöz and Toklu (2015) considered an assembly line balancing problem with parallel multi-manned stations, and following the problem definition, a mixed integer programming formulation and a new efficient constructive heuristic algorithm based on priority rules is proposed.
When considering the literature, it is clarified that there has been some published study about assembly line balancing and multi-manned approach together. In addition to these studies, the importance and advantage of resource constraints are mentioned in the literature (Ağpak & Gökçen, 2005, Corominas et al, 2011). There are many reasons what makes resource constraints important. One of the most important reason is a need for appropriate machine or skilled workforce.

Track or bus assembly tasks can be a good sample to figure out the difference of this study. This kind of line has many varied work requirements and tasks need skilled workforce or special equipment such as electrical, electronic, or engineering expertise to complete the line. Therefore, all tasks must be assigned to resources before assigned to a station. Hereby, the tasks that can be performed by the same resource should be assigned to same workstation. Proposed model enables resource saving and for this reason during this classification, number of resources is minimized as much as possible. This is what makes this method different from similar approaches in the literature.

3. The mathematical programming model

The developed mathematical programming model is shown below:

3.1 Notation

The notation used for the mathematical model is described as follows.

3.1.1 Indexes

i, i1: Tasks.
j: Workstations.
k, k1: Workers.
l: Resources.

3.1.2 Sets

J: Set of workstations. J={1,2,...,j,...,n}
K: Set of workers at each workstations. K={1,2,...,k,1,...,Nmax}
L: Set of resources. L = {1,2,...,l}
M(l): Set of tasks use l. resource.
P(i): Set of immediate predecessor of task i.
P1(i): Set of all predecessor of task i.
R(i): Set of immediate successor of task i.
R1(i): Set of all successor of task i.
M: A large positive number.

3.1.3 Decision variables

x_{ijk} = \begin{cases} 
1, & \text{if task } i \text{ is assigned to the worker } k \text{ in workstation } j. \\
0, & \text{otherwise} 
\end{cases}

w_{skj} = \begin{cases} 
1, & \text{if worker } k \text{ is used in workstation } j. \\
0, & \text{otherwise} 
\end{cases}

R_{ijk} = \begin{cases} 
1, & \text{if l. resource is used in workstation } j \text{ by } k.\text{th worker.} \\
0, & \text{otherwise} 
\end{cases}

w_{i,i1} = \begin{cases} 
1, & \text{if task } i \text{ is executed earlier than task } i1 \text{ on the same worker.} \\
0, & \text{otherwise} 
\end{cases}
3.2 Mathematical model

The proposed mathematical model is as follows:

\[
\begin{align*}
\min & \quad \sum_{j \in J} \sum_{k \in K} \sum_{i \in E} j \cdot ws_{jk} + \frac{1}{N_{\text{max}}} \sum_{i \in E} \sum_{j \in J} R_{ijk} \\
\text{s.t.} & \quad \sum_{j \in J} \sum_{k \in K} x_{ijk} = 1, \forall i \in Z \quad (1) \\
& \quad \sum_{j \in J} \sum_{k \in K} j \cdot x_{ijk} \leq \sum_{j \in J} \sum_{k \in K} j \cdot x_{ijk}, \forall i \in Z, i1 \in P(i) \quad (2) \\
& \quad \text{strt}_i + t_i \leq C_t \quad (3) \\
& \quad \text{strt}_i - \text{strt}_{i1} + M(1 - \sum_{k \in K} x_{i1jk}) \geq t_{i1}, \forall i \in Z, i1 \in P(i), j \in J \quad (4) \\
& \quad \text{strt}_{i1} - \text{strt}_i + M(1 - x_{ijk}) + M(1 - x_{ij1}) + M(1 - w_{i1l}) \geq t_i, \\
& \quad \forall i \in Z, i1 \in \{g \mid g \in l - (P1(i) \cup \{i\} \cup S1(i)) \text{ and } i < g\}, j \in J, k \in K \quad (5) \\
& \quad \text{strt}_i - \text{strt}_{i1} + M(1 - x_{i1jk}) + M(1 - x_{ijk}) + Mw_{i1l} \geq t_{i1}, \\
& \quad \forall i \in Z, i1 \in \{g \mid g \in l - (P1(i) \cup \{i\} \cup S1(i)) \text{ and } i < g\}, j \in J, k \in K \quad (6) \\
& \quad \sum_{i \in I(1)} x_{ijk} - ||M(\theta)||R_{ijk} \leq 0 \quad \forall j \in J, k \in K, l \in L \quad (7) \\
& \quad \sum_{i \in E} x_{ijk} \leq M \cdot ws_{jk}, \forall j \in J, k = 1, 2, \ldots, N_{\text{max}} - 1 \quad (8) \\
& \quad \text{strt}_i \geq 0, \forall i \in Z \quad (9) \\
& \quad x_{ijk} \in \{0, 1\}, \forall i \in Z, j \in J, k \in K \quad (10) \\
& \quad w_{i1l} \in \{0, 1\}, \forall i \in Z, i1 \in \{g \mid g \in l - (P1(i) \cup \{i\} \cup S1(i)) \text{ and } i < g\} \quad (11) \\
& \quad ws_{jk} \in \{0, 1\}, \forall j \in J, k \in K \quad (12) \\
& \quad R_{ijk} \in \{0, 1\}, \forall l \in L, j \in J, k \in K \quad (13)
\end{align*}
\]

Objective function Eq. (1) minimizes workers and stations as the primary objective and then try to minimize the number of resources assigned to workers in each workstation. The weighting factor ensures that the second term is a secondary objective (Fattahi, 2011). Constraint (2), ensures that each task is assigned to exactly one worker at one workstation. Constraint (3) is the precedence constraint and ensures that all precedence relations among tasks are satisfied. Constraint (4), cycle time constraint, ensures that each task should be finished before the end of the cycle time. Constraints (5), (6) and (7) control the sequencing constraints. For every pair of task i and i1, if task i1 is an immediate predecessor of task i, constraint (5) ensures that task i is started after finishing of task i1 in order to verify precedence constraints. If two tasks i and i1 have no precedence relation, they are assigned to the same worker, constraints (6) or (7) become active. If i is assigned earlier than i1 (wi1=1), then constraint (6) becomes active. Otherwise, constraint (7) becomes active. However, if two tasks assigned to different workstations, constraints 5, 6 and 7 are ignored by introducing a sufficiently larger number M. Constraint (8) ensures that if at least one task is done in workstation j with resource l by the worker k, then resource l is used in workstation j and R_{ijk} value gets 1. Constraint (9) is the worker constraint for straight simple assembly line balancing problem. It is modified for multi-manned assembly line balancing problems (Fattahi, 2011). Constraint (10) observes the sequence of workers’ index in a multi-manned workstation. Constraint (11) indicates that the start time of all tasks should be greater than or equal 0. Constraints (12, 13, 14, 15), show the binary and integer variables.
4. An illustrative example

Figure 2, shows an example of assembly line balancing problem, proposed by Mansoor (1964) in the literature, with cycle time of 45 and admitted maximum number of workers 2 in each workstation. This example is solved by existing (Fattahi et al. 2011) and proposed models. The precedence diagram contains 11 tasks for a single product and performance times of the tasks are presented in Table 1 with resources. The number inside the nodes shows the task ID.

![Precedence diagram of illustrative example](image)

Both models in this study have been coded in C# programming language and solved using GUROBI mathematical programming solver.

<table>
<thead>
<tr>
<th>Task Number</th>
<th>Task Time</th>
<th>Resource</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
<td>A</td>
</tr>
<tr>
<td>2</td>
<td>38</td>
<td>B</td>
</tr>
<tr>
<td>3</td>
<td>45</td>
<td>A</td>
</tr>
<tr>
<td>4</td>
<td>12</td>
<td>B</td>
</tr>
<tr>
<td>5</td>
<td>10</td>
<td>A</td>
</tr>
<tr>
<td>6</td>
<td>8</td>
<td>B</td>
</tr>
<tr>
<td>7</td>
<td>12</td>
<td>A</td>
</tr>
<tr>
<td>8</td>
<td>10</td>
<td>B</td>
</tr>
<tr>
<td>9</td>
<td>2</td>
<td>A</td>
</tr>
<tr>
<td>10</td>
<td>10</td>
<td>B</td>
</tr>
<tr>
<td>11</td>
<td>34</td>
<td>A</td>
</tr>
</tbody>
</table>
When Mansoor (1964) test problem is solved using traditional multi-manned assembly line balancing mathematical model presented by Fattahi et al. (2011) and proposed model, the results are shown in Table 2 and 3. Table 2 and Table 3 classified into 4 topics (Number of stations, number of workers, tasks in workstations and resources in workstations) and compared with 3 significant point. According to their importance, these are assigned worker, opened multi-manned workstation and current resources. As it seen in Table 2 and 3, number of workers and number of stations are same in both solutions. But, as it seen in Table 2 and 3, a total of 8 resources, with 4 units A and 4 units B, are being used in multi-manned assembly line balancing without resource constraints. In addition, the proposed mathematical model reduced the resources to 5 units (3 units A, 2 units B). Otherwise, optimal number of the workers as 5 and the optimal number of the workstations as 3 are found in both models. As a result, the solution of proposed model is the best in terms of number of resources used in stations. While reaching to best, optimum number of stations and workers are also achieved.

4. Conclusion

There are work pieces which need different worker skills or equipment in real-life assembly systems, which multi-manned workstations are used. Due to this reason, the problem studied in this paper differs from multi-manned assembly line problem in that, for any workstation, different types of resources are required in performing the tasks assigned to the workers in workstations. The goal of proposed approach is to establish balance of multi-manned assembly line with minimum number of workers, workstations and resources. Following the problem definition, a mathematical model were used to find optimal solutions and compared with existing model. In the future studies, the problem can be extended as some tasks are assembled by alternative resources. Due to the NP-hard nature of ALBP, the mathematical model is inefficient in solving large-scale problems. Consequently, further studies are also needed using heuristic methods to improve the quality of the solution algorithm. Heuristics which are well suited for assembly line balancing problems, are genetic algorithms, simulated annealing, ant colony optimization and tabu search. Simulated annealing is a powerful heuristics (Roshani et al., 2013) and can be used for future research.

References


**Biography**

Hamid Yılmaz is a Research Assistant in Industrial Engineering Department at Bayburt University, Bayburt, Turkey. He earned B.S. in Industrial Engineering from University of Gaziantep, Gaziantep, Masters in operations research from Atatürk University, he is still continuing to PhD in Industrial Engineering at University of Atatürk, Erzurum in Turkey. He has published journal and conference papers about assembly line balancing problems. His research interests include assembly lines, manufacturing systems, optimization, scheduling and mathematical modelling.
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